OPTIMAL VARIABLE-DENSITY K-SPACE SAMPLING IN MRI

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ABSTRACT

Magnetic resonance imaging (MRI) is often times limited by scan time. To reduce scan time, there have been various efforts to reduce the number of sampling points. In most cases, this is done by utilizing a priori knowledge of the signal of interest. In this paper, we propose an algorithm that will guide the determination of an optimal sampling pattern based on prior knowledge of the signal. Applications of this method include optimal variable-density k-space trajectory design or reduced phase encoding in 3DFT and spectroscopy. Preliminary results are shown as a proof of concept.

1. INTRODUCTION

In MRI, k-space sampling comes at a cost of time, and long scan times come with many disadvantages such as motion artifacts. Furthermore, in some imaging applications where the time constant of the physiological parameter of interest is shorter than the imaging time, MRI is not feasible. There have been many different approaches to reducing the number of sample points. Many of these approaches are based on the observation that the low frequency components of a signal are more important [1, 2]. In this work, we describe a general method for selecting a limited number of sample points, with the optimal interpolation kernel associated with the sampling pattern. The demonstration of the method was done on phantom and in-vivo data as a proof of concept. The algorithm can be applied to guide optimal variable-density trajectory design or reduced phase encoding in 3DFT and spectroscopy.

2. THEORY

Under the sole assumption that the signal is band-limited, the best thing to do is uniform sampling at the Nyquist sampling rate. However, to reduce the sampling rate below the Nyquist rate without introducing significant degradation, it is necessary to introduce prior knowledge of the signal to select sample points and reconstruct the signal optimally.

In the proposed algorithm, the prior knowledge of the signal is used to determine an orthogonal basis of the signal. The optimal sampling points are then determined by finding the best estimate of the coefficients of the orthogonal basis.

2.1. Finding the Orthogonal Basis

The orthogonal basis of the signal of interest can be found from the autocorrelation matrix of the signal. Since the autocorrelation is a statistical parameter, it has to be estimated from an ensemble of the signal. If we let X be a matrix with the member of the ensemble of the data as its columns,

$$X = [x_1 x_2 \dots x_L] \quad (1)$$

the autocorrelation matrix can be calculated as follows (assuming zero mean):

$$R_x = XX^T \quad (2)$$

Here, L is the number of representatives taken from the ensemble for the estimation of the autocorrelation matrix. L has to be significantly larger than the dimension of the signal N (L >> N). The eigenvalue decomposition of the autocorrelation matrix can then be used to find the orthogonal basis. If we write

$$R_x = VDV^T = \sum_{i=1}^{N} {}_{i} v_i v_i^T \quad (3)$$

then the eigenvectors v_i that corresponds to nonzero eigenvalues μ_i comprise an orthogonal basis [3]. In most signals of interest the number of orthogonal basis vectors for the signal (d) will be far less than the dimension of the signal (N). The inherent dimensionality of the signal is then d.

2.2. Finding the Basis Coefficients

The signal can be expressed as a linear combination of the basis vectors. Therefore, it can be completely described

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by the corresponding d coefficients of the basis vectors. However, the values of the signal as given, from which we draw the sample values, are from the original measurements in k-space and are not matched to the space spanned by the orthogonal basis determined in the first step. Finding the best set of samples that determine the coefficients of the orthogonal basis, that then specify the signal, becomes a difficult combinatorial problem. Fortunately, the problem can be relaxed into a convex problem that can be solved using a convex optimization routine [4]. The solution provides a good heuristic answer to our problem. The convex optimization problem was formulated as follows.

$$\text{Maximize } \det\left(\sum_{i=1}^{N} \lambda_i u_i u_i^T\right) \quad (4)$$

$$\text{Subject to } 0 \le \lambda_i \le 1$$

$$\sum_{i=1}^{N} \lambda_i = d$$

Where, u_i are vectors consisting of the ith components of the basis vectors. Therefore, they are d dimensional vectors. The inputs to this routine are the basis vectors, which will determine the vectors u_i, and the desired number of samples, d. The output is a vector λ which gives a heuristic as to how important each sample is in determining the coefficients of the orthogonal basis; each λ_i corresponds to a sample point. The reason why the values of λ_i are not simply 0 or 1 comes from the relaxation into a convex optimization problem. We then simply threshold this λ to determine whether to take the sample. Once the coefficients of the basis are found, the linear combination of the basis vectors provides the interpolation.

3. METHOD

The experiments were conducted using a GE 1.5 T whole-body scanner with a maximum gradient amplitude of 40 mT/m and maximum slew rate of 150 mT/m/ms.

Ideally, when characterizing the signal to find the optimal sampling point and the optimal interpolation kernel, an ensemble of the random process is required. For the demonstration of the concept, simple experiments were done on phantom and in-vivo data.

3.1. Phantom Experiment

Here, we try to demonstrate the method by using a simple one dimensional data that has some statistical correlation. To emulate this situation, we used a ball phantom data from a 3D projection reconstruction trajectory. The trajectory had 100 readout points with 31,744 projection angles. Since 31,744 >> 100, it provides enough data from the ensemble that can be used for the estimation of the autocorrelation matrix. Furthermore, the radial symmetry of the ball will provide apparent statistical correlation of the signal while, the nonzero phase of the data and other MR related imperfections provide the variations in the data. To make sure the signal we estimate is not included in the statistical estimation process, only half of the projection (which is still much bigger than the dimension of the signal, 15,872 >> 100) was used for the estimation. Then, the algorithm was used to reconstruct a signal from a projection angle that was not included in the statistical analysis.

3.2. In-Vivo Experiment

The same experiment was performed on an in-vivo knee data set. The different projections of the knee may not necessarily have significant statistical correlation. The purpose of this experiment was to demonstrate that even in signals without much apparent statistical correlation, the method can be used to reduce the sampling. Following the same procedure as in 3.1, all the projection angles were reconstructed and then the 3D image was reconstructed.

For both experiments, reconstruction from fully sampled data, as well as from non-optimal sub-sampled data was performed for comparison. The non-optimal sub-sampling points were chosen from the center of k-space with the same number of samples as the optimal case.

4. RESULT

4.1. Phantom Experiment

Figure 1 shows the λ parameter chosen by the optimization routine. The threshold for selecting the sampling point was chosen to be 0.5 for this demonstration. It results in 21 irregularly spaced samples. The reconstructions using three different methods are compared in Fig. 2. The solid line is the reconstruction from the full 100 samples. The bold-dashed line shows the reconstruction from 21 optimally selected sampling points. The dashed-dotted line shows the reconstruction from 21 uniform samples chosen from the center of k-space. The optimal sampling and reconstruction clearly shows improvement over uniform under-sampling. Finding the optimal sampling point took less than 10 sec in MATLAB with instant reconstruction.

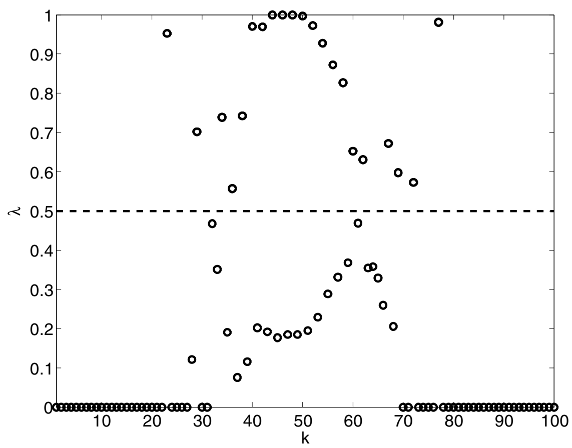

Figure 1 *λ for the phantom experiment. This value represents the importance of the corresponding sample point in k-space. When 0.5 is chosen as the threshold, 21 sample points are selected as indicated by the points above the threshold.*

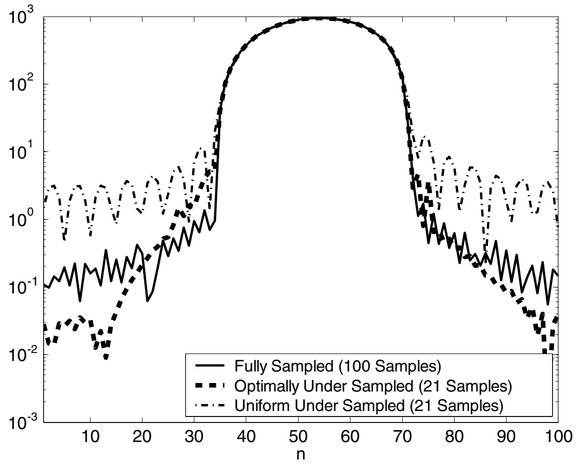

Figure 2 *Reconstruction of a ball phantom projection. The solid line shows the reconstruction from a full 100 samples. The bold- dashed line shows reconstruction from 21 optimally selected samples. The dashed-dotted line shows reconstruction from 21 samples at the center of k-space. The optimal sampling and reconstruction gives a curve close to the fully sampled case with one fifth of the sample points while the same number of samples at the center of k-space gives a reconstruction with much bigger error.*

4.2. In-Vivo Experiment

The reconstructions from the three methods are shown in Fig. 3. Compared to the full reconstruction from all 100 samples along the projection, the reconstruction with just 74 samples at the center of k-space appears more blurred. The reconstruction from the optimal 74 sampling points shows comparable sharpness as the reconstruction from the full 100 samples. It should be noted that reducing the number of sample points along the readout direction does not save scan time in 3DPR. The experiment is a demonstration for the feasibility of the method to reduce the number of samples in general. For selecting 74 samples out of a 100, the optimal sampling point selection algorithm took approximately 30 sec in MATLAB. Since the reconstruction is just a linear combination, it is done in an instant.

5. DISCUSSION

The advantage of this method is that it theoretically characterizes the optimal sampling points that need to be selected in order to obtain the best reconstruction. The practical difficulties may include characterizing the statistical parameters of the signal of interest. Since the statistical parameter needs to be set only once for a given protocol, it would be beneficial in cases where scan time is a highly limiting factor. This method can be utilized as guidance to design trajectories with optimal sampling. For example, it can be used to determine the optimal radial sampling locations of variable-density spiral trajectories. It can also be used for 3DFT and spectroscopic imaging to determine optimal phase encoding locations.

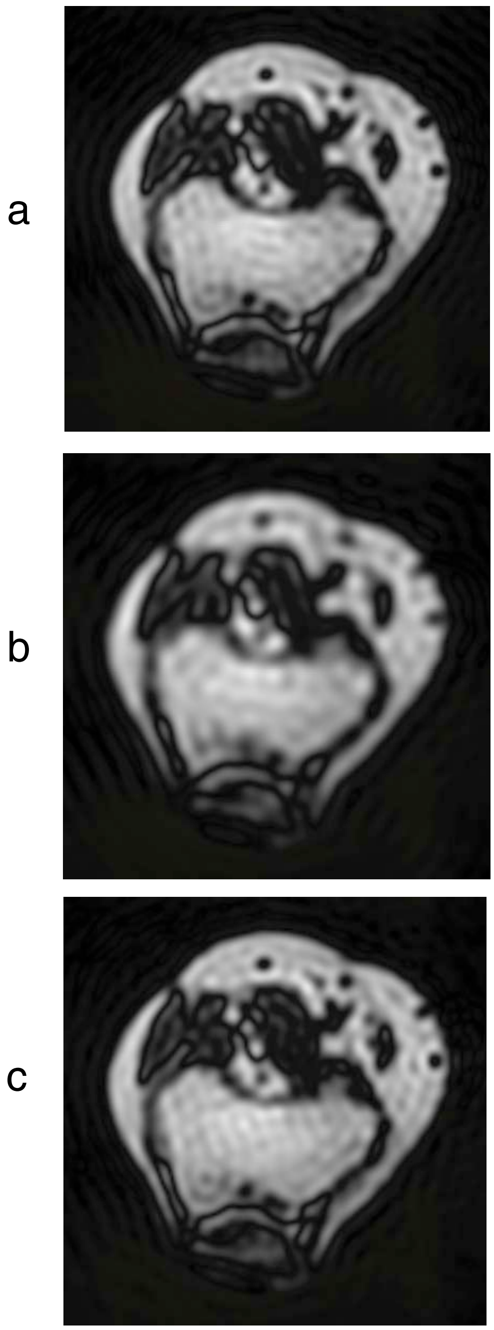

Figure 3 *Axial images from the 3D reconstruction of a knee. a) Reconstruction from full sampling. b) Reconstruction from 74% of the data at the central k-space region. The image is blurred from lacking higher spatial frequency components. c) Reconstruction from optimal sampling. It shows comparable sharpness to the fully sampled image.*

6. CONCLUSION

This method provides a way to characterize optimal sampling patterns given prior knowledge of the signal. It will be very beneficial in determining optimal sampling in both temporal and spatial dimensions for many different sampling situations in MRI.

7. REFERENCES

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