Capacity Growth of Multi-Element Arrays in Indoor and Outdoor Wireless Channels

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Abstract **– We demonstrate the capacity growth of multiple-element antenna arrays (MEAs) in a realistic propagation environment using WiSE, an experimental ray tracing tool. WiSE is used to construct the channel response for MEAs operating at 1.9 GHz for two situations: (a) (16-16)-MEAs located inside an office building and (b) (4, 4)-MEAs located in an outdoor fixed wireless loop. We define effective degrees of freedom (EDOFs) as parallel spatial modes of transmission for an MEA. We quantify the increase in both the number of EDOFs and capacity with transmit power, received SNR, and antenna spacing. More EDOFs are present when receiving MEAs are physically closer to the transmitting MEA, regardless of the scattering effect.**

I. INTRODUCTION

Recent information theory research has shown that multiple-element antenna arrays (MEAs) can achieve enormous capacity gains over single-antenna systems by exploiting the multipath in the rich-scattering wireless channel [1]-[4]. Theoretical analysis shows that using *n*-element MEAs at both the transmitter and the receiver leads to capacity that grows linearly (rather than logarithmically) with the number of antennas *n*, for fixed power and bandwidth [1]. This result is derived by assuming independent Rayleigh fadings between multiple antenna pairs. However, such statistical models do not capture correlations between antenna pairs that exist in practice, especially when a dominant line of sight (LOS) component is present. The question arises as to whether the linear capacity growth is retained in these cases.

In this paper, we explore the growth of MEA capacities in a more realistic propagation environment using Wireless System Engineering (WiSE) [5], an experimental ray tracing tool. We consider a single user case with *n* transmitting and *n* receiving antennas, denoted as an (*n*, *n*)-MEA structure. We use WiSE to characterize the channel response for MEAs placed in a) an office building, and b) an outdoor wireless loop in Rosslyn, Virginia. We show numerical results for (16,16)-MEAs operating at 1.9 GHz for comparisons to the laboratory prototype, BLAST (Bell Labs Layered Space-Time Communication Project) [6]. We address practical concerns in choosing our simulation parameters as an effort to anticipate possible implementation problems that might arise in the laboratory prototype, BLAST.

P. F. Dressen and G. J. Foschini found in [7] that spreading out the antennas well beyond a wavelength can achieve the linear capacity growth not only on Rayleigh channels with many scatterers, but also in environments with direct line of sight (LOS) paths only. We presented a statistical model of antenna correlation in [8] and derived the corresponding asymptotic capacity growth rate. In this paper, we further investigate the effect of antenna spacing on the MEA capacity and effective degrees of freedom (EDOFs) using the more realistic channel models generated by WiSE.

This paper is organized as the following: Section II reviews the channel model, MEA capacity and the relationship between capacity and EDOFs. In Section III, we list the basic assumptions for the capacity simulations using WiSE. The results are presented in Section III-B. We also include some preliminary results for a similar study of outdoor systems in Section III-C. This is followed by conclusions in Section IV.

II. CHANNEL MODEL AND MEA CAPACITIES

The following notations will be used throughout the paper: underline for vectors, † for transpose conjugate, det for determinant, and I_n for $n \times n$ identity matrix.

A. Channel Model

We consider a single user communication system with *n* transmitting and *n* receiving antennas. The communication bandwidth is assumed to be narrow enough that the Fourier transform of channel impulse response appears constant over the frequency band of interest. The path gain between *j*-th transmitter and *i*-th receiver is represented by H_{ij}, for $i = 1, 2, \ldots, n$. Assuming the channel is linear time invariant,

$$\underline{r} = H\underline{s} + \underline{v} \tag{1}$$

where $\underline{s}$ is an $n \times 1$ signal vector whose *j*-th component represents signal sent by *j*-th antenna; $\underline{r}$ and $\underline{v}$ are $n \times 1$ received signal and noise vectors, respectively.

We further assume that:

- The total power of the transmitted signal is constrained to $P_{\max}$, independent of *n*.
- The noise vector, $\underline{v}$, is additive white complex Gaussian, whose entries are statistically independent with identical power N_0.

B. MEA Capacities

Assume the channel is unknown to the transmitter-MEA, and the total transmit power P_{max} is equally allocated to all n antennas. The capacity of the (n, n)-MEA system has been derived in [3] as:

$$C(\rho_R) = \log_2 \det\left[I_n + \left(\frac{P_{max} \cdot \sigma^2}{N_0 \cdot n}\right) \cdot HH^\dagger\right] \text{ bits/Hz}, \quad (2)$$

where H is the normalized $n \times n$ channel matrix, whose entries, H_{ij}, are identically distributed with variance $\sigma^2 = E[|H_{ij} - E[H_{ij}]|^2]$. We assume a narrowband case where channel response H is constant over frequency band of interest. We define the average received signal-to-noise ratio (SNR) as $\rho_R = \sigma^2 \cdot P_{max}/N_0$.

By singular value decomposition, H can be written as $H = UDV^\dagger$, where $U \in C^{n \times n}$ and $V \in C^{n \times n}$ are unitary. D is diagonal and its entries are the non-negative square roots of the eigenvalues of $HH^\dagger$, λ_i for $i = 1, 2, \ldots, n$. Equation (2) becomes:

$$C(\rho_R) = \log_2 \det\left[I_n + \frac{\rho_R}{n} \cdot D^2\right] = \sum_{i=1}^{n} \log_2\left[1 + \frac{\rho_R}{n} \cdot \lambda_i\right] \quad (3)$$

C. *Effective Degrees of Freedom (EDOF)*

Equation (3) suggests that the (n-n)-MEA channel can be virtually decomposed into n parallel sub-channels, each of which contributes to the total capacity through

$$\log_2\left[1 + \frac{\rho_R}{n} \cdot \lambda_i\right]. \quad (4)$$

If $(\rho_R/n) \cdot \lambda_i \gg 1$, we say that this sub-channel provides an effective spatial mode of transmission, or *effective degree of freedom* (EDOF). Every EDOF provides one additional bps/Hz for every 3dB increase in ρ_R. Therefore, the practical definition of EDOF is the difference in capacity (bps/Hz) when ρ_R is doubled:

$$\text{EDOF} \equiv C(2\rho_R) - C(\rho_R). \quad (5)$$

The maximum possible number of EDOFs in this case is n.

III. Numerical Results From Ray Tracing Simulations

A. *Channel Modeling in WiSE*

We used WiSE to construct realistic realizations of the channel matrix H in our simulation study for both indoor and outdoor wireless environment. The power of rays impinging on the receiver is recorded when the carrier is launched from the base with power $10\log_{10} P_{max}$ dBm. The channel response is modeled as the vector sum of all the rays arriving at the receiving antenna locations. The predicted baseband channel impulse response is as follow:

TABLE I
Parameters Used In Wise Simulations

Parameters	Indoor Case	Outdoor Case
Prediction	Image	3-D Pincushion
Number of Bounces	6	0
Number of Diffractions	0	1
Antenna	Isotropic	Isotropic
Reception	Simulcast	Simulcast
Ray Summation Method	Vector Summation	Vector Summation
Ray Threshold (mW)	1.0×10^{-17}	1.0×10^{-12}
Diffraction	Reflection and diffraction included	Roof-top diffraction
Receiver Grid	Explicit Grid	Explicit Grid
Output Format	Impulse	Impulse

$$g_{ij}(t) = \sum_{k=0}^{M} \sqrt{P_k} \cdot e^{i\theta_k} \cdot \delta_k(t - \tau_k) \quad (6)$$

where P_k, θ_k and τ_k are the received power, phase angle and time delay of the k-th ray respectively. M is the total number of rays and $\delta_k(t)$ is a delta impulse function. With narrowband assumption, we compute the frequency response at infinitesimally small bandwidth centered at the carrier frequency,

$$h_{ij} = \sum_{k=0}^{M} \sqrt{P_k} \cdot e^{i\theta_k} \cdot e^{i2\pi f_0 \tau_k}. \quad (7)$$

H is computed using (7) and P_k, θ_k and τ_k obtained from WiSE simulation. All the n^2 entries, h_{ij}, are complex numbers in this case. Table I lists the choice of parameters used in WiSE to characterize propagation models for indoor and outdoor case.

B. *Simulation Study of Indoor MEA Systems*

We used WiSE to construct the propagation model for a (16,16)-MEA operating in the first floor of the Crawford Hill Office Building (Fig. 1). We consider 16 omni-directional antennas arranged in 4×4 square grids. We placed a (16,16) transmitter-MEA on the ceiling at the center of corridor (X in Fig. 1). The receiver-MEA was placed at 1000 test locations at

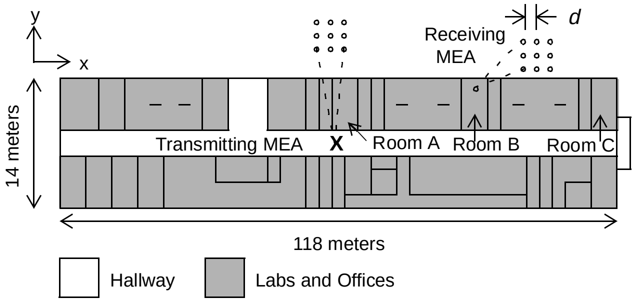

Fig. 1. **Indoor case:** Floor plan for the first floor of the Bell Laboratories office building at Crawford Hill, New Jersey. Receivers with antennas positioned in square grids are placed randomly at 1000 locations in room A, B and C.

desk-top height in each of the following three office rooms (A-closest to the transmitter, B-middle and C-furthest from the transmitter), and the corresponding H was recorded. The antennas in the MEAs were separated by d in multiples of λ (where λ is the carrier wavelength). For comparisons with the previous broadband studies at 5.2 GHz [9], we calibrated the transmit and noise power in dBm/10 MHz. For ease of notation, we write units of power in dBm instead of dBm/10MHz for the rest of our discussions. We considered N_0 = 100.8 dBm.

In this section, we present the simulation results from WiSE for indoor MEAs. The capacity $C(\rho_R)$, associated with each sample matrix H, is a random number. We are interested in the maximum bit rate a (16,16)-MEA system can support *consistently*, e.g. 95% of the time. We assume the communication is disrupted if $C(\rho_R)$ falls below this threshold, an event we refer to as a *channel outage*, and that its probability should be kept low (1-5%). For each of the three rooms, we determined the EDOFs as defined in Section II-C, and the MEA capacity at 5% *channel outage*, $C^{0.05}(\rho_R)$, where

$$\mathrm{Prob}(C(\rho_R) \le C^{0.05}) \;=\; 0.05\,.$$

Fig. 2 shows the complementary cumulative distribution function (CCDFs) of MEA capacity for $d = 0.5\ \lambda$, with P_{max} = 20 dBm (solid curves) and 23 dBm (dotted curves), respectively. Both ρ_R and $C(\rho_R)$ decrease when the receiver-MEAs are further away from the transmitter. For P_{max} = 20 dBm, $C^{0.05}$ = 167.7 bps/Hz in Room-A, 66.9 bps/Hz in Room-B and 24.3 bps/Hz in Room-C. Capacities like 167.7 bps/Hz, or 10.5 bps/Hz/EDOF may be too high for immediate practical considerations due to limitations of current antenna-array technology. However, these high capacities can be leveraged with advances in signal processing and antenna technologies. The additional bps/Hz obtained with 3 dB increase in P_{max} is equivalent to the number of EDOFs (as shown by the right shift of CCDF curves in Figure 2 when P_{max} is increased from 20 to 23 dBm). Although we have the full 16 EDOFs in Room-A, we only obtain 13 for Room-B, and 7 for Room-C (Table II).

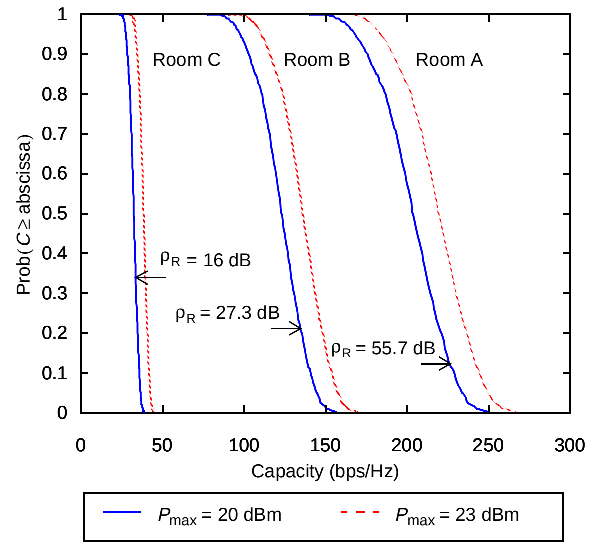

Fig. 2. **Indoor case:** CCDFs for MEA capacities in three different rooms with the transmitting MEA placed in the midway of corridor (X in Fig. 1). The average received SNR ρ_R is computed for each room and indicated above.

TABLE II

RESULTS FOR THREE DIFFERENT ROOMS WITH $d = 0.5\ \lambda$ AND P_{MAX} = 20 dBm

Location of Receivers	Room A	Room B	Room C
EDOF at 5% outage	16	13	6
Capacity at 5% outage (bps/Hz)	167.7	66.9	24.3
Average SNR ρ_R (dB)	55.7	27.3	16

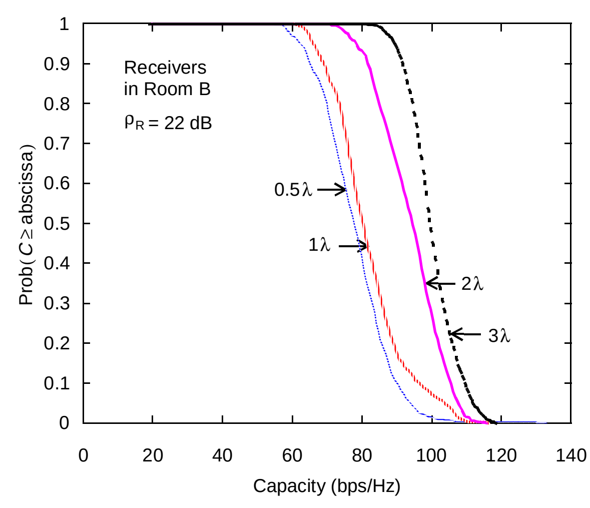

Fig. 3. **Indoor case:** CCDFs for MEA capacities in Room B when the antenna spacing is varied. The dashed curve shows the reference case for $d = 0.5\ \lambda$. The solid curves show the capacity distributions for MEAs with d = 1, 2 and 3 λs.

TABLE III

PERFORMANCE OF MEAS PLACED IN ROOM B, WITH ρ_R=22 dBm. THE TRANSMITTING MEA IS SPACED 0.5 λ APART.

Receiver Antenna Spacing (λ)	0.1	1	2	3
Average Capacity (bps/Hz)	78.2	81.8	93.9	100.4
Capacity at 5% outage (bps/Hz)	62.4	65.8	78.5	89.1
EDOF at 5% outage	7.4	7.7	8.8	9.7

From the implementation point of view, the average received SNR in Room-A (55.7 dB) and Room-B (27.3 dB) are so high that the capacity they provide cannot be fully realized with current technology. At the time of this writing, we consider 18-22 dB per antenna as reasonable. To maintain feasibility of implementation, we investigate various MEA arrangements that affect EDOFs and overall capacity while operating at reasonable SNR. Keeping ρ_R = 22 dBm, Fig. 3 shows the capacity distributions for Room-B when the antenna spacing is increased to 1, 2 and 3 λs (solid curves). For reference, the CCDF with d = 0.5 λ is plotted on the same figure (dashed curve). Both capacity and EDOFs increase with d (as shown in Table III). We get two additional EDOFs when d is increased from 1 to 3 λ. This implies that strong correlation does exist between the antenna pairs, and larger spacing helps reduce the loss in capacity due to destructive interference. Our results are consistent with the theoretical analysis of MEA capacities for correlated channels in [7] and [8]. Since we assume co-located arrays, we limit d to 3 λs to keep the array size small for immediate applications in fixed wireless and indoor LANs.

C. *Simulation Study of An Outdoor Fixed Wireless Loop*

There is increasing interest in fixed wireless loops for the outdoor environment, and we have started an initial investigation of achievable MEA capacities in this case. Fig. 4 shows the topology of Rosslyn City, Virginia that we modeled in WiSE. Typical coverage area is 0.1-1 km and transmitted power is 0.1-1 W. We simulated a (4, 4)-MEA transmitter placed on top of building A (the middle building marked with one circle that indicates the transmitter). The receiving MEA was placed at 128 random locations inside building B, (top left corner, with circles marked to indicate 3 out of 128 sample receiver locations). The transmitting antennas are separated by 3 λ, and the receiving antennas are separated by 1 λ.

One major problem we encountered in our WiSE simulation of outdoor case is the enormous computational time. The third column in Table I lists the choice of WiSE parameters used to construct the first-order approximation of the outdoor propagation model. The capacity distributions are plotted in Fig. 5. With P_{max}= 17 dBm, the capacity at 5% outage is 15.5 bps/Hz. The achievable capacity with (4,4)-MEAs is

Fig. 4. **Outdoor case:** The top and elevation view of Rosslyn, VA where we use a (4, 4)-MEA at both the transmitter and the receiver to obtain initial capacity estimates for outdoor wireless loops operating at 1.9 GHz.

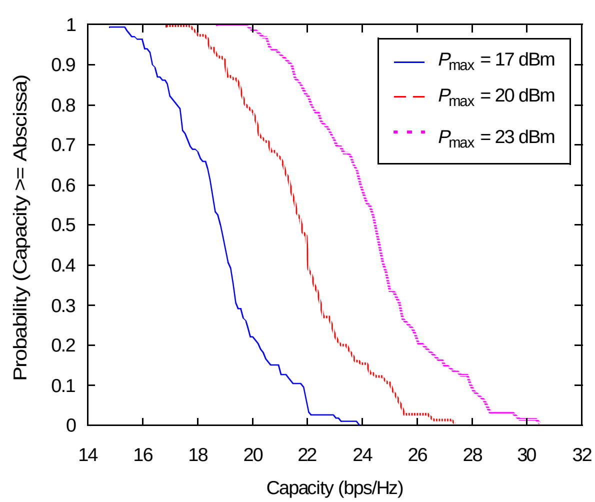

Fig. 5. **Outdoor case:** The capacity distributions for the outdoor (4, 4)-MEA systems located in the neighborhood of Rosslyn, Virginia. The receiving MEAs are placed at 128 random locations in Building B (Fig. 4). The transmitting MEA is placed on top of Building A. The transmitting antennas are separated by 3 λ and the receiving antennas are separated by 1 λ.

higher than single-antenna system, which achieves only 5.6 bps/Hz. When we increase P_{max} by 3 dB, $C^{0.05}$ increases to 18.3 bps/Hz, which implies that there are 2.8 EDOFs (as defined in (5)). When we double P_{max} again to 23 dBm, we observe that $C^{0.05}$ increases to 21.3 bps/Hz. This implies that (4, 4)-MEAs operating at 20 dBm can achieve 3 out of the 4 EDOFs. Further studies is needed to evaluate the capacity of (n, n)-MEAs in outdoor environment when *n* is large.

IV. Conclusions

We evaluated the capacity of multiple antenna arrays (MEAs) operating at 1.9 GHz and located in (a) an office building, and (b) an outdoor fixed wireless loop. We used the WiSE ray tracing tool to construct the channel response in both cases, based on previously measured data. Results show that higher capacity, received SNR and more EDOFs are observed when MEAs are placed closer to the transmitter. For example, (16, 16)-MEAs placed in Room A in Fig. 1 could achieve capacity as high as 167.7 bps/Hz (16 EDOFs) at 5% channel outage, which is almost 7 times more than the capacity in Room C, 24.3 bps/Hz (6 EDOFs). We also investigated various MEA arrangements that can increase capacity while operating at reasonable SNRs. From our observations, increasing antenna spacings at both the transmitting and receiving arrays to 3 λ helps to increase EDOFs and achieve higher capacity. Similar capacity growth has been demonstrated for MEAs located in outdoor fixed wireless loop in Rosslyn, Virginia.

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