Markov Chain Model in Maximum-Likelihood Sequence Detection for Free-Space Optical Communication Through Atmospheric Turbulence Channels

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***Abstract*—In free-space optical communication links using intensity modulation and direct detection (IM/DD), atmospheric turbulence-induced intensity fluctuations can significantly impair link performance. Communication techniques can be applied to mitigate turbulence-induced intensity fluctuations (i.e., signal fading) in the regime in which the receiver aperture D_0 is smaller than the fading correlation length d_0 and the observation interval T_0 is smaller than the fading correlation time τ_0. If the receiver has knowledge of the joint temporal statistics of the fading, maximum-likelihood sequence detection (MLSD) can be employed, but at the cost of high computational complexity. In this paper, we introduce a single-step Markov chain (SMC) model for the fading correlation and use it to derive two low-complexity, suboptimal MLSD algorithms based on per-survivor processing (PSP). Simulations are presented to verify the SMC model and the performance improvement achieved using these suboptimal PSP algorithms.**

***Index Terms*—Atmospheric turbulence, free-space optical communication, maximum-likelihood sequence detection (MLSD), per-survivor processing (PSP), single-step Markov chain.**

I. Introduction

FREE-SPACE optical communication has attracted considerable attention recently for a variety of applications [1]–[4]. Highly directed, coherent laser beams can propagate through the atmosphere with only moderate spreading and attenuation. Free-space links using directed beams and imaging receivers can achieve multigigabit-per-second transmission over kilometer ranges [1] or retrieve data simultaneously at kilobit-per-second bit rates from a collection of distributed, autonomous sensor nodes [2]–[4] using space-division multiplexing (SDM). Because of the complexity associated with phase or frequency modulation, current free-space optical communication systems typically use intensity modulation with direct detection (IM/DD).

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Atmospheric turbulence can degrade the performance of free-space optical links, particularly over ranges of the order of 1 km or longer. Inhomogeneities in the temperature and pressure of the atmosphere lead to variations of the refractive index along the transmission path. These index inhomogeneities can cause fluctuations in both the amplitude and the phase of the received signal. These effects can lead to an increase in the link error probability, limiting the performance of the communication systems.

Atmospheric turbulence has been studied extensively, and various theoretical models have been proposed to describe turbulence-induced image degradation and intensity fluctuations (i.e., signal fading) [5]–[8]. Two useful parameters describing turbulence-induced fading are d_0, the correlation length of intensity fluctuations, and τ_0, the correlation time of intensity fluctuations. When the receiver aperture D_0 can be made larger than the correlation length d_0, then turbulence-induced fading can be reduced substantially by aperture averaging [9].

Because it is not always possible to satisfy the condition $D_0 > d_0$, in a previous paper [10], we proposed alternative techniques for mitigating fading in the regime where $D_0 < d_0$. At the bit rates of interest in most free-space optical systems, the receiver observation interval T_0 during each bit interval is smaller than the turbulence correlation time τ_0. Throughout this paper, we will assume that $D_0 < d_0$ and $T_0 < \tau_0$.

In this paper, based on the statistical properties of turbulence-induced signal intensity fading, we propose an optimal maximum-likelihood sequence detection (MLSD). This MLSD requires complicated multidimensional integration, and its computational complexity is exponential in the length of the transmitted bit sequence. To simplify MLSD, we propose a single-step Markov chain (SMC) model for the fading temporal correlation, and we use the SMC model to derive an approximate higher order distribution of bit errors, as well as two reduced-complexity MLSD algorithms based on suboptimal per-survivor processing (PSP). We use simulations to investigate the accuracy of the SMC model and the effectiveness of the suboptimal MLSD techniques.

The remainder of this paper is organized as follows. In Section II, we first review the correlation of signal fading and the probability distribution of the light intensity received on turbulence channels. The MLSD for ON-OFF keying (OOK) in the turbulence channel is then introduced. In Section III, we propose the SMC model for fading temporal correlation. To verify its accuracy, simulation of the higher order burst error distribution with symbol-by-symbol detection [10] is presented. In Section IV, we derive two sets of suboptimal MLSD schemes that make use of PSP and the SMC model. These techniques only involve two-dimensional (2-D) integration and their computation complexity is of the order of $(n^2)/2$, where n is the length of the transmitted bit sequence. We present simulations showing that these two techniques provide a performance improvement over symbol-by-symbol detection. In Section V, we present our conclusions.

II. Probability Distribution for Turbulence-Induced Fading and MLSD in OOK Systems

In this section, we first review the spatial and temporal coherence of optical signals through atmosphere turbulence and the probability distribution for turbulence-induced fading. Then we introduce the model for OOK free-space optical communication systems and the MLSD for such systems.

A. Spatial and Temporal Coherence of Optical Signals Through Turbulence

To describe spatial coherence of optical waves, the so-called mutual coherence function (MCF) is widely used [6]

$$\Gamma(P_1,t_1;P_2,t_2) = E\left[u(P_1,t_1)\cdot u^*(P_2,t_2)\right] \tag{1}$$

where $u(P,t)$ is the complex optical field. Setting $t_1 = t_2$ in (1), we obtain the spatial MCF $\Gamma(P_1,P_2)$. The Rytov method is frequently used to expand the optical field $u(\overline{r})$, where $\overline{r}$ denotes the space vector of P as

$$u(\overline{r}) = A(\overline{r})\cdot \exp\left[i\phi(\overline{r})\right] = u_0(\overline{r})\cdot \exp(\Phi_1), \tag{2}$$

where $u_0(\overline{r})$ is the field amplitude without air turbulence

$$u_0(\overline{r}) = A_0(\overline{r})\cdot \exp\left[i\phi_0(\overline{r})\right]. \tag{3}$$

The exponent of the perturbation factor is

$$\Phi_1 = \log\left[\frac{A(\overline{r})}{A_0(\overline{r})}\right] + i\left[\phi(\overline{r}) - \phi_0(\overline{r})\right] = X + iS \tag{4}$$

where X is the log-amplitude fluctuation and S is the phase fluctuation. We assume X and S to be homogeneous, isotropic, and independent Gaussian random variables. This assumption is valid for long propagation distances through turbulence.

In order to characterize turbulence-induced fluctuations of the log-amplitude X, we use the log-amplitude variance function

$$B_X(P_1;P_2) = E\left[X(P_1)X(P_2)\right] - E\left[X(P_1)\right]\cdot E\left[X(P_2)\right]. \tag{5}$$

Since the random disturbance is Gaussian-distributed under the assumption of weak turbulence, we can use the Rytov method to derive the normalized log-amplitude variance function for two positions in a receiving plane perpendicular to the direction of propagation [9], [10]

$$b_X(d_{12}) = \frac{B_X(P_1,P_2)}{B_X(P_1,P_1)} \tag{6}$$

where d_{12} is the distance between P_1 and P_2. We define the correlation length of intensity fluctuations d_0 such that $b_X(d_0) = e^{-2}$. When the propagation path length L satisfies the condition $1_0 < \sqrt{\lambda L} < L_0$, where λ is the wavelength and l_0 and L_0 are inner and outer length scales, respectively, d_0 can be approximated by [9], [10]

$$d_0 \approx \sqrt{\lambda L}. \tag{7}$$

In most free-space optical communication systems with visible or infrared lasers and with propagation distance of a few hundred meters to a few kilometers, (7) is valid.

Atmosphere turbulence also varies with time and leads to intensity fluctuations that are temporally correlated. Modeling the movement of atmospheric eddies is extremely difficult, and a simplified "frozen air" model is normally employed, which assumes that a collection of eddies will remain frozen in relation to one another, while the entire collection is translated along some direction by the wind. Taylor's frozen-in hypothesis can be expressed as [8]

$$n(\overline{r},t) = n(\overline{r} - \vec{v}t, 0) \tag{8}$$

where n is the refractive index of the atmosphere. $\vec{v}$ is the velocity of the wind, which has an average $\vec{u}$ and a fluctuation $\vec{v}_f$. If $\vec{v}_f$ is negligible and $\vec{u}$ is transverse to the direction of light propagation, then temporal correlation becomes analogous to spatial correlation; in particular, the correlation time is $\tau_0 = (d_0)/u$. Assuming a narrow beam propagating over a long distance, the refractive index fluctuations along the direction of propagation will be well averaged and will be weaker than those along the direction transverse to propagation. Therefore, we need only consider the component of the wind velocity vector perpendicular to the propagation direction $u_\perp$. The turbulence correlation time is therefore

$$\tau_0 = \frac{d_0}{u_\perp}. \tag{9}$$

B. Probability Distributions of Turbulence-Induced Intensity Fading

As discussed previously, when the propagation distance is long, log-amplitude fluctuations can become significant. In this section, we will derive the statistical properties of the log-amplitude fluctuations, which we refer to as "intensity fading" or simply "fading." The marginal distribution of fading is derived in Section I, while the joint spatial and temporal distribution of fading are derived in Section II.

1) Marginal Distribution of Fading: In this section, we derive the marginal distribution of fading at a single point in space at a single instant in time.

For propagation distances less than a few kilometers, variations of the log-amplitude are typically much smaller than variations of the phase. Over longer propagation distances, where turbulence becomes more severe, the variation of the log-amplitude can become comparable to that of the phase. Based on the atmosphere turbulence model adopted here and assuming weak turbulence, we can obtain the approximate analytic expression for the variance of the log-amplitude fluctuation X of plane and spherical waves [8]

$$\sigma_X^2\big|_{\text{plane}} = 0.56\left(\frac{2\pi}{\lambda}\right)^{\frac{7}{6}} \int_0^L C_n^2(x)(L-x)^{\frac{5}{6}}dx \tag{10}$$

$$\sigma_X^2\big|_{\text{spherical}} = 0.56\left(\frac{2\pi}{\lambda}\right)^{\frac{7}{6}} \int_0^L C_n^2(x)\left(\frac{x}{L}\right)^{\frac{5}{6}}(L-x)^{\frac{5}{6}}dx \tag{11}$$

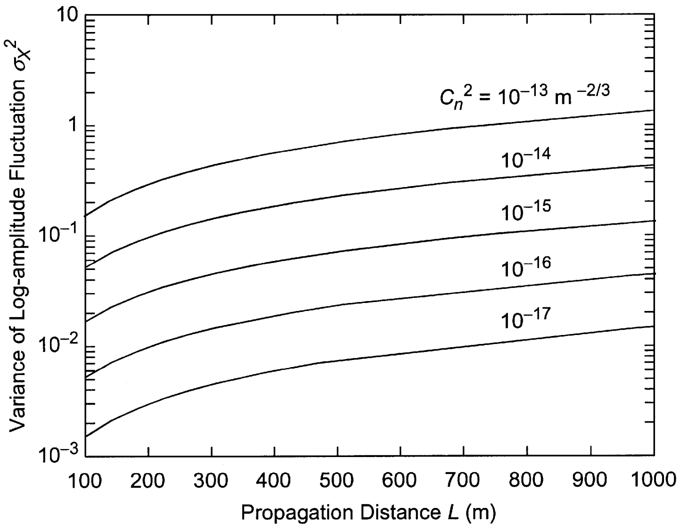

Fig. 1. Variance of the log-amplitude fluctuation versus propagation distance for a plane wave.

where C_n is the wavenumber spectrum structure parameter, which is altitude z-dependent. Hufnagel and Stanley gave a simple model for C_n [7] as

$$C_n^2(z) = K_0 z^{-\frac{1}{3}} \exp\left(\frac{-z}{z_0}\right) \tag{12}$$

where K_0 is parameter describing the strength of the turbulence and z_0 is effective height of the turbulent atmosphere. For atmospheric channels near the ground ($z < 18.5$ m)), C_n^2 can vary from $10^{-13}\text{m}^{-2/3}$ for strong turbulence to $10^{-17}\text{m}^{-2/3}$ for weak turbulence.

Fig. 1 shows the variance of the log-amplitude fluctuation σ_X^2 for a plane wave, computed using (10), as a function of the propagation distance L. In Fig. 1, we again assume a wavelength of 529 nm and assume $C_n^2(z)$ to be constant. Fig. 1 shows that, for propagation distances of a kilometer, σ_X^2 varies from 10^{-2} to 1 for different values of C_n^2.

Consider the propagation of light through a large number of elements of the atmosphere, each causing an independent, identically distributed phase delay and scattering. By the Central Limit Theorem, the marginal distribution of the log-amplitude is Gaussian

$$f_X(X) = \frac{1}{(2\pi\sigma_X^2)^{\frac{1}{2}}} \exp\left\{-\frac{(X - E[X])^2}{2\sigma_X^2}\right\}. \tag{13}$$

The light intensity I is related to the log-amplitude X by

$$I = I_0 \exp(2X - 2E[X]) \tag{14}$$

where $E[X]$ is the ensemble average of log-amplitude X.

From (13) and (14), the average light intensity is

$$E[I] = E\left[I_0 \exp(2X - 2E[X])\right] = I_0 \exp\left(2\sigma_X^2\right). \tag{15}$$

Hence, the marginal distribution of light intensity fading induced by turbulence is log-normal and is given by

$$f_I(I) = \frac{1}{2I} \frac{1}{(2\pi\sigma_X^2)^{\frac{1}{2}}} \exp\left\{-\frac{[\ln(I) - \ln(I_0)]^2}{8\sigma_X^2}\right\}. \tag{16}$$

2) Joint Temporal Distribution for Turbulence-Induced Fading: In a free-space optical communication system using OOK, we assume that an n-bit sequence $\vec{s} = [s_1\ s_2 \ldots s_n]$ is transmitted. We define the index subset of ON-state symbols $S_{\text{On}} = \{n_i \in \{1, 2, \ldots, n\}, s_{n_i} = 1\}_{i=1}^m$. We also have the index subset of OFF-state symbols $S_{\text{Off}} = \{1_j \in \{1, 2, \ldots, n\}, s_{l_j} = 0\}_{j=1}^{n-m}$. Ignoring intersymbol interference (ISI), the receiver would only receive signal light when the ON-state is transmitted. The joint distribution of the signal intensity of ON-state symbols is [10]

$$f(I_{n_1}, I_{n_2}, \ldots, I_{n_m}) = \frac{1}{2^m \prod_{i=1}^{m} I_{n_i}} \frac{1}{(2\pi)^{\frac{m}{2}} \left|C_X^{\text{On}}\right|^{\frac{1}{2}}} \times \exp\left\{-\frac{1}{8}\left[\ln\left(\frac{I_{n_i}}{I_0}\right) \ldots \ln\left(\frac{I_{n_m}}{I_0}\right)\right] \left(C_X^{\text{On}}\right)^{-1} \begin{bmatrix} \ln\left(\frac{I_{n_i}}{I_0}\right) \\ \cdots \\ \ln\left(\frac{I_{n_m}}{I_0}\right) \end{bmatrix}\right\} \tag{17}$$

where the ith ON-state symbol intensity [6], [7]

$$I_{n_i} = I_0 \exp(2X_{n_i} - 2E[X]) = I_0 \exp(2x_{n_i}). \tag{18}$$

Here, $x_{n_i} = X_{n_i} - E[X]$ can be modeled as a Gaussian random variable with zero mean and variance σ_X^2. For a string of bits, the variance matrix of ON-state bits is given in (19), shown at the bottom of the page, where T is the bit interval. The correlation time is given in (9).

One can also show that the joint distribution of $\vec{x} = [x_{n_1}, x_{n_2}, \ldots, x_{n_n}]$ [10] is

$$f\left(\vec{x}\right) = \frac{1}{(2\pi)^{\frac{m}{2}} \left|C_X^{\text{On}}\right|^{\frac{I}{2}}} \exp\left[-\frac{1}{2} x \cdot \left(C_X^{\text{On}}\right)^{-1} \cdot x^T\right]. \tag{20}$$

C. OOK System Model

In this paper, we consider IM/DD links using OOK. In most practical systems, the receiver signal-to-noise ratio (SNR) is limited by shot noise caused by ambient light which is much stronger than the desired signal and/or by thermal noise in the electronics following the photodetector. In this case, the noise can usually be modeled to high accuracy as additive, white Gaussian noise that is statistically independent of the desired signal. Assume that the receiver integrates the received photocurrent for an interval $T_0 \leq T$ during each bit interval and

$$C_X^{\text{On}} = \begin{bmatrix} \sigma_X^2 & \sigma_X^2 b_X\left[\frac{|n_1 - n_2|^T}{\tau_0} d_0\right] & \cdots & \sigma_X^2 b_X\left[\frac{|n_1 - n_m|^T}{\tau_0} d_0\right] \\ \sigma_X^2 b_X\left[\frac{|n_1 - n_2|^T}{\tau_0} d_0\right] & \sigma_X^2 & \cdots & \sigma_X^2 b_X\left[\frac{|n_1 - n_m|^T}{\tau_0} d_0\right] \\ \cdots & \cdots & \cdots & \cdots \\ \sigma_X^2 b_X\left[\frac{|n_m - n_1|^T}{\tau_0} d_0\right] & \sigma_X^2 b_X\left[\frac{|n_m - n_2|^T}{\tau_0} d_0\right] & \cdots & \sigma_X^2 \end{bmatrix}_{n \times n} \tag{19}$$

that $T_0 \ll \tau_0$. Therefore, the light intensity can be viewed as constant during exposure interval. At the end of the integration interval, the resulting electrical signal can be expressed as

$$r_e = \eta(I_s + I_b) + n_w \tag{21}$$

where I_s is the received signal light intensity and I_b is the ambient light intensity. The optical-to-electrical conversion efficiency is given by

$$\eta = \gamma T_0 \cdot \frac{e\lambda}{hc} \tag{22}$$

where γ is the quantum efficiency of the photodetector, e is the electron charge, λ is the signal wavelength, h is Plank's constant, and c is the speed of light. The additive noise n_w is white and Gaussian and has zero mean and covariance $N/2$, independent of whether the received bit is off or on. In most applications, the ambient light intensity remains constant and can be easily subtracted from the signal light. Therefore, in this paper, we simply define the received signal as

$$r = \eta I_s + n_w. \tag{23}$$

D. MLSD of OOK Systems

The MLSD exploits the temporal correlation of turbulence-induced fading and is thus expected to outperform the symbol-by-symbol ML detector. For a sequence of n transmitted bits, the MLSD computes the likelihood ratio of each of the 2^n possible bit sequences $\vec{s} = [s_1\ s_2 \dots s_n]$ and the received signal sequence $\vec{r} = [r_1\ r_2 \dots r_n]$ and chooses [11]

$$\begin{aligned}\bar{\vec{s}} = \arg\max_{\vec{s}} P(\vec{T}|\vec{s}) = \arg\max_{\vec{s}} \\ \times \int_{\vec{x}} f\left(\vec{x}\right) \exp\left[-\sum_{i=1}^{n} \frac{\left(r_i - \eta s_i I_0 e^{2x_i}\right)^2}{N}\right] d\vec{x}.\end{aligned} \tag{24}$$

Here, each s_i can take the value OFF or ON. The complexity of MLSD is proportional to $n \cdot 2^n$, because it requires computing an n-dimensional integral for each of 2^n bit sequences.

III. SMC Model for Fading Correlation

In this section, we propose an SMC model for fading correlation. While we consider only temporal correlation, the model can be used to treat spatial correlation, provided that the receivers are located along a line perpendicular to the direction of propagation. We then validate the SMC model by computing the burst-error probability for symbol-by-symbol ML detection.

A. SMC Model

Here, we consider the SMC model describing the correlation of fading at a sequence of equally spaced times. It is straightforward to extend the treatment to spatial correlation provided that the receivers are equally spaced along a line perpendicular to the direction of propagation. Let x_n denote the log-amplitude at time n, and let x_1^{n-1} denote $[x_1, \dots, x_{n-1}]$. Assuming the turbulence-induced fading is an SMC, we have

$$P\left(x_n | x_1^{n-1}\right) = P(x_n | x_{n-1}). \tag{25}$$

Define $\rho_T = b_x(d_0 T/\tau_0)$. If x_1^n follows a joint Gaussian distribution, the conditional distribution of x_n given x_1 is

$$\begin{aligned}f(x_n|x_1) = &\frac{1}{\left[2\pi\left(1-\rho_T^{2n-2}\right)\sigma_X^2\right]^{\frac{1}{2}}} \\ &\times \exp\left[-\frac{\left(x_n - \rho_T^{n-1}x_1\right)^2}{2\left(1-\rho_T^{2n-2}\right)\sigma_X^2}\right].\end{aligned} \tag{26}$$

Comparing the variance term in (19) with that in (26) and from the simulation results presented later in this paper, we see that the correlation in the SMC model is stronger than in the exact correlation model.

Since the SMC model only takes into consideration the probabilistic distribution of the most adjacent ON-bits which contain the turbulence information, its applicability to model general spatial correlation is restricted to the special case when the multiple receivers are aligned along a line perpendicular to the direction of propagation. However, since the number of receivers is typically small, the computational complexity of ML detection with diversity reception will typically be reasonable, as we have shown in [10] and [12].

B. Burst-Error Distribution for Symbol-by-Symbol Detection

To test the validity of the SMC model, we compute the burst error probability for symbol-by-symbol detection of OOK, which also reveals the higher order statistics of the bit-error distribution. Assume that a sequence of ON bits are transmitted. The probability of having m consecutive erasures is

$$P_{\text{On}}^m = \text{Prob}\left\{r_0 > r_{\text{th}}; r_{m+1} > r_{rmth}; r_i < r_{\text{th}}, \forall i, 1 \le i \le m\right\} \tag{27}$$

where r_{th} denotes the receiver decision threshold. Using the chain rule, (27) can be written as

$$\begin{aligned}P_{\text{On}}^m =& P_{\text{On}}(r_0 > r_{\text{th}}) P_{\text{On}}(r_1 < r_{\text{th}} | r_0 > r_{\text{th}}) \dots \\ & P_{\text{On}}(r_{m+1} > r_{\text{th}} | r_0 > r_{th}; r_i < r_{\text{th}}, \forall i, 1 \le i \le m),\end{aligned} \tag{28}$$

which is upper-bounded by

$$\begin{aligned}P_{\text{On}}^m \le & P_{\text{On}}(r_0 > r_{\text{th}}, r_1 < r_{\text{th}}) P_{\text{On}}(r_2 < r_{\text{th}} | r_1 < r_{th}) \dots \\ & P_{\text{On}}(r_{m+1} > r_{\text{th}} | r_m < r_{\text{th}}) \\ =& P_{\text{On}}(r_0 > r_{\text{th}}, r_1 < r_{\text{th}}) \left[P_{\text{On}}(r_2 < r_{\text{th}} | r_1 < r_{\text{th}})\right]^{m-1} \\ & P_{\text{On}}(r_{m+1} > r_{\text{th}} | r_m < r_{\text{th}}).\end{aligned} \tag{29}$$

To simplify the calculation, let us first ignore additive white Gaussian noise (AWGN) and focus on errors caused by turbulence-induced fading. In the absence of AWGN, we have

$$\begin{aligned}P_{\text{On}}(r_n < r_{\text{th}} | r_1 < r_{\text{th}}) &= \frac{P_{\text{On}}(r_1 < r_{\text{th}}, r_n < r_{\text{th}})}{P_{\text{On}}(r_1 < r_{\text{th}})} \\ &= \frac{\int_{-\infty}^{x_{\text{th}}} f(x_1) \int_{-\infty}^{x_{\text{th}}} f(x_n|x_1) d_{x_n} dx_1}{\int_{-\infty}^{x_{\text{th}}} f(x_1) dx_1}\end{aligned} \tag{30}$$

where x_{th} can be calculated by r_{th} as

$$r_{\text{th}} = \eta I_0 \exp(2x_{\text{th}}). \tag{31}$$

Note that, to compute (29) and (30), we need only perform two-dimensional (2-D) integration, independent of the number of bits in the sequence.

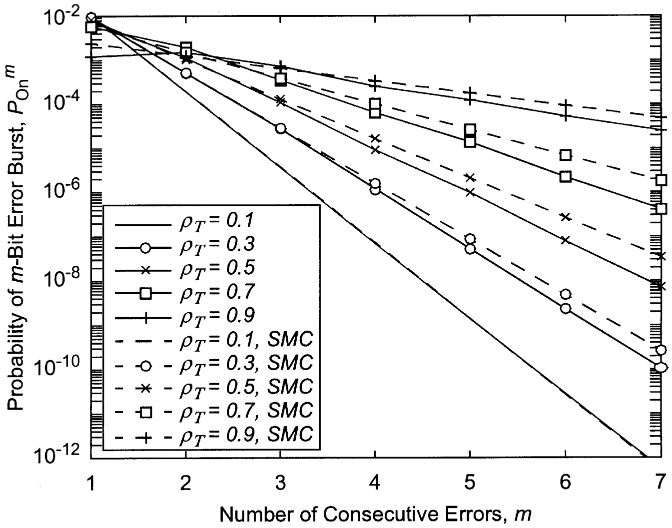

Fig. 2. Distribution of consecutive bit errors and its upper bound derived using the SMC model. For simplicity, AWGN is ignored here.

In Fig. 2, we present the distribution of consecutive bit errors for several different correlation parameters, comparing the SMC model to the exact correlation model. Fig. 2 has been computed assuming $E[X] = 0$, $\sigma_X = 1$ and a decision threshold $r_{\text{th}} = 1/10 \cdot \exp(2E[X]) = 0.1$. As we would expect, the probability of a burst of length $m > 1$ increases as we increase the correlation parameter ρ_T. We see that the SMC model yields a fairly tight upper bound on the exact burst-error probability, validating its use in modeling turbulence-induced fading correlation. Note that, as the correlation ρ_T increases, the probability of a single bit error $(m = 1)$ decreases. That is because when the correlation increases, implying a larger coherence time, the fading state will vary more slowly. It then becomes more likely that a burst of more than one bit error will occur during a large fading state.

Considering AWGN, (30) is modified to

$$P_{\text{On}}(r < r_{\text{th}}) = \int_{-\infty}^{r_{\text{th}}} P(r|\text{On}) dr \tag{32}$$

$$P_{\text{On}}(r_1 < r_{\text{th}}, r_n < r_{\text{th}}) = \int_{-\infty}^{r_{\text{th}}} \int_{-\infty}^{r_{\text{th}}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x_1) f(x_n|x_1) \frac{1}{\pi\sqrt{N_1 N_n}} \times \Phi(r_1, r_n, x_1, x_n) \times dx_n dx_1 dr_n dr_1 \tag{33}$$

where

$$\Phi(r_i, r_j, x_i, x_j) = \exp[-\frac{(r_i - \eta I_0 e^{2x_i})^2}{N_i} - \frac{(r_j - \eta I_0 e^{2x_j})^2}{N_j}]. \tag{34}$$

Note that a four-dimensional (4-D) integration is required to compute (33), independent of the length of the sequence.

In the above, we focused on missed detection of ON bits. When a sequence of OFF bits is transmitted, the probability of m consecutive false alarms (falsely detecting a sequence of m ON bits) is simply

$$P_{\text{Off}}^m = (1 - P_{\text{Off}})^2 (P_{\text{Off}})^m \tag{35}$$

where

$$P_{\text{Off}} = \int_{r_{th}}^{\infty} P(r|\text{Off}) dr. \tag{36}$$

Because the correlation of atmosphere turbulence only affects the detection of ON bits, a burst of m missed ON bits is much more likely than a burst of m false alarms.

It is obvious that

$$P_{\text{On}}(r > r_{\text{th}}) = 1 - P_{\text{On}}(r < r_{\text{th}}) \tag{37}$$

$$P_{\text{On}}(r_1 < r_{\text{th}}, r_n > r_{\text{th}}) = P_{\text{On}}(r_1 < r_{\text{th}}) - P_{\text{On}}(r_1 < r_{\text{th}}, r_n < r_{\text{th}}). \tag{38}$$

We can modify (29)–(36) to derive the higher order bit-error probability distribution for a sequence of bits with symbol-by-symbol detection. Since only ON-state bits are affected by atmospheric turbulence, based on the SMC model, the ON-state bit-error probability would depend on the joint probability distribution of the most recent preceding ON-state bit and the current ON-bit.

IV. SUBOPTIMAL PSP FOR MLSD

The MLSD, as expressed in (26), is optimal for detecting a sequence of bits that is i.i.d. and uniform on the set {Off, On}. Detecting a sequence of n bits requires a complexity of order $n \cdot 2^n$, because it requires computing an n-dimensional integral for each of 2^n bit sequences. To reduce the complexity, we consider PSP, which was proposed by Polydoros to extend the Viterbi algorithm to uncertain environments [13]. The key idea is to use the received intensity of recently detected ON bits to reduce uncertainty about the state of the turbulence-induced fading. If we consider AWGN and use the SMC model for the fading temporal correlation, then knowing the correlation between two consecutive ON bits is sufficient to perform MLSD. Even under the SMC assumption, however, the likelihood function in (24) cannot be decoupled into a sum of per-branch metrics, which is required for a reduced-complexity implementation. We will modify the metric function to allow us to implement a sub-optimal MLSD using PSP. This suboptimal MLSD can decode an n-bit sequence with a complexity of order n^2, as compared to the optimal MLSD described in Section II-D, which has a complexity of order $n \cdot 2^n$.

Assume a transmitted n-bit sequence $\vec{s}[s_1 \ s_2 \dots s_n]$. Define the index subset of ON-state symbols $S_{\text{On}} = \{n_i \in \{1, 2, \dots, n\}, s_{n_i} = 1\}_{i=1}^m$ with $n_1 = 1$, i.e., assume the first bit is ON. We also have the index subset of OFF-state symbols $S_{\text{Off}} = \{1_j \in \{1, 2, \dots, n\}, s_{1_j} = 0\}_{j=1}^{n-n}$. The exact likelihood function in (24) is

$$L(\vec{s}) = \int_{\vec{x}} f(\vec{x}) \exp\left[-\sum_{i=1}^{n} \frac{\left(r_i - \eta s_i I_0 e^{2x_i}\right)^2}{N_i}\right] d\vec{x}$$

$$= \exp\left[-\sum_{\substack{i=1\\ l_i\in S_{\rm Off}}}^{n-m}\frac{r_{l_i}^2}{N_{l_i}}\right]\cdot\int_{\vec{x}_{\rm On}} f(\vec{x}_{\rm On}) \times \exp\left[\sum_{\substack{i=1\\ l_i\in S_{\rm On}}}^{m}\frac{\left(r_{n_i}-\eta I_0 e^{2x_{n_i}}\right)^2}{N_{n_i}}\right] d\vec{x}_{\rm On} \quad (39)$$

where $\vec{x}_{\rm On}=\{x_{n_i} n_i \varepsilon S_{\rm On}\}_{i=1}^{m}$. Based on the SMC model, we have

$$L(\vec{s}) \cong \exp\left[-\sum_{\substack{i=1\\ l_i\in S_{\rm Off}}}^{n-m}\frac{r_{l_i}^2}{N_{l_i}}\right]\cdot\int_{\vec{x}_{\rm On}} f(x_{n_1}) f(x_{n_2}|x_{n_1})\ldots f(x_{n_m}|x_{n_{m-1}})\exp\left[-\sum_{\substack{i=1\\ l_i\in S_{\rm On}}}^{m}\frac{\left(r_{n_i}-\eta I_0 e^{2x_{n_i}}\right)^2}{N_{n_i}}\right] d\vec{x}_{\rm On}. \quad (40)$$

In order to decouple (40), we modify the weighted integration of (40) by

$$\int_{\vec{x}_{\rm On}} f(x_{n_1})\exp\left[-\frac{\left(r_1-\eta I_0 e^{2x_1}\right)^2}{N_1}\right]\prod_{\substack{i=1\\ n_i\in S_{\rm On}}}^{m-1} \times\frac{f(x_{n_i},x_{n_{i+1}})\Phi(r_{n_i},r_{n_{i+1}},x_{n_i},x_{n_{i+1}})}{f(x_{n_i})\exp\left[-\frac{\left(r_{n_i}-\eta I_0 e^{2x_{n_i}}\right)^2}{N_{n_i}}\right]} d\vec{x}_{\rm On}. \quad (41)$$

We modify (41) by decoupling the integration as shown in (42), at the bottom of the page, since the first term in the denominator is identical in the likelihood function for all codewords. We can write the modified likelihood function of $\vec{s}$ as given in (43), shown at the bottom of the page. From (43), we can define the metric function of the kth branch $(k>1)$ as

$$BM_k(\text{On}) = \frac{\int f(x_{n_{i-1}},x_{n_i})\Phi(r_{n_{i-1}},r_{n_i},x_{n_{i-1}},x_{n_i})dx_{n_{i-1}}dx_{n_i}}{\int f(x_{n_{i-1}})\exp\left[-\frac{\left(r_{n_{i-1}}-\eta I_0 e^{2x_{n_{i-1}}}\right)^2}{N_{n_{i-1}}}\right]dx_{n_{1-1}}} \quad (44)$$

$$BM_k(\text{Off}) = \exp\left(-\frac{r_k^2}{N_k}\right) \quad (45)$$

where $k=n_i$ and n_{i-1} denotes the position of the most recent ON-state bit.

In terms of the branch metrics, the MLSD can be expressed as

$$\vec{s} = \arg\max_{\vec{s}} \prod_{\substack{i=2\\ n_i\in S_{\rm On}}}^{m} BM_{n_i}(\text{On}) \prod_{\substack{i=1\\ l_i\in S_{\rm Off}}}^{n-m} BM_{l_i}(\text{Off}). \quad (46)$$

Since the MLSD considers path metrics that are the product of branch metrics, it can be implemented using the Viterbi algorithm, with a complexity of the order of $n^2/2$. Note that computation of each branch metric of the form (44) requires only a 2-D integration, independent of n. In (44), we see that computing the branch metric for an ON bit requires information obtained during the most recently transmitted ON bit, so we can only choose a survivor path when the previous bit is known to be ON; otherwise, we must keep track of the amplitude of the most recently received ON bit, and must also keep track of all survivor paths whose last bit is OFF. An example of the asymmetric PSP is shown in Fig. 3. The number on each branch is the branch metric computed using (44) and (45). The first and sixth bits correspond to the ON state. We see that, in this asymmetric PSP, we can only eliminate nonsurvivor paths when the most recent bit corresponds to the ON state. As in this example, to reduce the complexity of this algorithm, we can add an ON bit at the beginning and ending of each n-bit sequence. We can

$$\frac{\left\{\int_{\vec{x}_{\rm On}} f(x_{n_1})\exp\left[-\frac{\left(r_1-\eta I_0 e^{2x_1}\right)^2}{N_1}\right]dx_{n_1}\right\}\left[\prod_{\substack{i=1\\ n_i\in S_{\rm On}}}^{m-1}\int f(x_{n_i},x_{n_{i+1}})\Phi(r_{n_i},r_{n_{i+1}},x_{n_i},x_{n_{i+1}})dx_{n_i}dx_{n_{i+1}}\right]}{\prod_{\substack{i=1\\ n_i\in S_{\rm On}}}^{m-1}\int_{-\infty}^{\infty} f(x_{n_i})\exp\left[-\frac{\left(r_{n_i}-\eta I_0 e^{2x_{n_i}}\right)^2}{N_{n_i}}\right]dx_{n_i}} \quad (42)$$

$$L(\vec{s}) = \frac{\left[\prod_{\substack{i=1\\ n_i\in S_{\rm On}}}^{m-1}\int f(x_{n_i},x_{n_{i+1}})\Phi(r_{n_i},r_{n_{i+1}},x_{n_i},x_{n_{i+1}})dx_{n_i}dx_{n_{i+1}}\right]\exp\left[-\prod_{\substack{i=1\\ n_i\in S_{\rm Off}}}^{n-m}\frac{r_{l_i}^2}{N_{l_i}}\right]}{\prod_{\substack{i=1\\ n_i\in S_{\rm On}}}^{m-1}\int_{-\infty}^{\infty} f(x_{n_i})\exp\left[-\frac{(r_{n_i}-\eta I_0 e^{2x_{n_i}})^2}{N_{n_i}}\right]dx_{n_i}} \quad (43)$$

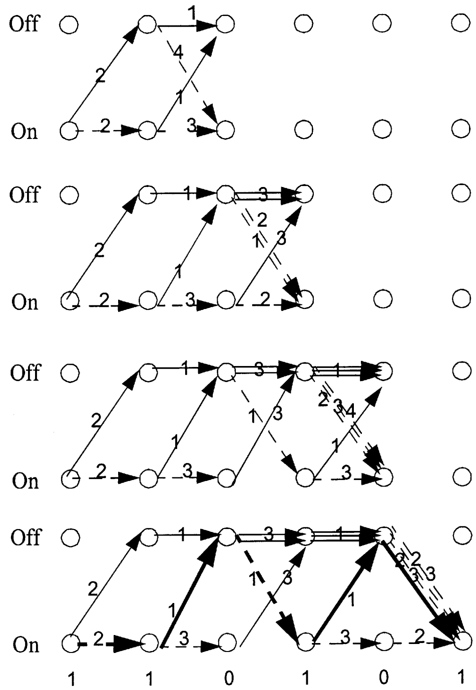

Fig. 3. Viterbi algorithm for MLSD with turbulence-induced fading. Solid lines denote OFF bits and dashed lines denote ON bits. The branch metric is marked on each branch. The decoded bit sequence is indicated at the bottom.

simply employ the starting ON bit of the next sequence as the ending ON bit of the previous sequence. The complexity of such an algorithm would be of order $n^2/2$, and the integration is only 2-D. However, we need extra memory to keep track of the survivor path information. Also, some bit overhead is required for implementation of this algorithm.

The algorithm described above still requires a large computational load to perform the 2-D integration. To reduce this complexity, we can estimate x_n using r_{n-1}, since

$$\begin{aligned}\vec{\hat{s}} &= \arg\max_{\vec{s}} \int_{\vec{x}} f(\vec{X}) \exp\left[-\sum_{i=1}^{n} \frac{\left(r_i - \eta s_i I_0 e^{2x_i}\right)^2}{N_i}\right] d\vec{x} \\ &= \arg\max_{\vec{s}} \int_{\vec{x}} f(x_i)\phi(s_1, r_1, x_1) \\ &\quad \times \prod_{i=2}^{n} \left[f(x_i|x_{i-1})\phi(s_i, r_i, x_i)\right] d\vec{x} \end{aligned} \tag{47}$$

$$\phi(s_i, r_i, x_i) = \exp\left[-\frac{(r_i - \eta s_i I_0 e^{2x_i})^2}{N_i}\right]. \tag{48}$$

Replacing $f(x_i|x_{i-1})$ with $f(x_i|r_{i-1})$, we can define the metric function for branch i as

$$BM_i(\text{On}) = \begin{cases} \int_{x_1} f(x_1)\phi(s_1 = 1, r_1, x_1)dx_1, & i = 1 \\ \int_{x_i} f(x_i|r_{1-1})\phi(s_i = 1, r_i, x_i)dx_i, & i > 1 \end{cases} \tag{49}$$

$$BM_i(\text{Off}) = \int_{x_i} f(x_i|r_{i-1})\phi(s_i = 0, r_i, x_i)dx_i. \tag{50}$$

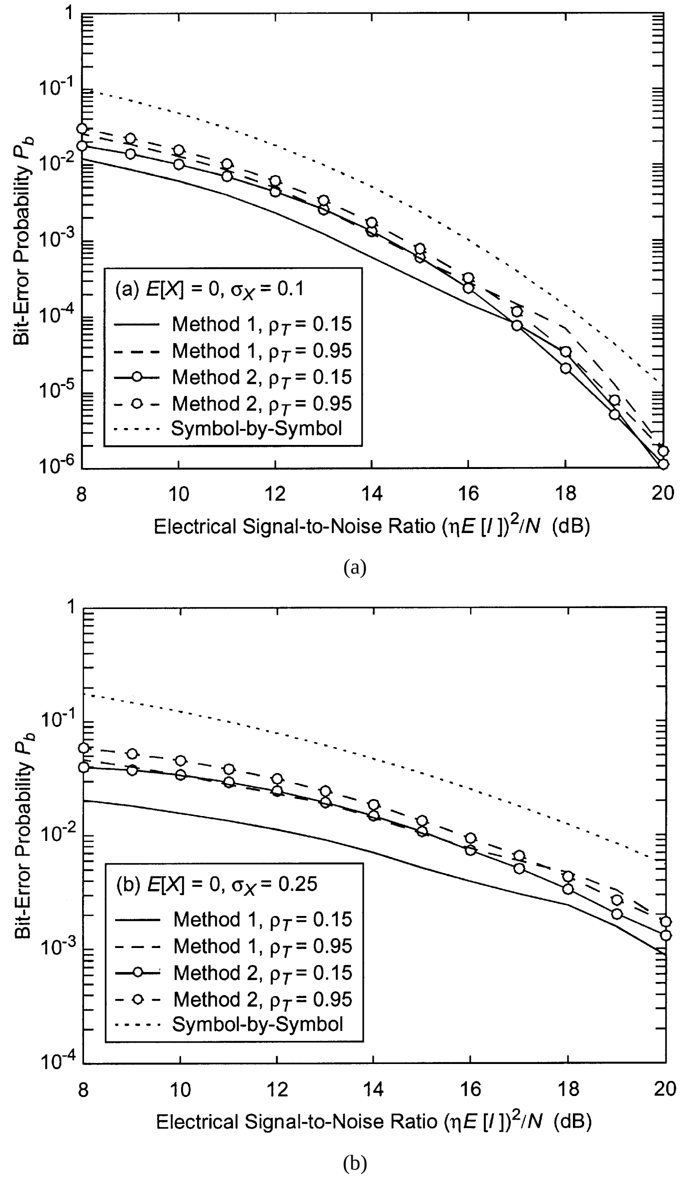

Fig. 4. Bit-error probability of different decoding schemes versus average electrical SNR with turbulence-induced fading. The dotted line represents the bit-error probability using a symbol-by-symbol decoding scheme. The solid lines consider a PSP algorithm based on the SMC model using branch metric functions (44) and (45), while the dashed lines represent a similar algorithm using (49) and (50).

In terms of the branch metrics, the suboptimal MLSD can be expressed similar to (46) with branch metric functions in (49) and (50).

To demonstrate the effectiveness of the suboptimal PSP algorithm, we present the simulation results in Fig. 4. In this simulation, we assume there is at most one error between two correctly decoded ON bits, and we set $E[X] = 0$, $I_0 = 1$. In Fig. 4(a) and (b), we assume $\sigma_X = 0.1$ and $\sigma_X = 0.25$, respectively. We plot the bit-error probability versus average electrical SNR, given by $SNR = (\eta E[I])^2/N$. We consider the two choices of branch metric discussed above; Method 1 uses (44) and (45), while Method 2 uses (49) and (50), both with a sequence length of 32. The temporal correlation coefficient ρ_T is chosen to be 0.15 and 0.95. In Fig. 4, we see that both Methods 1 and 2 can achieve much better bit-error probability performance than the symbol-by-symbol decoding scheme. Method 2 is subject to a penalty of a few decibels compared to Method 1, but avoids the 2-D integration required by Method 1.

From the discussions above, we see the SMC model can help to greatly simplify the implementation of MLSD with the suboptimal PSP algorithm, leading to a significant improvement in bit-error performance.

V. Conclusion

In free-space optical links through long-range atmospheric turbulence channels, turbulence-induced log-amplitude fluctuations can degrade link performance when $D_0 \leq d_0$ and $T_{0 \leq} \tau_0$. If the temporal correlation of fading is known, we can apply MLSD, leading to a performance improvement over symbol-by-symbol detection. To reduce the computational complexity, we have proposed a simple, single-step Markov channel for fading. We have verified its accuracy by considering the higher order bit-error probability distribution and we have used this model to derive two algorithms for suboptimal, low-complexity MLSD based on PSP. Simulations have shown that both algorithms provide a significant performance improvement over symbol-by-symbol detection.

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