Mode Coupling Effects in Multi-Mode Fibers

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Abstract: In mode-division-multiplexed systems using coherent detection, strong mode coupling is beneficial. Mode coupling reduces modal dispersion, minimizing signal processing complexity. In combination with modal dispersion, mode coupling creates frequency diversity, mitigating the mode-dependent gain of optical amplifiers.

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1. Introduction

In an optical fiber, an *ideal mode* [1] is an eigenvector of a *propagation operator*. *Mode coupling* [1] enables transfer of energy from one ideal mode to another during propagation. Mode coupling can be induced by random or intentional index perturbations, bends and stresses. The pairwise coupling strength between two modes depends on a dimensionless ratio between the coupling coefficient (per unit length) and the difference between the two modal propagation constants. Hence, a given perturbation may strongly couple modes having nearly equal propagation constants, but weakly couple modes having highly unequal propagation constants.

In multi-mode fiber (MMF), a plurality of modes typically leads to *modal dispersion*, limiting the bit rate × distance product of direct-detection systems, so it was long viewed as a strictly negative effect. This plurality is now seen as increasing capacity through *mode-division multiplexing* (MDM) [2,3]. Models for mode coupling in MMF were first developed nearly 40 years ago. Early MMF systems used spatially and temporally incoherent light-emitting diodes, so most studies employed *power coupling models* [4-6], in which redistribution of energy among modes is described by non-negative real coefficients. Power coupling models can explain certain effects, such as a reduced group delay (GD) spread in plastic MMF [5]. However, most modern MMF systems use spatially and temporally coherent laser sources, and power coupling models cannot explain certain observations [7].

Single-mode fiber (SMF) supports propagation in two polarization modes. Polarization-mode dispersion (PMD) [8] and polarization-dependent loss (PDL) [9] have long been described by *field coupling models*, in which phase-dependent coupling of modal fields is described by complex coefficients. Field coupling models describe not only a redistribution of energy among modes, but also how eigenvectors and their eigenvalues depend on the mode coupling coefficients. In the case of PMD, field coupling models describe *principal states of polarization* (PSPs), which are eigenvectors of a group delay operator, and are free of GD dispersion to first order in frequency [8]. These PSPs and their GDs change in response to random changes in the amplitude and phase of the mode coupling coefficients. PSPs form the basis for adaptive optical techniques to compensate PMD in systems using direct detection [8]. Over the past few years, field coupling models have been applied to MMF, demonstrating the existence of *principal modes* (PMs) [7,10], which are free of modal dispersion to first order in frequency. PMs form the basis for adaptive optical techniques to compensate modal dispersion in direct-detection systems [11].

To characterize the strength of the overall coupling caused by random perturbations along a fiber, we consider a *correlation length*, over which the local eigenvectors [1] can be assumed constant. In the *weak-coupling regime*, the fiber length is not much longer than the correlation length. While there may be significant coupling between modes having nearly equal propagation constants, there is limited coupling between modes having highly unequal propagation constants. Glass MMFs have tightly controlled index profiles, and even graded-index fibers up to several km long exhibit weak coupling between modes in different spatial mode groups [6], so the overall GD spread scales linearly with fiber length. In the *strong-coupling regime*, the fiber length far exceeds the correlation length, and there is significant coupling between all modes. As a result, the overall GD spread is reduced, and scales with the square-root of fiber length. Plastic MMFs have index perturbations causing strong mode coupling, which reduces the GD spread [5], but couples power to lossy higher-order modes, greatly increasing the loss [12].

In view of a significant renewed interest in MDM [3], there is a need for a detailed understanding of how mode coupling affects modal GDs and modal losses and gains in MMF. We review recent work addressing these issues. We show that in long-haul MDM systems using coherent detection, strong mode coupling reduces the GD spread to minimize signal processing complexity, and mitigates the impact of mode-dependent losses and gains.

2. Modal dispersion

In a MMF with mode coupling, a pulse launched into an ideal mode couples into other modes, and a superposition of pulses with various GDs is received. Given a fiber supporting D ideal modes (in two polarizations), one can find a set of D PMs, such that a pulse launched into a PM yields a single output pulse. A PM is defined as an input field distribution such that, as the optical frequency is varied, the output field distribution exhibits minimal variation [10]. Alternatively, given a propagation operator $\mathbf{M}$ describing lossless propagation and mode coupling, one can define a *group delay operator* $\mathbf{G} = j(\partial \mathbf{M}/\partial \omega)\mathbf{M}^H$, whose eigenvectors and eigenvalues are the PMs and their GDs, respectively [10]. These first-order PMs are valid over a bandwidth sufficiently small that the phase of $\mathbf{M}$ can be considered to vary linearly with frequency. For the case $D = 2$ (PMD in SMF), the PMs correspond to the PSPs [8].

The PMs in graded-index glass MMF were studied in [7], where spatial- and polarization-mode coupling were induced by subdividing a fiber into numerous short sections with random curvatures and orientations. Fig. 1 shows the GDs of PMs in a 1-km fiber. A standard deviation (STD) of curvature $\sigma_\kappa < 1$ m^{-1} corresponds to weak coupling, where spatial and polarization degrees of freedom mix weakly, the GDs depend weakly on σ_κ, and the overall GD spread scales linearly with the fiber length. As σ_κ increases to 10 m^{-1}, strong coupling is approached, where spatial and polarization degrees of freedom mix strongly, the GDs spread is reduced significantly, and the GD spread scales with the square-root of fiber length. Higher-order modal dispersion was studied in [13], again using random curvature to generate mode coupling. For signal bandwidths of tens of GHz and $\sigma_\kappa > 1$ m^{-1}, higher-order effects cause a filling-in of the impulse response (as observed in [6]), spatial mode-dependent chromatic dispersion, and mode pattern blurring, which are all analogous to higher-order PMD effects [8]. Higher-order effects limit compensation of modal dispersion by any frequency-independent device, such as an SLM [11].

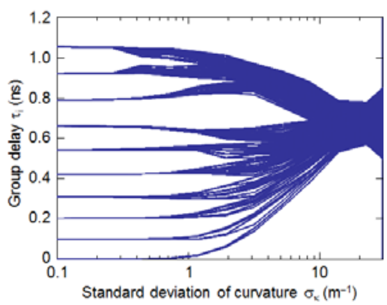

Fig. 1. Group delays (GDs) versus curvature for a 1-km graded-index fiber with $D = 110$ modes in two polarizations [7].

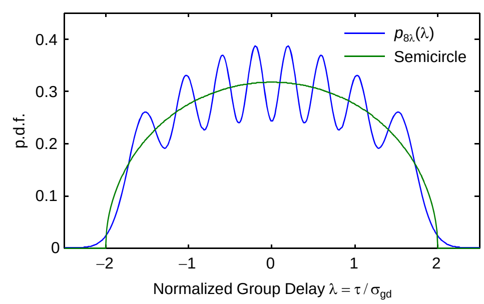

Fig. 2. Analytical marginal p.d.f. of normalized GD τ/σ_{gd} for a fiber with $D = 8$ modes compared to the semicircle distribution [14].

In MDM using coherent detection, one can employ a fixed set of D orthogonal modes (e.g., ideal modes) as transmit and receive bases, and perform frequency-dependent $D \times D$ matrix signal processing to approximately invert the propagation operator $\mathbf{M}$, thereby demultiplexing signals and compensating modal and chromatic dispersions. In the absence of mode-dependent loss/gain, $\mathbf{M}$ is a constant times a unitary operator, and these dispersions have no impact on performance. This approach does not rely upon PMs (just as coherent polarization multiplexing in SMF does not rely upon PSPs), but the signal processing complexity is proportional to the peak-to-peak spread between their GDs. With weak coupling, this GD spread can span thousands of symbol intervals, so it is desirable to have strong coupling to reduce signal processing complexity. The statistics of the PM GDs in the strong-coupling regime were studied in [14]. Assuming a MMF in strong-coupling regime can be modeled as K independent sections, each with STD of GD σ_τ, the overall fiber has a STD of GD $\sigma_{gd} = \sqrt{K}\sigma_\tau$, which scales with the square-root of total fiber length. A matrix representing the GD operator $\mathbf{G}$ is the sum of many i.i.d. random matrices having independent eigenvectors and so, by the Central Limit Theorem, is a zero-trace Gaussian unitary ensemble. The distribution of the GDs is the eigenvalue distribution of the ensemble, which can be computed analytically for any D. For $D = 2$, this corresponds to the Maxwellian distribution [8]. As D increases, the tails of the distribution shrink rapidly, and as $D \to \infty$, the distribution approaches a semicircle. Fig. 2 shows the distributions of the GDs for $D = 8$ and $D = \infty$. From a practical perspective, Fig. 2 suggests that for $D \geq 8$, the receiver signal processing can be designed with a temporal memory slightly longer than the peak-to-peak spread of the semicircle, $\Delta\tau_{p\text{-}p} = 4\sigma_{gd}$.

3. Mode-dependent loss and gain

Long-haul MDM systems will employ numerous inline optical components, such as amplifiers and fibers, which can introduce *mode-dependent loss and gain*, collectively referred to here as MDL. Unlike modal dispersion, MDL is a fundamental performance-limiting factor [15,16]. In the extreme case, MDL is equivalent to a reduction of the number of propagating modes, leading to a proportional reduction of data rate or channel capacity.

The statistical properties and system impact of MDL in the strong-coupling regime were studied in [15]. Given a propagation operator $\mathbf{M}$ including modal losses and gains, one can define a *modal gain operator* $\mathbf{MM}^H$. At a single frequency, MDL is characterized by the logarithms of the eigenvalues of $\mathbf{MM}^H$, which are the modal gains measured in log power gain units (proportional to decibels). In [15], two fundamental propositions were shown to hold true in the strong-coupling, low-to-moderate-MDL regime of practical interest. (a) The distribution of the modal gains is equivalent to the eigenvalue distribution of a zero-trace Gaussian unitary ensemble, which is the same as the distribution of the modal GDs [14]. For example, the MDL distribution is Maxwellian for $D = 2$ (as known for PDL in SMF [9]), and is a semicircle for $D = \infty$. (b) The STD of *overall MDL* σ_{mdl} depends solely on the STD of *accumulated MDL* ξ via $\sigma_{\mathrm{mdl}} = \xi\sqrt{1+\xi^2/12}$. If there are K MDL sources (e.g., amplifiers), each with STD of MDL σ_g, then the accumulated MDL is $\xi = \sqrt{K}\sigma_g$, which scales with the square-root of the number of MDL sources. Although the noise from a single amplifier with MDL is spatially nonwhite, the noise becomes spatially white as K increases [15].

The performance or channel capacity of MDM at a single frequency depends only on the distribution of MDL, even if modal dispersion is present. Achieving optimal performance requires knowledge of the instantaneous eigenvectors and eigenvalues of $\mathbf{MM}^H$, which represent *channel state information* (CSI). In long-haul systems, CSI is likely to be unavailable, so a transmitter must allocate equal power to each mode, and MDL always reduces capacity. MDL causes the capacity to fluctuate randomly and, with or without CSI, the outage capacity is always lower than the average capacity. Fig. 3 shows outage capacity at 10^{-3} outage probability vs. accumulated MDL ξ for MDM using $D = 2, 4, 8, 16$ modes, at SNR $\rho_t = 10$ dB (defined as the total power over all modes divided by noise variance per mode). For example, at $\xi = 5$ dB (corresponding to $K = 25$ amplifiers, each with MDL $\sigma_g = 1$ dB), MDL is observed to decrease capacity significantly (except when $\rho_t < D$ with CSI [15]).

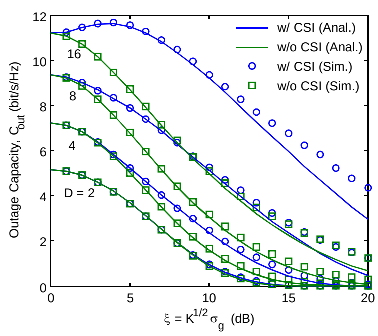

Fig. 3. Outage capacities, with or without CSI, for MDM with $D = 2, 4, 8, 16$ modes as a function of accumulated MDL ξ for SNR $\rho_t = 10$ dB and 10^{-3} outage probability [15].

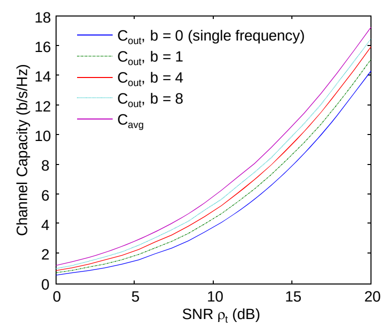

Fig. 4. Outage capacity vs. SNR ρ_t for various normalized bandwidths $b = B_{\mathrm{sig}}\cdot\sigma_{\mathrm{gd}}$ for MDM with $D = 10$ modes, assuming no CSI, at 10^{-3} outage probability. Average capacity is also shown [16].

When modal dispersion and strong mode coupling are present, MDL varies over frequency, with a coherence bandwidth inversely proportional to the STD of GD σ_{gd}. When an MDM signal occupies a bandwidth B_{sig} much greater than the coherence bandwidth, *frequency diversity* is obtained [16]. Fluctuations in modal gains are averaged over frequency, and the variance of the capacity is reduced by a factor called the *diversity order*, which is roughly equal to a *normalized signal bandwidth* $b = B_{\mathrm{sig}}\sigma_{\mathrm{gd}}$. A rigorous definition of diversity order is given in [16]. Fig. 4 shows outage capacities vs. SNR at different values of b for MDM using $D = 10$ modes. As b increases, the outage capacity approaches the average capacity. Frequency diversity enhances reliability without incurring overhead. Achieving a high diversity order requires substantial modal dispersion and strong mode coupling.

4. References

[1] D. Marcuse, *Theory of Dielectric Optical Waveguides* (Academic Press, 1974).

[2] S. Berdagué and P. Facq, Appl. Opt. **21**, 1950-1955 (1982).

[3] S. Randel *et al*, Opt. Express 19, 16697-16707 (2011).

[4] D. Gloge, Bell Sys. Tech. J. **51**, 1767-1780 (1972).

[5] A. F. Garito, J. Wang, and R. Gao, Science **281**, 962-967 (1998).

[6] K. Balemarthy, A. Polley and S. E. Ralph, J. Lightw. Technol. **24**, 4885-4894 (2006).

[7] M. B. Shemirani, W. Mao, R. A Panicker and J. M. Kahn, J. Lightw. Technol. **27**, 1248-1261, (2009).

[8] H. Kogelnik, R. M. Jopson, and L. E. Nelson, "Polarization-mode dispersion" in *Optical Fiber Telecommunications IVB: Systems and Impairments*, I. Kaminow and T. Li, eds. (Academic Press, 2002).

[9] A. Mecozzi and M. Shtaif, Photon. Technol. Lett. **14**, 313-315 (2002).

[10] S. Fan and J. M. Kahn, Opt. Lett. **30**, 135-137 (2005).

[11] X. Shen, J. M. Kahn and M. A. Horowitz, Opt. Lett. **30**, 2985-2987 (2005).

[12] P. J. Decker, A. Polley, J. H. Kim and S. E. Ralph, J. Lightw. Technol. **29**, 305-315 (2011).

[13] M. B. Shemirani and J. M. Kahn, J. Lightw. Technol. **27**, 5461-5468 (2009).

[14] K.-P. Ho and J. M. Kahn, "Statistics of Group Delays in Multimode Fiber with Strong Mode Coupling", to appear in J. Lightw. Technol.

[15] K.-P. Ho and J. M. Kahn, Opt. Express **19**, 16612-16635 (2011).

[16] K.-P. Ho and J. M. Kahn, "Frequency Diversity in Mode-Division Multiplexing Systems", subm. to J. Lightw. Technol.