Optimal Predistortion of Gaussian Inputs for Clipping Channels

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Abstract— **For a clipped channel with a Gaussian input, a predistortion/restoration technique to reduce clipping-induced nonlinear distortion is proposed and analyzed. The input signal is processed by a nonlinear predistorter circuit, reducing the probability of clipping. The receiver output signal passes through a restorer having the inverse transfer characteristic, which yields the original signal. For both one-sided and two-sided limiter channels, the optimal predistortion curves are determined analytically. A limiter channel with Gaussian input may be used to model clipping-induced nonlinear distortion in optical-fiber common antenna television (CATV) distribution systems using multiple intensity-modulated subcarriers. When applied to an amplitude-modulated vestigial-sideband (AM-VSB) CATV system, the optimal predistortion curves yield sensitivity improvements of 5.3 and 4.4 dB for one- and two-sided limiter channels, respectively.**

I. Introduction

FREQUENCY-division subcarrier multiplexing (SCM) with intensity modulation and direct detection is a promising technique for transmitting a large number of television signals over a single optical fiber. The channel-carrying capacity of a laser is large, but is fundamentally limited by the fact that if the product of the number of channels and the modulation-current depth per channel exceeds the threshold current of the laser, occasionally the input current will drop below the laser threshold current, turning the laser off (see Fig. 1). Since the power of each subcarrier channel is proportional to the modulation depth, in an effort to maximize the number of channels transmitted by a single laser, some amount of clipping distortion is usually tolerated [1], [2]. Usually, the input signal of a SCM system can be modeled as a Gaussian random process because a large number of channels renders the central limit theorem valid. Therefore, mathematically, we deal with a clipped channel with Gaussian input.

A pretransmission nonlinear distortion and postdetection restoration scheme to reduce the effect of clipping-induced nonlinear distortion (NLD) has been proposed and analyzed previously [3]–[5]. In this scheme, the laser drive current at the transmitter is predistorted before passing to the semiconductor laser, and a post-detection restorer at the receiver yields the original signal. The transfer characteristic of the predistorter and restorer are inverse functions of each other, so that no NLD will be caused by the predistortion technique. The principle behind this predistortion/restoration technique is similar to the companding or nonuniformly spaced quantizer used in pulse-code modulation system [6]–[7]. Previous work has considered use of a simple piecewise-linear (PWL) curve to predistort the input signal [3]. Since an arbitrary predistortion curve can be used (as long as its inverse function exists), it is of interest to determine the form of the optimal predistortion curve and the resulting maximum possible sensitivity improvement. In this paper, we find the optimal predistortion curves for both one- and two-sided limiter channels, and we show that these provide significant sensitivity improvements as compared to systems without predistortion.

The remainder of this paper is organized as follows. Section II presents a mathematical model of the SCM transmission system. Section III and Section IV present the optimal predistortion curves of one- and two-sided limiter channel, respectively, and describe the resulting sensitivity improvements obtained by using predistortion. Discussion and conclusions are presented in Section V and Section VI, respectively.

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II. System Modeling

In this section, we discuss the modeling of noise and NLD in SCM systems, deriving results for the one-sided limiter channel (corresponding results for the two-sided limiter channel are presented in Section IV). For an N-channel SCM system, the combined analog signal is

$$i(t) = I_b\left(1 + \sum_{n=1}^{N} m_n \cos(\omega_n t + \phi_n)\right) \tag{1}$$

where m_n, ω_n, ϕ_n are the amplitude-modulation index, angular frequency and modulation angle, respectively, of the nth channel. The product $m_n I_b$ is the modulation depth of the nth channel. In most systems, each channel will have an equal modulation index, so that $m_n = m$. For the case $N > 10, i(t)$ can be approximated as a Gaussian random process with mean value I_b and variance $(\mu I_b)^2$, where the root-mean squared (RMS) modulation index is defined as [1]

$$\mu = m\sqrt{\frac{N}{2}}. \tag{2}$$

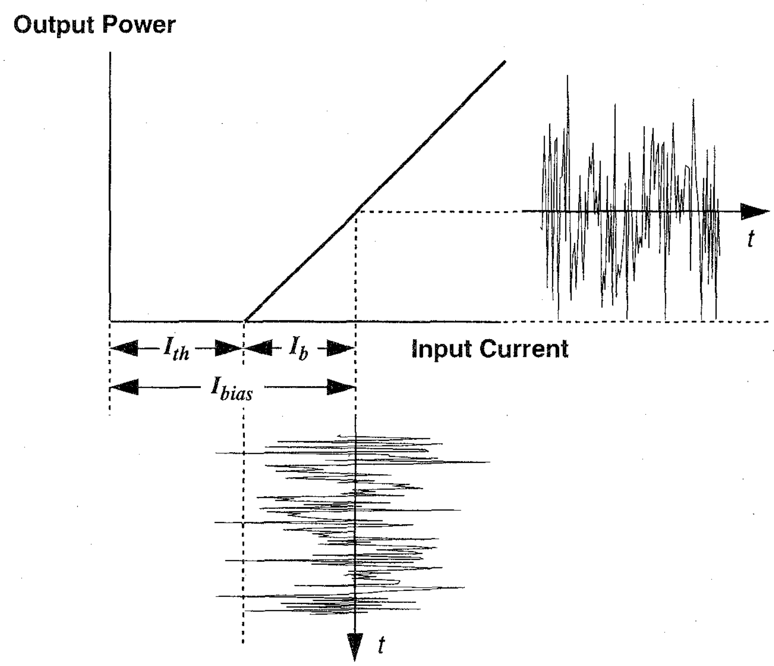

Fig. 1. Semiconductor laser input-output characteristic and input and output waveforms. Laser output exhibits clipping distortion at threshold.

Fig. 2 shows a general mathematical model of the predistortion/restoration scheme. The channel is normalized to have unit clipping amplitude, and the original dc bias point is shifted to zero in both predistorter and restorer. The signal and noise are normalized accordingly. For simplicity, our analysis neglects fiber loss and dispersion, receiver bandwidth limitations, predistorter–restorer mismatch, and other complicating factors in practical systems, which have been considered elsewhere [3], [5].

We let $y(\cdot)$, $g(\cdot)$, and $y^{-1}(\cdot)$ denote the transfer characteristics of predistorter, clipping channel and restorer, respectively; all are assumed to be memoryless, nonlinear functions. In the absence of predistortion, the output of the system is $z(t) = g(i(t)) + n(t)$. If $g(\cdot)$ is an one-sided limiter, this will induce clipping noise if $i(t) < -1$. Predistortion/restoration will extend the clipping limit and will affect the noise. If $y(i(t)) > -1$, the restorer output signal is $z(t) = y^{-1}(y(i(t)) + n(t)) = i(t) + n(t) \cdot dy^{-1}/dx|_{i(t)}$. If $y(i(t)) < -1$, the output will still be clipped. Therefore, the restorer output is

$$z(t) = \begin{cases} i(t) + \dfrac{n(t)}{\left.\frac{dy}{dx}\right|_{i(t)}} & i(t) \geq -\gamma \\ -\gamma + \dfrac{n(t)}{\left.\frac{dy}{dx}\right|_{-\gamma}} & i(t) < -\gamma \end{cases} \tag{3}$$

where γ is the new clipping boundary, which is defined by the expression $y(-\gamma) = -1$. Here, $n(t)$ is the additive Gaussian noise with variance σ_n^2. In writing down (3), we have assumed that the noise $n(t)$ is small compared to the signal $i(t)$; this assumption is clarified in Appendix A.

The total amount of noise at the restorer output will depend on the predistortion curve and the probability distribution of the input signal. Although predistortion will extend the clipping boundary, some residual amount of NLD will not be cancelled. The total amount of noise in a one-sided limiter system will be

$$N[y] = \sigma_n^2 \int_{-\gamma}^{\infty} \frac{p(x)}{\dot{y}^2} dx + \frac{\sigma_n^2}{[\dot{y}(-\gamma)]^2} \int_{-\infty}^{-\gamma} p(x) dx + \text{NLD} \tag{4}$$

where the first and second terms correspond to the Gaussian noise contributed by the nonclipping and clipping regions, respectively, the third term is the clipping-induced NLD. Here, $\dot{y} = dy/dx$ is the slope of the predistorter transfer characteristic, $p(x)$ is the distribution of the input signal, which is Gaussian with zero mean and variance μ^2

$$p(x) = \frac{1}{\sqrt{2\pi}\mu} e^{-x^2/2\mu^2}. \tag{5}$$

In this paper, the modified Saleh formula is used to calculate the carrier-to-NLD ratio (CNLD) as [1]–[2]

$$\text{CNLD} = D(\mu) = \sqrt{2\pi} \frac{1 + 6\mu^2}{\mu^3} e^{1/2\mu^2}. \tag{6}$$

This modified Saleh formula has been verified in [8] and generalized in [9]. In Appendix A, (4) is derived in a more rigorous way.

When predistortion/restoration is employed, the Gaussian noise will have a different variance than in the original system. We define the power penalty for Gaussian noise induced by predistortion/restoration

$$\delta_e = \int_{-\gamma}^{\infty} \frac{p(x)}{\dot{y}^2} dx + \frac{1}{[\dot{y}(-\gamma)]^2} \int_{-\infty}^{-\gamma} p(x) dx. \tag{7}$$

This power penalty can be smaller or larger than unity, leading to gain or loss, respectively. Then overall carrier-to-noise ratio

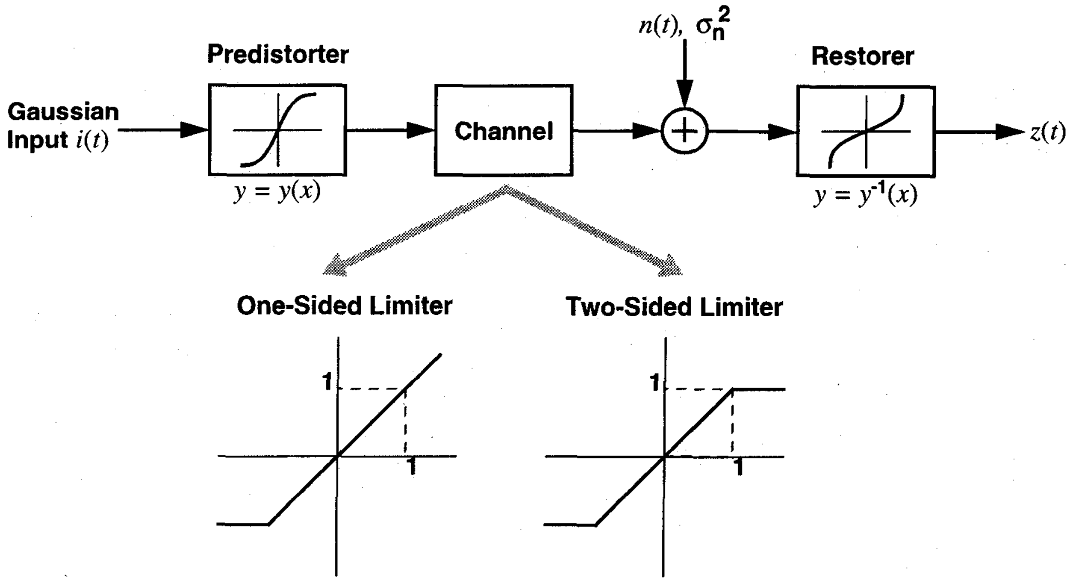

Fig. 2. General model of predistortion/restoration on a one- or two-sided clipping channel with Gaussian-distributed input signal. In the nonclipping region, the channel is normalized to have a transfer characteristic with unit slope, unit clipping boundary and zero intercept.

(CNR) per channel of the system with predistortion/restoration is

$$\frac{1}{\rho_t} = \frac{\delta_e}{\mu^2 \rho_\Sigma} + \frac{1}{D(\mu/\gamma)} \tag{8}$$

where $\mu^2 \rho_\Sigma = 0.5m^2/\sigma_n^2$ is the signal-to-Gaussian-noise ratio of each channel, and ρ_Σ is the total signal to total Gaussian noise ratio

$$\rho_\Sigma = \frac{1}{N\sigma_n^2} \tag{9}$$

i.e., the total average normalized received power of unity (without normalization, it is I_b^2), divided by the total Gaussian noise of all N channels. The quantity ρ_Σ is a good figure-of-merit to characterize the overall performance of a SCM system. In addition to its dependence on ρ_Σ, the overall CNR per channel depends on the modulation depth and on the amount of NLD. The overall system performance is better when the system requires a smaller value of ρ_Σ to achieve a given overall CNR per channel with a given number of channels. The relationship between ρ_Σ and the design parameters of a lightwave system is described in [5].

III. Optimal Predistortion of One-Sided Limiter Channel

An intensity-modulated semiconductor laser represents a one-sided limiter system, since it cannot produce negative output power. While a straightforward equalization method would just shift the laser input signal up to a higher level to reduce the probability of clipping, this will decrease the average optical-power efficiency of the system. Furthermore, the laser will also have a peak output-power limitation (not shown in Fig. 1) which has the potential to induce NLD; this peak-power limiting is typically much "softer" than the clipping at zero output power. Therefore, a reasonable constraint for the predistortion curve is that it preserve the average value of the input signal so that, in Fig. 2, both the predistorter input and output signals have zero average value. We note that the portion of predistortion curve for $y(x) < -1$ does not enter into calculation of the noise in (4), and this portion of curve can be chosen arbitrary, as long as its derivative exists at the point $y(-\gamma) = -1$. We assume that the output of the clipped channel has the same average signal level as the predistorter input, which is expressed mathematically as

$$\int_{-\gamma}^{\infty} y(x)p(x)dx - \int_{-\infty}^{-\gamma} p(x)dx = 0 \tag{10}$$

where the first term corresponds to the nonclipped region and the second term to the clipped region.

In Appendix B, we use a variational method [10] to derive the predistortion curve that minimizes the output noise (4), subject to the constraint (10). The predistortion curve is given by

$$y(x) = \kappa \int_{-\gamma/\mu}^{x/\mu} \frac{e^{-t^2/6}dt}{[\mathrm{erfc}(t/\sqrt{2})]^{1/3}} - 1 \tag{11}$$

where, as a function of $\gamma/\mu, \kappa$ is given by

$$\kappa^{-1} = \frac{1}{2}\int_{-\gamma/\mu}^{\infty} e^{-t^2/6}[\mathrm{erfc}(t/\sqrt{2})]^{2/3}dt. \tag{12}$$

Fig. 3 shows examples of predistorter curves $y(x)$ for three different values of the ratio γ/μ. The portion of the predistortion curve for $y(x) < -1$ is defined mathematically by (11), but physically has no effect on the system. Because the optimal predistorter curve has been calculated for normalized input x/μ and the output is the value of $y(x)$ itself, then the probability density of the predistorter output is completely determined by the ratio γ/μ. In other words, regardless of the distribution of the input, for a given ratio γ/μ the optimal

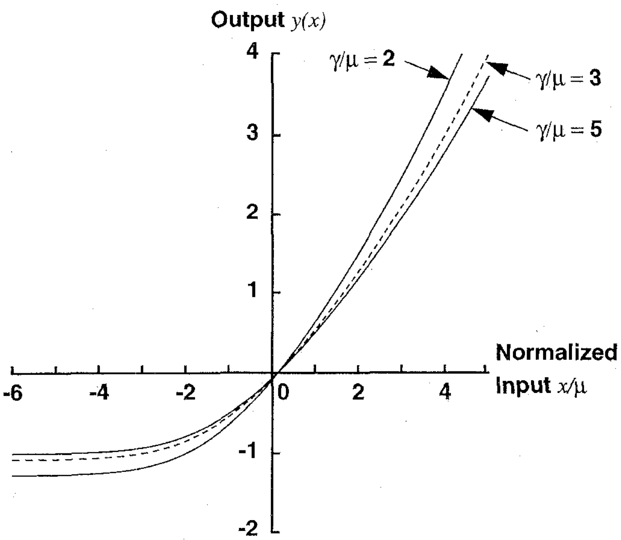

Fig. 3. Examples of optimal predistortion curves for a one-sided limiter channel for various values of the ratio γ/μ.

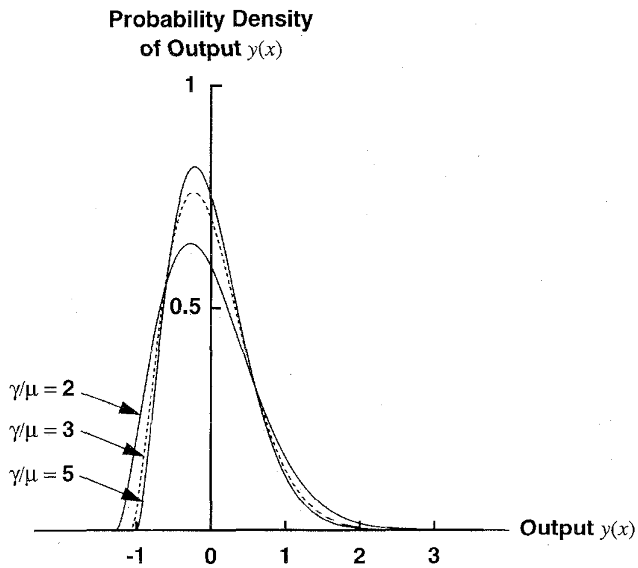

Fig. 4. Probability density function of predistorter output for one-sided limiter channel, for various values of the ratio γ/μ, assuming the predistorter of Fig. 3.

predistortion curve will either scale-up or scale-down the signal such that the distribution of predistorter output will be the same. Fig. 4 shows the probability density of the predistorter output for the same values of the ratio γ/μ as in Fig. 3. There is still a small probability that $y(x) < -1$ and the signal will be subject to some NLD.

The optimal predistortion curve described by (11) and (12) can be substituted into (7) and (8) to determine the Gaussian noise penalty and CNR per channel. For a specific value of the total SNR ρ_Σ, the overall CNR per channel of the system depends on γ and μ through the ratio γ/μ only. Using numerical optimization, the optimal choice of γ/μ can be easily found.

Fig. 5 displays the overall CNR per channel that can be obtained using the optimal predistortion curve, and also shows the CNR per channel obtained without predistortion. For the system with predistortion, Fig. 6 shows the corresponding values of the new clipping boundary γ/μ as a function of ρ_Σ. In Fig. 5, we see that predistortion yields a sensitivity improvement that increases as ρ_Σ and ρ_t increase. For a system that require a large value of overall CNR per channel ρ_t, a relatively small amount of NLD will deteriorate the system greatly. Because our predistortion scheme strongly reduces the effect of NLD, the sensitivity improvement will be larger for system that can tolerate only a small amount of NLD. Seeing that for large values of the ratio γ/μ, the effect of clipping is relatively small, the value of γ/μ increases with an increase of ρ_Σ and ρ_t, as shown in Fig. 6. For large values of γ/μ, from (8), $\rho_t = \mu^2\rho_\Sigma/\delta_e$. From Appendix B, for large value of $\gamma/\mu, \delta_e$ is also proportional to μ^2, leading to the asymptotic expression

$$\rho_t = \rho_\Sigma - 7.91 \quad (\text{dB}). \tag{13}$$

In Fig. 5, we see that for $\rho_\Sigma > 50$ dB, the overall CNR per channel is well approximated by (13). This means that the residual NLD is very small, and may be neglected.

For an amplitude-modulated vestigial-sideband (AM-VSB) system which requires $\rho_t = 55$ dB [1], the system without predistortion requires a $\rho_\Sigma = 68.2$ dB, but the predistortion scheme reduces the required ρ_Σ to 62.9 dB, which represents a 5.3-dB sensitivity improvement or a 240% increase in permissible number of channels. For a FM system that requires $\rho_t = 17$ dB [2], the sensitivity improvement is approximately 1 dB (reducing ρ_Σ from 23.3 dB to 22.2 dB). Therefore, our system offers a greater advantage for AM-VSB SCM systems, due to their smaller tolerable NLD.

IV. Optimal Predistortion of Two-Sided Limiter Channel

Due to the practical difficulties in achieving sufficiently high CNR per channel in encountered in directly modulated AM-VSB CATV systems, it has been proposed to employ external modulation [11]–[12]. An externally modulated system may be modeled as a two-sided limiter. The total noise of a two-sided limiter is almost the same as (4), and is given by

$$N[y] = 2\sigma_n^2 \int_0^\gamma \frac{p(x)}{\dot{y}^2}dx + \frac{2\sigma_n^2}{[\dot{y}(\gamma)]^2}\int_\gamma^\infty p(x)dx + 2\cdot \text{NLD} \tag{14}$$

where the first and second terms correspond to the Gaussian noise contributed by the nonclipping and clipping regions, respectively. The third term is the residual clipping-induced NLD, which is about twice that of a one-sided limiter when the NLD is small [8].

The boundary condition of a two-sided limiter is simpler than that of a one-sided limiter, because of the symmetry property of the channel. The predistortion curve $y(x)$ must be an odd function of x; this will guarantee that the average signal after the predistorter will remain zero.

We first consider a system in which no clipping occurs when the clipping boundary $\gamma \to \infty$. In this case, the second and

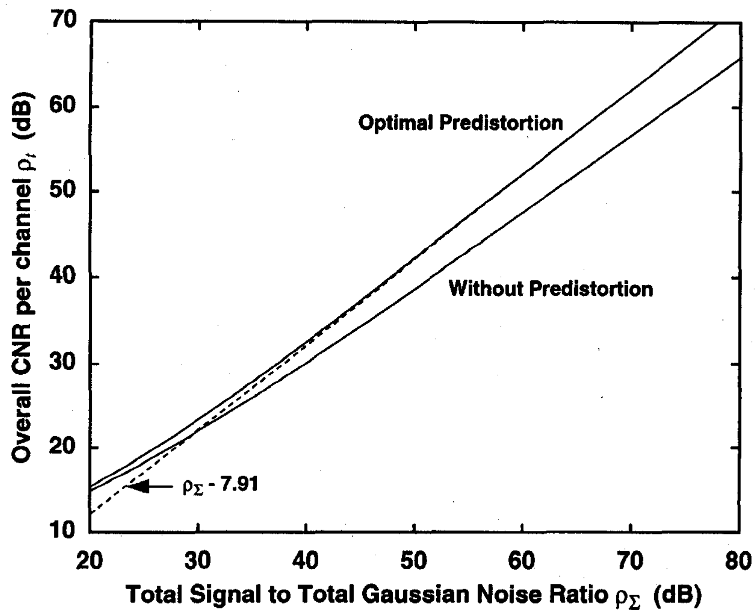

Fig. 5. Overall CNR per channel ρ_t versus total SNR ρ_Σ, with and without predistortion, for a one-sided limiter channel.

third terms of (14) vanish, and $y(x)$ must be bounded between ± 1. Mathematically, the problem is to find a predistortion curve to minimize

$$N[y] = 2\sigma_n^2 \int_0^\infty \frac{p(x)}{\dot{y}^2} dx. \qquad (15)$$

Using the simple calculus of variation [13] and solving the Euler equation, the optimal transfer characteristic of the predistortion curve is obtained

$$y(x) = \mathrm{erf}\left(\frac{x}{\sqrt{6}\mu}\right). \qquad (16)$$

As required, the solution (16) is an odd function of x, and satisfies $\lim_{x\to\pm\infty} y(x) = \pm 1$, so that after predistortion, the signal swing will be confined to the range between ± 1. Similar to the case of one-sided limiter, the predistortion curve either scale-up or scale-down the signal to fit the channel. The noise at the restorer output can be obtained by substituting (16) into (15), yielding the power penalty

$$\delta_e = 3\sqrt{3}\pi\mu^2/2. \qquad (17)$$

If $\mu < 0.35$, then $\delta_e < 1$ and the noise penalty will be less than unity. The maximum overall CNR per channel is independent of the choice of μ and equal to

$$\rho_t = \rho_\Sigma - 9.12 \quad \text{(dB)}. \qquad (18)$$

This gives a value $\rho_t = 55$ dB when $\rho_\Sigma = 64.1$ dB.

We now consider the more general case in which the clipping boundary γ is finite. In the variational method [13], (14) will have the same Euler equation as (15), though we will have different boundary conditions. With the boundary condition $y(\pm\gamma) = \pm 1$, we obtain the optimal predistorter curve

$$y(x) = \frac{\mathrm{erf}(x/\sqrt{6}\mu)}{\mathrm{erf}(\gamma/\sqrt{6}\mu)}. \qquad (19)$$

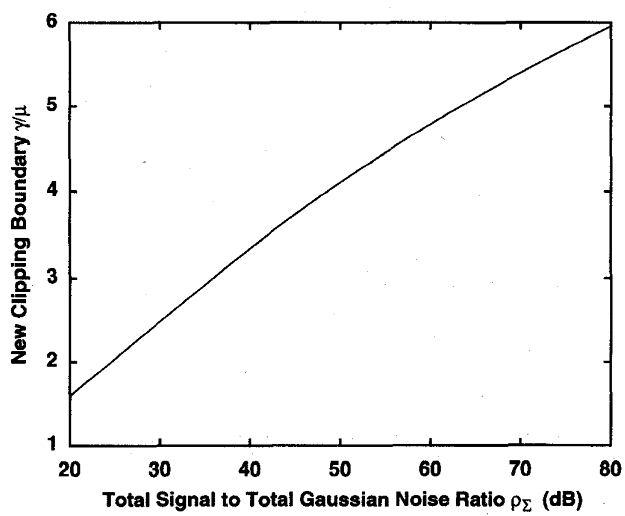

Fig. 6. New clipping boundary γ/μ as a function of total SNR ρ_Σ for a one-sided limiter channel.

Using the variational method directly to treat the boundary condition and determine the optimal value of γ is complicated. As an alternative, we substitute (19) into (14), yielding the power penalty

$$\delta_e = \frac{3\pi\mu^2}{2}[\mathrm{erf}(\gamma/\sqrt{6}\mu)]^2 \times \left[\sqrt{3}\mathrm{erf}(\gamma/\sqrt{6}\mu) + e^{\gamma^2/3\mu^2}\mathrm{erfc}(\gamma/\sqrt{2}\mu)\right]. \qquad (20)$$

Instead of minimizing the noise, it is equivalent to maximize the CNR. The overall CNR per channel is

$$\frac{1}{\rho_t} = \frac{\delta_e}{\mu^2\rho_\Sigma} + \frac{2}{D(\mu/\gamma)}. \qquad (21)$$

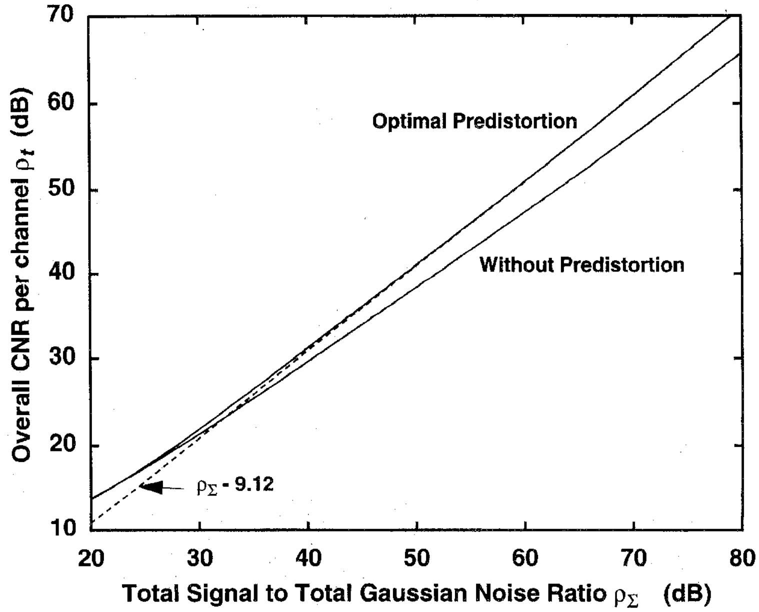

Fig. 7. Overall CNR per channel ρ_t versus total SNR ρ_Σ, with and without predistortion, for a two-sided limiter channel.

As for a one-sided limiter, for a given value of ρ_Σ, the maximum obtainable ρ_t depends on γ and μ only through the ratio γ/μ.

Fig. 7 shows the overall CNR per channel ρ_t versus ρ_Σ for a two-sided limiter system. For $\rho_\Sigma > 50$ dB, the overall CNR per channel can be approximated as $\rho_t = \rho_\Sigma - 9.12$ (dB). At high ρ_t and ρ_Σ, the system cannot tolerate a large amount of NLD, so that after predistortion, the system penalty due to clipping is negligible. The small NLD in the predistorted system is verified in Fig. 8, which shows the new clipping boundary γ/μ as a function of ρ_Σ. When $\rho_\Sigma > 50$ dB, $\gamma/\mu > 4$, and only a small amount of NLD will remain.

For an AM-VSB system that requires $\rho_t = 55$ dB, the system without predistortion requires $\rho_\Sigma = 68.5$ dB, which is 0.3 dB more than in the case of the one-sided limiter channel. Predistortion reduces the required ρ_Σ to 64.1 dB, which is 1.2 dB more than for the one-sided limiter channel. The overall result represents a 4.4-dB sensitivity improvement or an 175% increase in the permissible number of channels. This is not as large as the 5.3-dB improvement obtained for a one-sided limiter system.

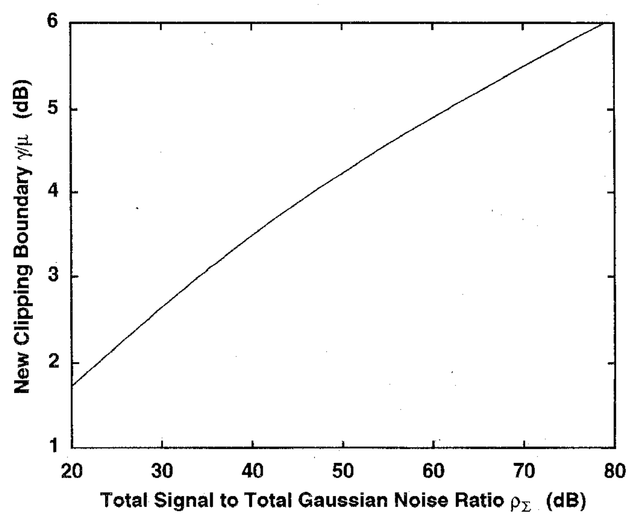

Fig. 8. New clipping boundary γ/μ as a function of total SNR ρ_Σ, for a two-sided limiter channel.

V. Discussion

The predistortion technique discussed here is perhaps applicable only to lightwave systems. Nonlinear distortion due to limiting can occur in conventional free-space or coaxial-cable communication systems. However, it might be difficult to apply the proposed technique in these systems, since predistortion broadens the signal spectrum, creating out-of-band signals that may represent interference, or may otherwise not be transmittable through the system. On the other hand, lightwave systems have available a huge bandwidth but limited power, so that transmission techniques with high power efficiency are important. The importance of power efficiency makes the predistortion method a promising technique to extend system performance.

The predistortion technique analyzed here yields a significant sensitivity improvement as compared to the system without predistortion. However, the sensitivity improvement may be over-estimated. The CNLD given by the modified Saleh formula (6) may over-estimate the clipping-induced NLD. Further analyzes and experiments [9], [14]–[16] have demonstrated smaller NLD and larger CNLD. These refined analyzes [9], [14]–[15] consider the frequency allocation of the input signal and are rather complicated to employ in numerical optimization. Table I summarizes the performance of the predistortion scheme as compared to systems without predistortion for a typical 42-channel AM-VSB system [9].

TABLE I
SUMMARY OF PREDISTORTION SCHEMES COMPARED TO THE RESULTS OF FRIGO MODEL, THE SENSITIVITY IMPROVEMENTS ARE 4.4 AND 3.2 DB FOR ONE- AND TWO-SIDED LIMITER CHANNELS, RESPECTIVELY. COMPARED TO THE RESULTS OF FRIGO MODEL, THE INCREASES ARE 275% AND 209% FOR ONE- AND TWO-SIDED LIMITER CHANNELS, RESPECTIVELY

Type of Channel	Scheme	Minimum ρ_Σ for ρ_t = 55 dB (dB)	Sensitivity Improvement (dB)	Relative Number of Channels
One-Sided Limiter	Saleh Model	68.2	0	100%
	Frigo Model	67.3	0.9	123%
	Optimal Predistortion	62.9	5.3†	340%‡
Two-Sided Limiter	Saleh Model	68.5	0	100%
	Frigo Model	67.3	1.2	131%
	Optimal Predistortion	64.1	4.4†	275%‡

†Compared to the results of Frigo Model, the sensitivity improvements are 4.4 and 3.2 dB for one- and two-sided limiter channels, respectively.
‡Compared to the results of Frigo Model, the increases are 275% and 209% for one- and two-sided limiter channels, respectively.

Various methods to analyze the system without predistortion are included. Using the most conservative results, predistortion still yields sensitivity improvements of 4.4 and 3.2 dB for one- and two-sided limiter channels, respectively. The models of Frigo [9], [17]–[18] and Shi [15] can be derived by a transform method [18], [19, Chap. 13] and by a direct method [19, Chap. 12] for analysis of nonlinear devices, respectively. It can be proved that the models of Frigo and Shi are accurate and identical [5], [20].

Future CATV systems will transmit compressed digital signals. It has been found that laser clipping induces impulse noise, which leads to a bit-error-rate (BER) floor in digital systems [21]–[24]. Because impulse noise cannot be modeled as a Gaussian process, the BER of a digital system cannot be predicted from knowledge of the CNR alone. In principle, the use of predistortion techniques can eliminate clipping-induced NLD, rendering the Gaussian model accurate, and allowing the BER to be predicted from the CNR alone. From [21]–[24], when clipping-induced NLD is eliminated, the RMS modulation index can be made 50%–100% larger. Therefore, in digital systems, predistortion can not only improve the system CNR, but can improve system performance by the elimination of impulse noise.

VI. CONCLUSION

The optimal predistortion transfer characteristics for clipped systems with Gaussian-distributed inputs have been found by using variational methods. The optimal predistortion curves of both one- and two-sided limiter channel can be expressed analytically, with the new clipping boundary as a parameter. For a system with a specific signal power, that new clipping boundary can be found by using numerical optimization to achieve the highest possible overall CNR per channel.

The predistortion technique is most useful in systems that require a high CNR per channel. For an AM-VSB system requiring overall CNR per channel $\rho_t = 55$ dB, the predistortion technique yields sensitivity improvements of 5.3 and 4.4 dB for one- and two-sided limiter channels, respectively.

APPENDIX A
POWER PENALTY OF GAUSSIAN NOISE

In this appendix, we derive (4) and (7) for the one-sided limiter channel in a more rigorous way. First, we rewrite (3) as

$$z(t) = g(i(t)) + n_o(t), n_o(t) = f(i(t)) \cdot n(t) \quad (22)$$

where

$$f(\xi) = \begin{cases} (dy/dx)^{-1}|_\xi & \xi \geq -\gamma \\ (dy/dx)^{-1}|_{-\gamma} & \xi < -\gamma. \end{cases} \quad (23)$$

Because the signal $i(t)$ and the noise $n(t)$ are independent, the autocorrelation function of the Gaussian noise at the system output $n_o(t)$ is

$$R_o(t+\tau, t) = \langle f(i(t+\tau))f(i(t))\rangle\langle n(t+\tau)n(t)\rangle \quad (24)$$

where $\langle\cdot\rangle$ denotes ensemble average. If the Gaussian noise is white, $R_n(\tau) = \langle n(t+\tau)n(t)\rangle = N_0\delta(\tau)$, and the output noise $n_o(t)$ is also white with autocorrelation function

$$R_o(t+\tau, t) = \langle [f(i(t))]^2\rangle N_0\delta(\tau). \quad (25)$$

Substitution of (23) into (25) yields (4).

Because white noise has infinite power, the derivation of (25) violates the assumption that the Gaussian noise is much smaller than the signal, under which (3) was derived. In practice, the input noise has a finite power much smaller than the signal power and is confined to a bandwidth which, in general, is larger than the signal bandwidth. We assume that the Gaussian noise is band limited to a bandwidth of W and

$$\phi_n(f) = \int_{-\infty}^{\infty} R_n(\tau)e^{-j2\pi f\tau}d\tau = \begin{cases} N_0 & |f| \leq W \\ 0 & |f| > W. \end{cases} \quad (26)$$

The autocorrelation function and power spectral density (PSD) of $f(i(t))$ can be determined [19, Chap. 12–13]. Assuming that the PSD of $f(i(t))$ is $\phi_f(f)$, the PSD of $n_o(t)$ is

$$\phi_o(f) = \int_{-W}^{W} \phi_n(f')\phi_f(f-f')df' \leq N_0 \int_{-\infty}^{\infty} \phi_f(f')df'. \tag{27}$$

Because $\int_{-\infty}^{\infty} \phi_f(f')df' = \langle [f(i(t))]^2 \rangle$, (4) provides the worst-case estimate of the output Gaussian noise, i.e., for $W \gg$ signal bandwidth. The same explanation can be applied to (14) for two-sided limiter channels.

Appendix B
Derivation of Predistortion Curve for One-Sided Limiter Channel

In this appendix, we find a function $y = y(x)$ to minimize (4) subject to the constraints (10) and

$$y(-\gamma) = -1. \tag{28}$$

The problem defined by (4), (10), and (28) is similar to the Bolza problem in optimal control [10], which is somewhat more complicated than the classical calculus of variations [13]. The Euler equation for this problem is $H_y - dH_{\dot{y}}/dx = 0$, where $H = p(x)/\dot{y}^2 + \lambda_1 y \cdot p(x)$ and λ_1 is a multiplier. Therefore, the Euler equation is

$$\lambda_1 p(x) + 2\frac{d}{dx}\frac{p(x)}{\dot{y}^3} = 0. \tag{29}$$

After some algebra, defining $\lambda = \sqrt[3]{\lambda_1/4}$, the solution to (29) is

$$y(x) = \frac{1}{\lambda}\int_{-\gamma}^{x} [p(x)/\mathrm{erfc}(x/\sqrt{2}\mu)]^{1/3} dx - 1 \tag{30}$$

where $y(x)$ has been made to satisfy (28) and the transversality condition at one of the boundaries $(x \to \infty)$, but the exact value of γ is still unknown.

Substituting (30) into (10), we can express λ as a function of γ

$$\lambda = \frac{1}{2}\int_{-\gamma}^{\infty} (p(x))^{1/3}[\mathrm{erfc}(x/(\sqrt{2}\mu))]^{2/3} dx. \tag{31}$$

If (31) is substituted into (30), after changing variables, we obtain (11) and (12).

From (11) and (12), the curve of $y(x)$ versus x/μ depends on γ and μ only through the ratio γ/μ. Fig. 9 displays κ of (12) as a function of γ/μ, which monotonically decreases as γ/μ increases, approaching the value 0.51 at large values of γ/μ.

At large values of γ/μ, the power penalty defined by (7) reaches an asymptotic limit. Substituting (11) and (12) with $\gamma/\mu \to \infty$ into (7), the asymptotic power penalty is

$$\delta_e = \frac{\mu^2}{8}\sqrt{\frac{2}{\pi}}\left(\int_{-\infty}^{\infty} e^{-t^2/6}[\mathrm{erfc}(t/\sqrt{2})]^{2/3} dt\right)^3 = 6.18 \times \mu^2. \tag{32}$$

If $\mu < 0.40, \delta_e < 1$, and the noise is smaller than the original noise. If $\gamma/\mu \to \infty, y(x) \geq -1$ for all x. Therefore, the predistorter output is always within the clipping limit, and no NLD will be induced into the system.

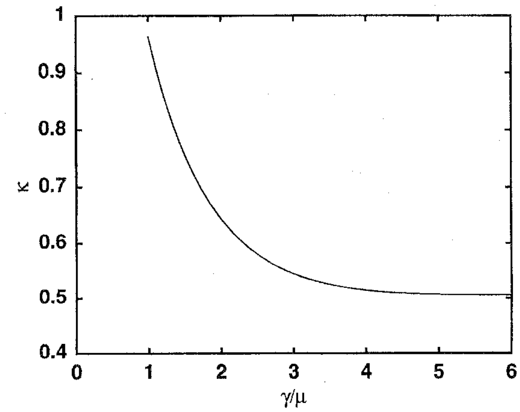

Fig. 9. Optimal predistortion curve parameter κ as a function of the ratio γ/μ.

The remainder of the problem is to find the value of γ that minimizes the noise (4). It is very difficult to apply the transversality condition directly. As an alternative, one can substitute (11) into (4), so that $N[y]$ becomes a function of γ, i.e., $N[y] = N(\gamma)$. By using numerical optimization, or by finding the root of the equation $dN(\gamma)/d\gamma = 0$, the value of γ that minimizes the noise can be found. In our calculations, we choose a value of γ to maximize the CNR given by (8), which is equivalent.

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Keang-Po Ho (S'92–M'95), for a photograph and biography, see this issue, p. 1443.

Joseph M. Kahn (M'87), for a photograph and biography, see this issue, p. 1443.