Achievable Rates of Space-Division Multiplexed Submarine Links Subject to Nonlinearities and Power Feed Constraints

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***Abstract*—We study the achievable rates of submarine fiber systems in the high-dimensional design space of variables including span length, launch power, number of spatial channels, and power feed current. We identify the regimes in which nonlinearities or power feed equipment constraints become dominant, and demonstrate that optimized system design evolves toward the linear regime as the system scales to a high number of spatial channels. We calculate the bit rate achievable by uniform and probabilistically shaped M-ary Quadrature Amplitude Modulation constellations to identify potential capacity-achieving implementations.**

***Index Terms*—Optical communications, power feed constraints, submarine transmission.**

I. Introduction

SUBMARINE fiber systems carry nearly 95% of the data traffic between different countries and continents [2]. The capability to support such massive traffic volume is enabled by space-division multiplexing (SDM). In current submarine systems, SDM is realized as a bundle of single-mode fibers (SMFs). To increase the spatial information density and enable compact scaling of components, multi-core and multi-mode fibers are of interest as alternatives for the future [3].

In long-haul systems, typically the bit rate is limited by amplified spontaneous emission (ASE) noise at low power levels, and by fiber nonlinear interference (NLI) at high power levels. Bit rate limitations due to these two effects have been extensively studied, and optimal operating regimes have been identified [4]. On the other hand, further limitations arise in modern submarine systems, as a large number of spatial channels are required to maximize the bit rate subject to power feed equipment (PFE) constraints.

Recent works on submarine optical transmission have addressed the design of optical fibers with low attenuation and large effective area [5], [6]. An example is the work by Makovejs *et al.* [7], [8], which proposes a silica-core fiber with 0.146 dB/km attenuation and 148 μm^2 effective area. While low attenuation substantially increases reach, a large effective area brings little gain to submarine links with ultra-long distances and a high number of spatial dimensions. These systems are limited by PFE constraints, and the available electric power is not enough to drive amplifiers at the desired optical power levels.

Capacity limits of submarine links subject to PFE constraints have been investigated in [9]–[11], in limited scenarios and for discrete link lengths. In submarine systems, the optimum system design may correspond to the linear regime, as described by Desbrulais in [11]. The paper proposed an analytic method to optimize the number of channels allowed by the system for a given link length. Bit rate calculations were based on a signal-to-noise ratio (SNR) threshold that includes ASE noise and Kerr nonlinearities. In [9], Pilipetskii *et al.* showed that in submarine systems with SDM, the optimum regime of operation corresponds to lower launch powers than in terrestrial systems. The work used the information-theoretical capacity as a figure of merit, but for limited scenarios and one-dimensional optimization. In [10], Maher *et al.* computed the mutual information of a submarine link subject to nonlinearities and PFE constraints, addressing potential gains of digital back propagation and probabilistic shaping. In [12], [13] and [14], achievable information rates of long-haul fiber-optic communications were investigated in different scenarios, but the impact of power feed limitations is not taken into account.

In [1], we applied multi-dimensional optimization to obtain the rate-maximizing system design, considering NLI and PFE constraints. Using numerical methods, we calculated the maximum bit rate attainable by different link lengths and operation

Manuscript received March 16, 2017; revised May 17, 2017; accepted June 20, 2017. Date of publication June 26, 2017; date of current version August 7, 2017. This paper was presented in part at the Optical Fiber Communication Conference, Los Angeles, CA, March 2017. Additional material include a deeper analysis of the solution and results concerning the achievable bit rates of different modulation formats. This work was supported in part by the FAPESP under Grant #2015/04382-0 and in part by the PIBIC-CNPq under Grant #129103/2016-0 at the University of Campinas, Campinas, Brazil. *(Corresponding author: Darli A. A. Mello.)*

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Digital Object Identifier 10.1109/JLT.2017.2720541

regimes. The present work extends the results of [1], providing a deeper analysis of the problem, and applying the same technique to different M-quadrature amplitude modulation (QAM) modulation formats, with uniform and shaped constellations.

The remainder of this paper is structured as follows. Section II presents the system model and explains the design limitations imposed by PFE constraints to submarine links. Section III provides a brief introduction to information theoretical concepts. Section IV explains the optimization procedure used to design the submarine system. Section V presents the simulation results. Section VI concludes the paper.

II. Electrical Power Aspects

Consider a submarine cable composed of n_{fp} standard single-mode fiber (SMF) pairs. Each SMF is assumed to support n_{ch} wavelength channels of equal rate, bandwidth and launch power. Signals are amplified by n_r repeaters using erbium-doped fiber amplifiers (EDFA). Each repeater has a power consumption, P_r, given by [11]:

$$P_r = \frac{2 n_{\text{fp}} n_{\text{ch}} P_{\text{ch}}}{\eta_c} + P_0 \tag{1}$$

where η_c is the electrical-to-optical power conversion efficiency, and P_{ch} is the channel launch power. Parameter P_0 is the extra overhead power consumption, which is not related to the optical amplification process.

Given its PFE constraints, a submarine cable has limitations on the power it can deliver to EDFAs to supply transmission. The power feed voltage, V_{PFE}, and P_r, are related by:

$$V_{\text{PFE}} = I R_c + \frac{n_r P_r}{I} \tag{2}$$

where I is the power feed current, and R_c is the cable resistance.

The PFE imposes voltage and current constraints, and thus limits the maximum transmission rate given other system parameters.

III. Information Theoretical Aspects

Consider an additive white Gaussian noise (AWGN) channel, where the input variable X is added to the noise random variable N, of zero mean and variance σ_N^2, yielding the output random variable $Y = X + N$. The maximal mutual information of this channel is expressed by Shannon capacity:

$$C = \log_2\left(1 + \frac{P}{\sigma_N^2}\right) = \log_2(1 + \text{SNR}) \tag{3}$$

where P is the upper bound on the average input signal power, i.e., $P \geq E[|X|^2]$ and $\text{SNR} = P/\sigma_N^2$ is the corresponding signal-to-noise-ratio. The capacity C is achieved for a continuous input random variable X with Gaussian distribution and $E[|X|^2] = P$.

Two important theoretical bounds of a communication system, the achievable bit rate, and the achievable spectral efficiency, can be directly obtained from the Shannon capacity, as $R_b^{\max} = C \cdot R_s$, and $\text{SE}^{\max} = C \cdot R_s / B_{\text{ch}}$, respectively, where R_{s} is the symbol rate, and B_{ch} is the channel bandwidth.

If X is discrete, the mutual information between X and Y, called the constrained capacity, C^*, is obtained as:

$$C^* = I(X,Y) = \sum_{S \in \mathcal{A}} \mathrm{P}_X(S) \int_{\mathbb{C}} \mathrm{p}_{Y|X}(y|S) \log_2\left(\frac{\mathrm{p}_{Y|X}(y|S)}{\mathrm{p}_Y(y)}\right) dy \tag{4}$$

where $\mathcal{A}$ is the finite alphabet of X (e.g. an M-QAM constellation), $\mathrm{P}_X(S)$ is the a priori probability of generating symbol S at the input, and $\mathrm{p}_Y(y)$ is the output probability density function. In an AWGN channel, the probability density function of output y, given input symbol S, $\mathrm{p}_{Y|X}(y|S)$, is described by a complex Gaussian distribution:

$$\mathrm{p}_{Y|X}(y|S) = \frac{1}{\pi \sigma_N^2} e^{-\frac{|y-S|^2}{\sigma_N^2}} \tag{5}$$

Although not explicit in (4), the constrained capacity C^* is also a function of the SNR. This can be seen by assuming that X has zero mean, and writing $Y = X + N = \sqrt{P}Z + \sigma_N U$, where Z and U are unit-variance random variables. Then, C^* can be written as:

$$C^* = I(X,Y) = I\left(\frac{X}{\sigma_N}, \frac{Y}{\sigma_N}\right) = I\left(\sqrt{\text{SNR}} \cdot Z, \sqrt{\text{SNR}} \cdot Z + U\right) \tag{6}$$

where the first expression is due to the invariance of the mutual information to scale [15]. For fixed probability distributions of Z and U, C^* is only a function of the SNR.

Shannon capacity constitutes a theoretical upper bound on the achievable mutual information. On the other hand, recent advances in modulation and coding techniques for optical systems demonstrate that the Shannon capacity can be approached in practice within a small gap [10], [16], [17].

In most communications systems with M-QAM constellations, $\mathrm{P}_X(S)$ is uniform. However, the mutual information with these constellations is maximized by the Maxwell-Boltzmann distribution [18]:

$$\mathrm{P}_X(S) = \frac{1}{\sum_{S \in \mathcal{A}} e^{-\lambda |S|^2}} e^{-\lambda |S|^2},\ \lambda \geq 0 \tag{7}$$

where λ is a parameter that needs to be optimized for each SNR level [19]. The technique of transmitting symbols with a nonuniform distribution to approach the Shannon capacity is known as probabilistic shaping (PS) [13], [20], [21]. Note that $C > C^*_{\text{PS}} > C^*_{\text{Uniform}}$, where C^*_{PS} and C^*_{Uniform} are the constrained capacities for the shaped and uniform constellations. The calculation of the mutual information can also be adapted to grasp the limitations of specific coding and decoding techniques [14].

IV. Optimizing Submarine Systems

We address the design of submarine links as an optimization problem with the following heuristic objective function:

$$\max_{n_r, n_{\text{ch}}, P_{\text{ch}}, I} \ 2 n_{\text{fp}} n_{\text{ch}} C' - p n_r \tag{8}$$

where $C' = C$, if an optimum input distribution is assumed, or $C' = C^*$, for specific discrete distributions. The first term of (8) maximizes the total mutual information conveyed unidirectionally over the cable (the factor 2 accounts for polarization multiplexing). Shannon capacity is an increasing function of the SNR, and the first term alone is enough to yield a single solution. Thus, in this case, we set $p = 0$. However, with M-QAM constellations, there are infinite values of SNR yielding practically the maximal constrained capacity $C^* = \log_2 M$. Consequently, there can exist several solutions achieving the optimal C^*, and term pn_r is added to select the solution that minimizes the number of repeaters. In our simulation scenario, setting $p = 0.1$ is enough to ensure that pn_r is always smaller than 1% of the first term.

There are PFE constraints on voltage and current. In addition, from an economic standpoint, the span length $L_s = L/(n_r + 1)$, where L is the total link length, should not be too short. Thus, the optimization problem is subject to the following constraints:

$$V_{\text{PFE}} - V_{\text{PFE}}^{\max} \leq 0 \tag{9}$$

$$I - I^{\max} \leq 0 \tag{10}$$

$$L_s^{\min} - L_s \leq 0 \tag{11}$$

$$n_{\text{ch}} - n_{\text{ch}}^{\max} \leq 0 \tag{12}$$

$$n_r \geq 1,\ n_{\text{ch}} \geq 1,\ P_{\text{ch}} \geq 0,\ I > 0 \tag{13}$$

where V_{PFE} is the feed voltage, I is the electrical current and n_{ch} the number of supported channels. The nonlinear SNR is given by [4], [22]:

$$\text{SNR} = \frac{P_{\text{ch}}}{R_s \left[n_s \left(F e^{2\alpha L_s} - 1 \right) h\nu + \frac{a n_s^{1+\epsilon} P_{\text{ch}}^3}{B_{\text{ch}}^3} \right]} \tag{14}$$

where α is the field attenuation constant, F is the amplifier noise figure, B_{ch} is the channel bandwidth, $n_s = n_r + 1$ is the number of spans, ϵ is the coherence factor [4], h is Planck's constant, and ν is the operating frequency. The NLI coefficient, a, is given by [11] [4]:

$$a = \frac{8}{27} \frac{\gamma^2 L_{\text{eff}}^2}{\pi |\beta_2| L_{\text{eff,a}}} \operatorname{asinh}\left(\frac{\pi^2}{2} |\beta_2| L_{\text{eff,a}} B_{\text{ch}}^2 n_{\text{ch}}^{2\frac{B_{\text{ch}}}{\Delta f}} \right) \tag{15}$$

where $\beta_2 = -\lambda^2 D/(2\pi c)$ is the dispersion parameter, $\gamma = 2\pi n_2/(\lambda A_{\text{eff}})$ is the nonlinear coefficient, L_{eff} is the effective length, $L_{\text{eff,a}} = 1/(2\alpha)$ is the asymptotic effective length, and Δf is the channel spacing.

We solve the submarine link design problem described in (8) using a global optimization package [23]. We consider three transmission scenarios:

1) The system is limited by nonlinearities and PFE constraints [problem (8) with all constraints (9) to (13)];
2) The system is only limited by nonlinearities [problem (8) with constraints (11), (12) and (13) only];
3) The system is only limited by PFE constraints [problem (8) with all constraints (9) to (13), and the NLI coefficient (15) set to zero].

We also estimate the data rates achievable by several uniform and shaped M-QAM modulation formats. In these cases, all constraints (9) to (13) have been taken into account.

TABLE I
SYSTEM PARAMETERS

Parameter	Symbol	Value
Attenuation coefficient	α	0.16 dB/km
Dispersion coefficient	D	20 ps/nm/km
Nonlinear effective area	A_{eff}	130 µm^2
Nonlinear coefficient	n_2	$2.6 \cdot 10^{-20}$ m^2/W
Noise figure	F	4.5 dB
Power conversion efficiency	η_c	1.5%
Cable resistance	r_o	1 Ω/km
Coherence factor	ϵ	0.07
Power overhead	P_0/P_r	10%
Channel bandwidth	B_{ch}	34 GHz
Channel spacing	Δf	37.5 GHz
Wavelength	λ	1550 nm
Symbol rate	R_s	34 Gsymbol/s
Maximum PFE voltage	$V_{\text{PFE}}^{\max}$	12 kV
Maximum PFE current	$I^{\max}$	1 A
Minimum span length	$L_s^{\min}$	40 km
Maximum number of channels	$n_{\text{ch}}^{\max}$	130

V. Simulation Results

The simulated system parameters are shown in Table I. In all scenarios and for all link lengths, the optimum number of channels obtained was $n_{\text{ch}} = n_{\text{ch}}^{\max} = 130$.

A. Optimization Based on the Shannon Capacity

Fig. 1 shows the maximum supported bit rate and spectral efficiency for a submarine system containing 6 unidirectional fiber pairs, which is a typical number in the current infrastructure. Shorter links are limited by nonlinearities, and PFE constraints become dominant after 7,000 km. Current submarine systems operate typically with Quadrature Phase Shift Keying (QPSK) modulation, which offers a maximal spectral efficiency of 4 bit/s/Hz/fiber, far below the 8 bit/s/Hz/fiber achievable by a submarine system around 8,000 km. Therefore, although PFE constraints do not impair current transmission systems, they may become a limiting factor in future upgrades to higher-order modulation formats.

Fig. 2(a)–(d) detail the design parameters of optimized submarine links. The optimum span length is shown in Fig. 2(a). In the range dominated by NLI, the optimization process selects the shortest possible link length allowed by the model (40 km). After the PFE constraint starts to influence the design, the span length increases up to 55 km. This has an impact in the number of repeaters, as depicted in Fig. 2(b). Although PFE constraints reduce the number of repeaters, which helps reduce capital expenditures, longer span lengths worsen signal quality and reduce the achievable bit rate. Fig. 2(c) shows the optimal channel power. At shorter links, the optimum launch power remained around −5 dBm. However, PFE constraints cause a significant reduction in per-channel launch power after 7,000 km. This effect can be also observed in Fig. 2(d), which shows the ratio of ASE to NLI powers in the link. Note that the PFE (red) curve does not appear because, in this case, the nonlinear power goes to zero. In NLI-limited systems, as in terrestrial networks, the optimum ratio, predicted by Gaussian noise model [4], [12], [24], is 3 dB. This is confirmed in the figure for distances below 6,000 km. For longer links, NLI drops quickly, as a consequence of reduced launch

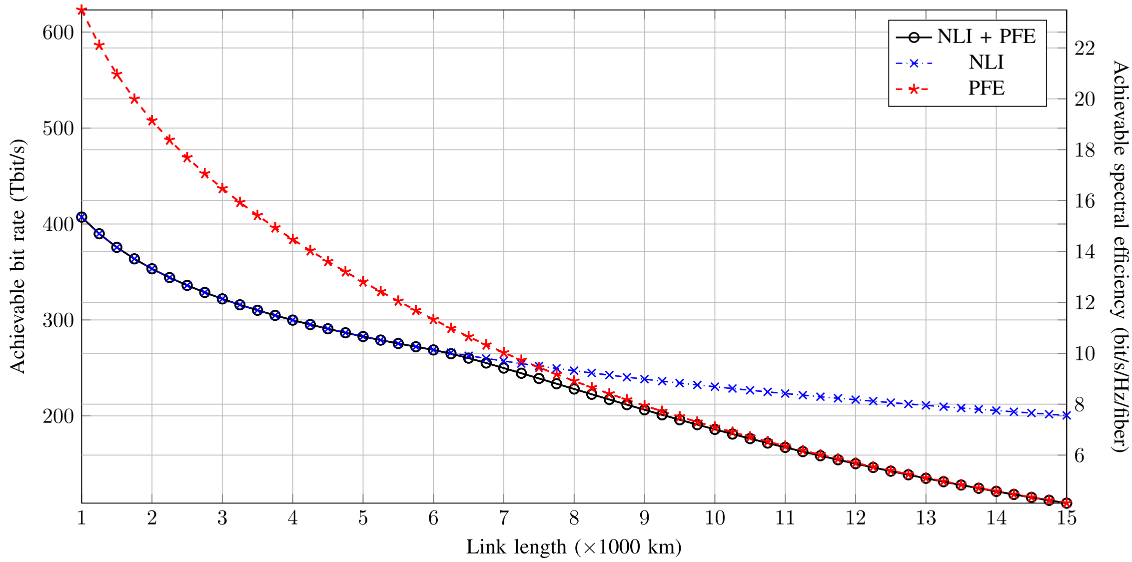

Fig. 1. Achievable bit rate (left) and achievable spectral efficiency (right) of a 6-fiber-pairs submarine link. NLI (blue): the influence of nonlinearities is considered and PFE constraints are neglected. PFE (red): PFE constraints are considered and the influence of nonlinearities is neglected. NLI+PFE (black): both PFE constraints and the influence of nonlinearities are considered.

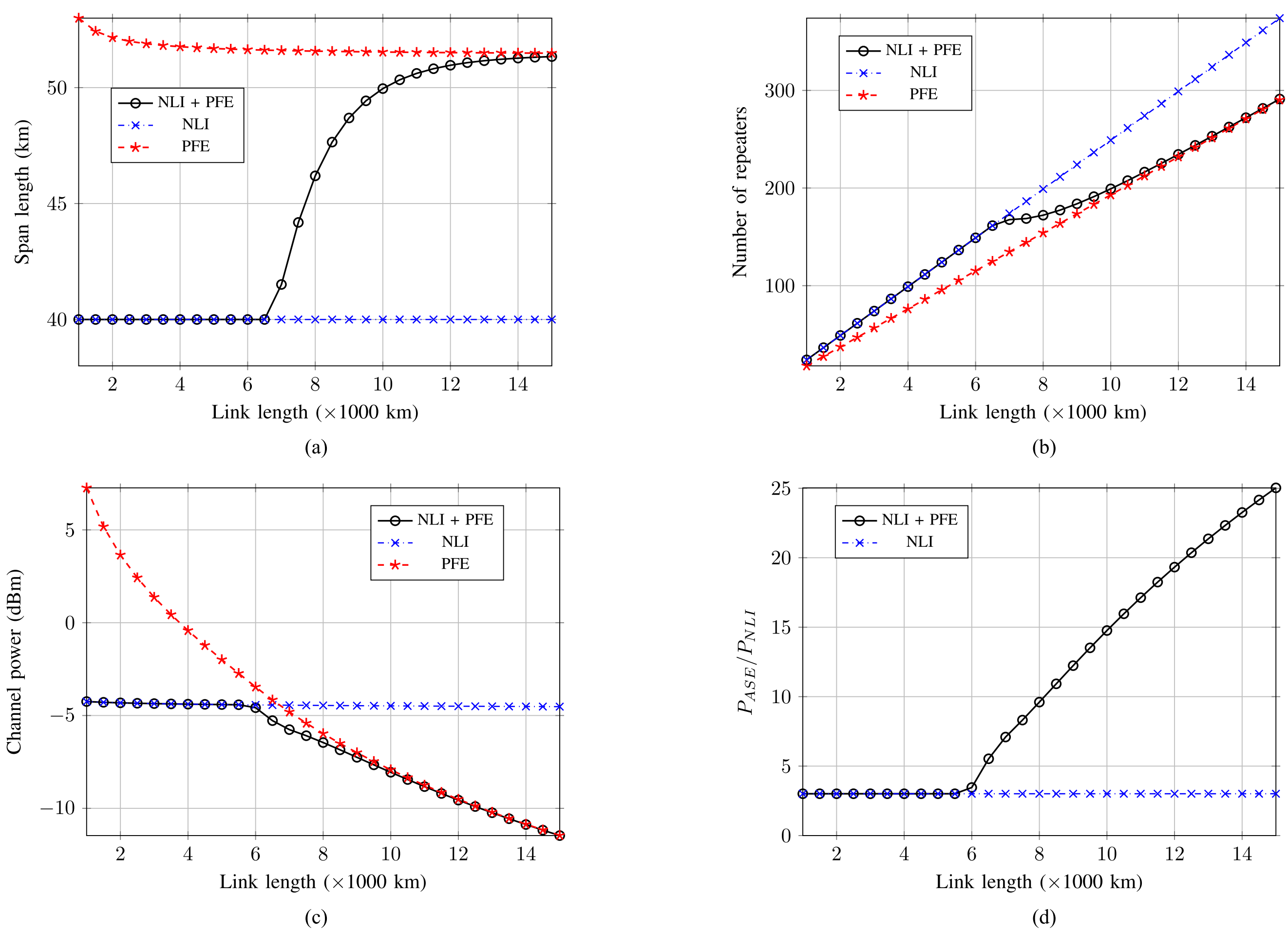

Fig. 2. Parameters of the optimum design of a submarine link containing 6 fiber pairs, assuming an optimum signal input distribution. NLI: the influence of nonlinearities is considered and PFE constraints are neglected. PFE: PFE constraints are considered and the influence of nonlinearities is neglected. NLI+PFE: both PFE constraints and the influence of nonlinearities are considered. (a) Span length. (b) Number of repeaters. (c) Channel power. (d) Ratio of ASE to NLI powers.

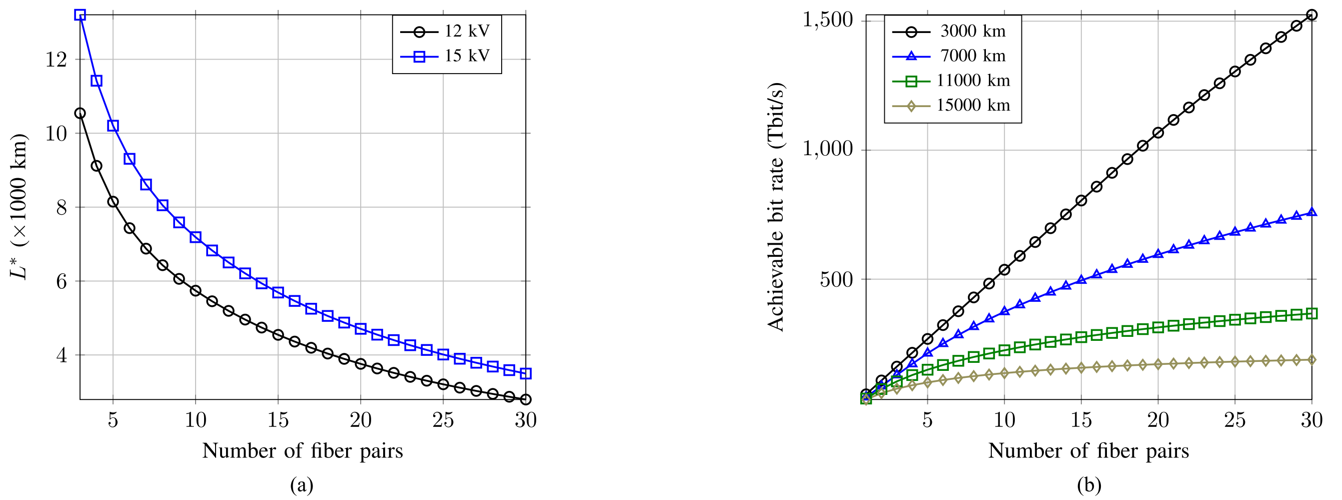

Fig. 3. Simulation results obtained for a varying number of fiber pairs. (a) Intersection between PFE and NLI curves (L^*) versus the number of fiber pairs. (b) Achievable bit rate versus number of fiber pairs.

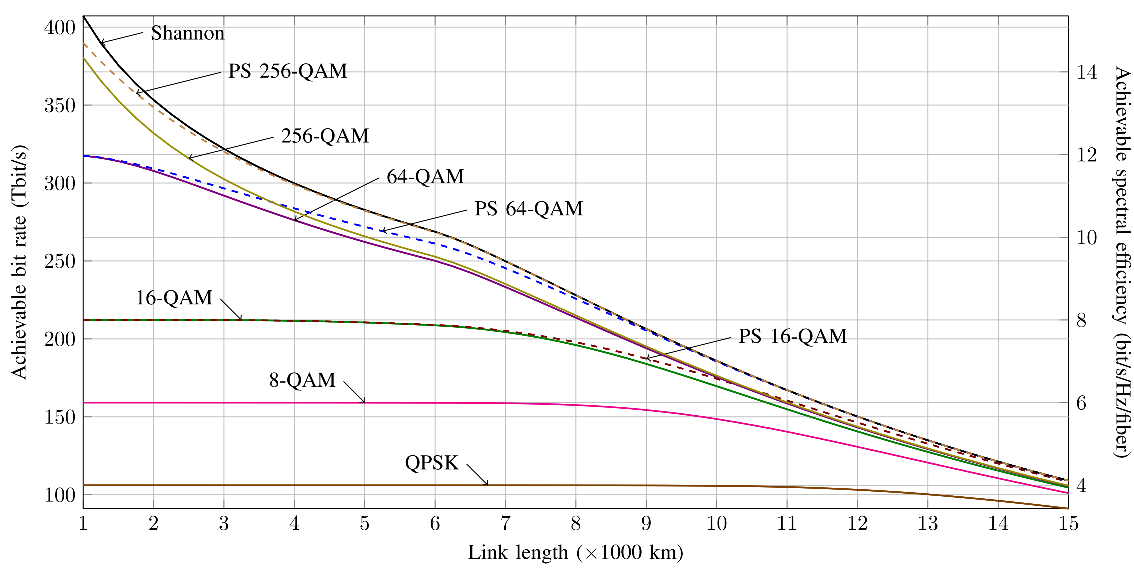

Fig. 4. Achievable bit rate and spectral efficiency of the optimum design of a submarine link containing 6 fiber pairs. In the figure, PS stands for probabilistic shaping.

power. This indicates that links longer than 6,000 km should operate in a linear regime so as to maximize the transmission rates. Note that, at these distances and operating conditions, large effective areas should bring a little gain to transparent reach.

The results obtained with the Shannon capacity can be scaled for a higher number of fiber pairs in the cable. Fig. 3(a) depicts the evolution of the intersection between the PFE and NLI (L^*) with the number of fiber pairs. While this intersection occurs at about 7,000 km for 6 fiber pairs and 12 kV feed voltage (as observed in Fig. 2), L^* can be as low as 4,000 km for cables with 18 fiber pairs. Although this number of fiber pairs is extremely high in comparison to current systems, the exponential increase in data traffic may make it typical in the long term. Relief of this problem may come from increases in feed voltages. For example, raising $V_{\max}$ to 15 kV increases L^* to 5,000 km. Fig. 3(b) shows the maximal unidirectional bit rate conveyed by a submarine cable as a function of the number of fiber pairs of the cable. For a 3,000 km link, the achievable bit rate increases almost linearly with the number of fibers up to 30 fiber pairs. For an 11,000 km link, however, increasing the number of fiber pairs from 10 to 30 increases the achievable bit rate by only 60%.

B. Achievable Bit Rate Optimization for M-QAM Modulation Formats

Fig. 4 shows the achievable bit rates for different modulation formats. Note that the Shannon capacity curve is the same as that presented in Fig. 1. For links longer than 8,000 km, the 64-QAM modulation format with PS is enough to approach the Shannon limit. It is interesting to observe that PS-64-QAM offers a reach gain of about 800 km over the 64-QAM, or even 256-QAM.

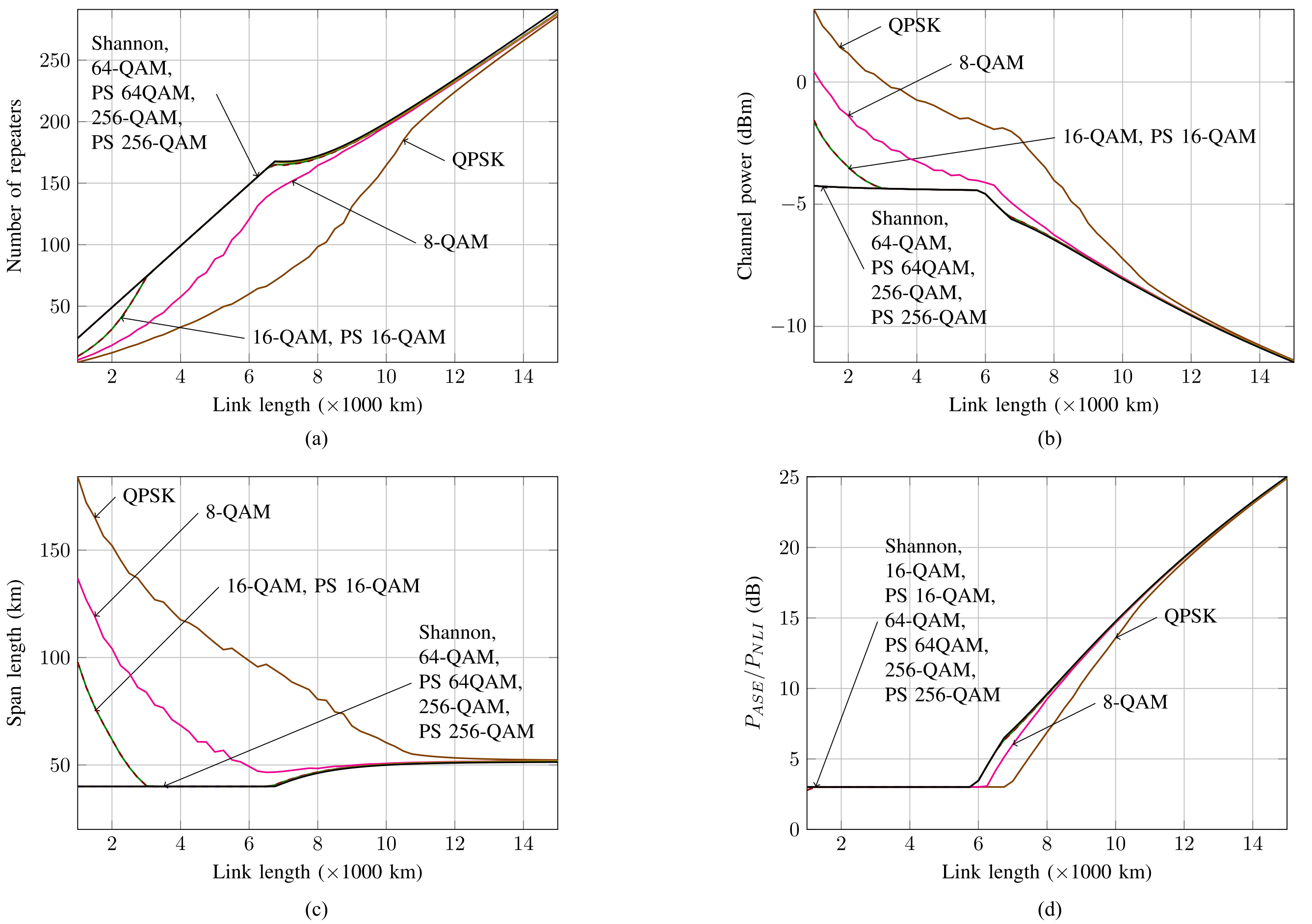

Fig. 5. Parameters of the optimum design of a submarine link containing 6 fiber pairs. In the figure, PS stands for probabilistic shaping. (a) Number of repeaters. (b) Channel power. (c) Span length. (d) Ratio of the ASE power to the NLI power.

Links shorter than 8,000 km, however, may have to go up to 256-QAM with PS to approach capacity. As an example, at 300 Tbit/s, and with 256-QAM modulation, PS increases the transparent reach from 3,000 km to 4,000 km, and approaches the Shannon limit. Fig. 5(a)–(d) present four design parameters involved in the submarine network design. In all four figures, the curves related to the Shannon limit, and the modulation formats 64-QAM and 265-QAM, with and without PS, coincide, because these modulation formats require an optimized SNR to operate. The lower-order modulation formats, however, allow a tradeoff between SNR and cost, enabling system designs with fewer repeaters, evidencing the role of term $-pn_r$ in (8). As an example, at 6,000 km, operating the system with QPSK modulation allows a repeater spacing as high as 100 km, instead of the SNR-maximizing value of 40 km, but prohibits upgrading the system to higher-order modulation formats, e.g. 8-QAM.

VI. Conclusion

The achievable bit rate of submarine systems is limited due to NLI and PFE constraints. The optimum regime of operation depends on the span length, launch power, number of spatial channels, and power feed current. Through numerical optimization, we have studied the achievable bit rates for practical uniform and shaped M-QAM constellations. The results for a submarine cable with 6 fiber pairs indicate that the transmission rate of links longer than 7,000 km is already limited by PFE constraints. Capacity analyses show that using PS-64-QAM transmission for links longer than 8,000 km, and PS-256-QAM for links between 1,000 km and 8,000 km, are sufficient to approach the Shannon limit. The results also demonstrate that uniform or shaped M-QAM constellations with $M \geq 64$ correspond to the same optimum link design.

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