Power Profile Optimization in Phase-Modulated Systems in Presence of Nonlinear Phase Noise

Alan Pak Tao Lau, *Student Member, IEEE*, and Joseph M. Kahn, *Fellow, IEEE*

***Abstract*—In optical fiber transmission systems using inline amplifiers, the interaction of signal and amplifier noise through the Kerr effect leads to nonlinear (NL) phase noise that impairs detection of phase-modulated signals. We study the minimization of the total phase noise variance (linear, NL, and receiver) by using a power profile along the link that is linear (on a decibel scale) with distance. For DPSK systems with direct detection, a total variance reduction of 65% can be achieved by designing the transmitted power to be about 8 dB higher than the received power in typical terrestrial links. In addition, for a given transmitted power and received power, the optimal power profile along the link is numerically determined.**

***Index Terms*—Optical fiber amplifier, optical Keer effect, phase noise.**

I. Introduction

OPTICAL fiber transmission systems using coherent or differentially coherent detection of phase-modulated signals are subject to impairment by phase noise. Amplified spontaneous emission (ASE) from inline amplifiers is a major source of phase noise, and is referred to here as *linear phase noise*. Gordon and Mollenauer [1] showed that signal and ASE can interact via the fiber Kerr nonlinearity to produce *nonlinear (NL) phase noise*. Thermal and shot noise generated at the receiver result in additional phase noise and will be referred to collectively as *receiver phase noise*. Since the NL phase noise increases with increasing signal power while linear phase noise and receiver phase noises decrease with signal power, it is possible to design signal power levels to optimize the trade-off between the various effects. This problem has been studied under various optimization criteria, such as Q factor, path-average power, mean NL phase shift, and optical signal-to-noise (SNR) ratio [2]–[5]. In particular, for *M*-ary PSK or DPSK systems, the phase noise statistics completely characterize system performance, and the variance of the total phase noise (linear plus NL plus receiver) is a good predictor of system performance. Therefore, in this letter, we study the design of transmit power $P_{\rm in}$, received power $P_{\rm rec}$ and the power profile along the link to minimize the variance of the total phase noise. Laser phase noise (from the transmitter and from the local osciallator present in some systems) is ignored here; at sufficiently low linewidth-to-bit rate ratios, it can have negligible impact in well-designed sytems. Note that designing the power profile along the link is equivalent to designing the gains of the inline amplifiers. We will focus on the case of distributed amplification where the incremental gain does not necessarily compensate for the incremental fiber loss, and the gains of all the amplifiers along the link can be varied independently. The common configuration with $P_{\rm in} = P_{\rm rec} = 0$ dBm with a power profile that is kept constant along the link will then be referred to as the *conventional distributed amplification profile* for the rest of the letter.

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The authors are with the Department of Electrical Engineering, Stanford University, Stanford, CA 94305 USA (e-mail: alanlau@stanford.edu; jmk@ee.stanford.edu).

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II. Power Profile Design for Phase Noise Minimization

We begin the analysis with an optical transmission system of fixed length L with N lumped amplifiers uniformly spaced along the fiber and take the limit $N \to \infty$ to obtain the analysis for distributed amplification. We assume that dispersion and multichannel effects are negligible in the system. For an inline amplifier located at l km from the transmitter where $0 \le l \le L$, let $A(l)$ and $P(l)$ denote the signal amplitude and power after the amplifier respectively. The linear phase noise at the receiver is given by

$$\phi_L = \arg\left(A(L) + \sum_{i=1}^{N} \frac{A(L)}{A\left(\frac{iL}{N}\right)} n\left(\frac{iL}{N}\right)\right) \quad (1)$$

where $n(iL/N) = x(iL/N) + jy(iL/N)$ is the ASE noises introduced by the amplifier located at $l = iL/N$. The x and y are independent identically distributed zero-mean Gaussian random variables with $(x(iL/N), y(iL/N)) \sim \mathcal{N}(0, \sigma^2(iL/N)\mathcal{I})$ and

$$\sigma^2\left(\frac{iL}{N}\right) = b\left(\frac{P\left(\frac{iL}{N}\right)}{P\left(\frac{(i-1)L}{N}\right)e^{-\alpha L/N}} - 1\right). \quad (2)$$

The constant $b = 2h\nu n_{\rm sp}\Delta\nu_{\rm opt}$, where $\Delta\nu_{\rm opt}$ is the bandwidth of an optical filter at the receiver, $n_{\rm sp}$ is the spontaneous emission factor, and ν is the signal frequency. If we let $\mathbf{x} = [x(L/N)\ x(2L/N)\ldots x(L)]$ and $\mathbf{y} = [y(L/N)\ y(2L/N)\ \ldots\ y(L)]$, the NL phase noise can be written as

$$\phi_{\rm NL} = \gamma L_e\left(\sum_{i=1}^{N} P\left(\frac{iL}{N}\right) + 2\mathbf{w}^T\mathbf{x} + \mathbf{x}^T C\mathbf{x} + \mathbf{y}^T C\mathbf{y}\right) \quad (3)$$

where γ is the NL coefficient, $L_e = (1 - e^{-\alpha L/N}/\alpha)$ is the effective length, and $\mathbf{w}$ and C are defined according to [6]. For DPSK systems with direct detection at high SNR, we denote the phase noise variance corresponding to thermal and shot noise from the receiver as

$$\sigma_{\text{rec}}^2 = \frac{1}{2}\frac{\sigma_T^2 + \sigma_s^2}{(R\Lambda P_{\text{rec}})^2} \tag{4}$$

where R is the responsivity of the receiver, σ_T^2 and σ_s^2 are the variances of the thermal and shot noises generated at the receiver, respectively, and Λ is an attenuation factor that takes into account the insertion loss of any demultiplexers placed before the receivers in the system. Note that for PSK systems, a local oscillator with power P_{LO} is present at the receiver. In this case, P_{rec} in (4) should be replaced by $\sqrt{P_{\text{rec}}P_{\text{LO}}}$. Also, the local oscillator generates additional shot noise at the receiver, resulting in a larger value of σ_s^2. For concreteness, we will focus on DPSK systems with direct detection in this letter, but our conclusions are all expected to be valid for PSK systems with coherent detection as well. Including σ_{rec}^2, the total phase noise variance σ_{tot}^2 is given by [6]

$$\begin{aligned}\sigma_{\text{tot}}^2 &= \sigma_L^2 + \sigma_{\text{NL}}^2 + \sigma_{\text{rec}}^2 \\ &= 4\gamma^2\left[\sum_{i=1}^{N} c_{ii}^2\sigma^4\left(\frac{iL}{N}\right)\right. \\ &\quad + 2\sum_{i,j,i>j} c_{ij}^2\sigma^2\left(\frac{iL}{N}\right)\sigma^2\left(\frac{jL}{N}\right) \\ &\quad \left.+\sum_{i=1}^{N}\mathbf{w}_i^2\sigma^2\left(\frac{iL}{N}\right)\right] + \frac{1}{2}\sum_{i=1}^{N}\frac{\sigma^2\left(\frac{iL}{N}\right)}{P\left(\frac{iL}{N}\right)} + \sigma_{\text{rec}}^2.\end{aligned} \tag{5}$$

If we define $T(l) = \int_l^L P(l')dl'$ and $\dot{P}(l) = dP/dl$, then as $N \to \infty$

$$\begin{aligned} L_e &\to dl, \\ \sigma^2\left(\frac{iL}{N}\right) &\to b\left(\frac{\dot{P}\left(\frac{iL}{N}\right)}{P\left(\frac{iL}{N}\right)} + \alpha\right)dl \\ c_{ii} &\to \frac{\int_{iL/N}^{L} P(l')dl'}{P\left(\frac{iL}{N}\right)} = \frac{T\left(\frac{iL}{N}\right)}{P\left(\frac{iL}{N}\right)} \\ c_{ij} &\to \frac{T\left(\frac{iL}{N}\right)}{\sqrt{P\left(\frac{iL}{N}\right)P\left(\frac{jL}{N}\right)}}, \quad i<j \\ \mathbf{w}_i^2 &\to T^2\left(\frac{iL}{N}\right)/P\left(\frac{iL}{N}\right).\end{aligned}$$

With summations replaced by integrals, the total phase noise variance given by (5) becomes

$$\begin{aligned}\sigma_{\text{tot}}^2 = 4\gamma^2 b^2 &\left[\int_0^L \frac{T^2(l)}{P^2(l)}\left(\frac{\dot{P}(l)}{P(l)} + \alpha\right)^2 dl\right. \\ &+ \int_0^L\int_{l_j}^L \frac{2T^2(l_i)}{P(l_i)P(l_j)}\left(\frac{\dot{P}(l_i)}{P(l_i)} + \alpha\right) \\ &\left.\times\left(\frac{\dot{P}(l_j)}{P(l_j)} + \alpha\right)dl_i dl_j\right] \\ + b&\int_0^L\left(\frac{\dot{P}(l)}{P(l)} + \alpha\right)\left(\frac{4\gamma^2T^2(l) + \frac{1}{2}}{P(l)}\right)dl \\ + \sigma_{\text{rec}}^2&.\end{aligned} \tag{6}$$

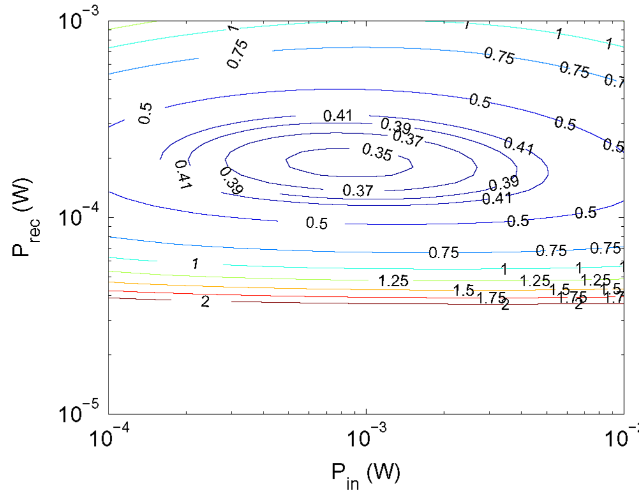

Fig. 1. Variance ratio of total linear and NL phase noise as a function of input and received powers for $L = 3000$ km, $\gamma = 1.2$ and $\alpha = 0.25$ dB/km.

Using (6), we can evaluate the total phase noise variance for any given P_{in} and P_{rec} with an arbitrary $P(l)$. For simplicity, we will focus on a class of power profiles in which $P(l)$ (on a decibel scale) is a linear function of l. An example of such a profile is shown in Fig. 2 and will be referred to as the *linear power profile* for the rest of the letter. For such a profile, $P(l)$ is given by

$$P(l) = P_{\text{in}}e^{\frac{\alpha(\hat{L}-L)}{L}l}, \quad P_{\text{rec}} = P(L) = P_{\text{in}}e^{\alpha(\hat{L}-L)}. \tag{7}$$

The quantity $\hat{L}$ can be thought of as the length of a fiber whose loss is exactly compensated by the total amplifier gain in the system. When $\hat{L} > L$, then the overall gain overcompensates the total fiber loss, and *vice versa*. Note that if $P_{\text{in}} = P_{\text{rec}}$, the system is simply the conventional distributed amplification profile and its performance has been briefly studied by Mecozzi [7]. With (7), $T(l)$ can also be evaluated accordingly. Defining $z = \alpha(\hat{L} - L)$, the total phase noise variance is given by

$$\begin{aligned}\sigma_{\text{tot}}^2 = &\frac{8\gamma^2 b^2\hat{L}^2L^2\alpha^2}{z^3}\left[\frac{e^{2z}-1}{2z} + 2\frac{e^z-1}{z} - 1 - 2e^z\right] \\ &+ \frac{4\gamma^2 bP_{\text{in}}\hat{L}L^2\alpha}{z^2}\left[\frac{e^{2z}-1}{z} - 2e^z\right] \\ &+ \frac{b\hat{L}\alpha(1-e^z)}{2P_{\text{in}}z} + \sigma_{\text{rec}}^2.\end{aligned} \tag{8}$$

Denote the variance of total phase noise in the conventional distributed amplification profile as $\tilde{\sigma}_{\text{tot}}^2$. For a typical terrestrial system with $L = 3000$ km and a receiver with sensitivity -19.5 dBm, responsivity $R = 0.85$ A/W and assuming a 6-dB insertion loss for demultiplexers, the ratio $\sigma_{\text{tot}}^2/\tilde{\sigma}_{\text{tot}}^2$ as a function of P_{in} and P_{rec} is shown in Fig. 1. The values on the contour curves represent the ratio values. From the figure, we deduce that a 65% reduction in total phase noise variance can be achieved by using an input power of about 0 dBm and received power of -8 dBm. The dependence of variance reduction on other system parameters has also been investigated, revealing that the reduction increases with NL coefficient γ and fiber length L. This is because ϕ_{NL} is a stronger function of signal power than ϕ_L, and ϕ_{NL} dominates as the system length

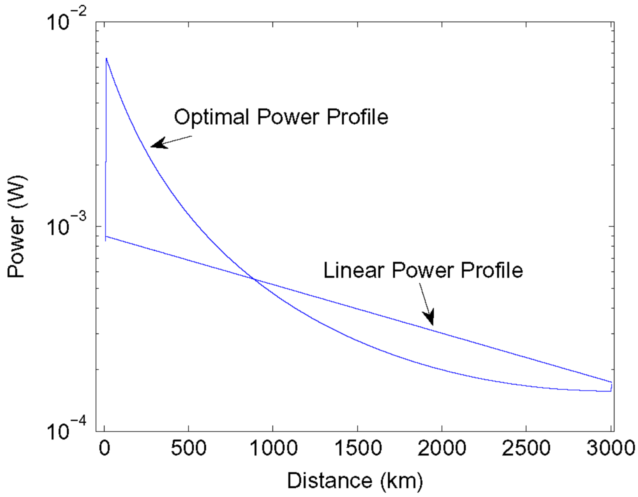

Fig. 2. Linear and optimal power profiles for $L = 3000$ km. The path-averaged powers are -3.5 and -1.5 dBm for the linear profile and the optimal profile, respectively.

increases [8], [9]. In all cases, the transmit power should be about 6–10 dB higher than the received power in order to minimize the total phase noise variance. In a practical setting with lumped amplifiers uniformly spaced along the link, the linear power profile can be approximated by a scheme in which the signal loss per span is constant. For an amplifier spacing of 50 km using the optimal P_{in} and P_{rec} obtained above, the resulting total phase noise variance is 20% lower than the case with $P_{\text{in}} = P_{\text{rec}} = 0$ dBm with per-span loss compensation. Finally, it should be noted that with a power profile that drops from the transmitter to receiver, the amount of pump power required for the inline amplifiers will also be reduced.

III. Optimal Power Profile for Phase Noise Minimization

In the previous section, we studied the total phase noise variance reduction achieved by using the optimal P_{in} and P_{rec} with a linear power profile. However, for a given P_{in} and P_{rec}, the linear power profile studied above does not necessarily result in the minimum σ^2_{tot}. It is of interesting to determine the optimal power profile $P^*(l)$ and the corresponding minimum total phase noise variance σ^{2*}_{tot}. To formulate this optimization problem, we will make the approximation that the first two integral terms in (6) are negligible compared to the third term. This is because the first two terms correspond to the beating of ASE noise with itself, and are typically orders of magnitude smaller than the third term, which corresponds to the beating of signal with ASE noise [9]. Using this insight and rewriting the total phase noise variance in terms of $T(l)$ only, we obtain

$$\begin{aligned}\sigma^2_{\text{tot}} &= b\int_0^L \frac{[\ddot{T}(l)/\dot{T}(l)+\alpha][4\gamma^2T^2(l)+\frac{1}{2}]}{-\dot{T}(l)}dl \\ &= \int_0^L F(T,\dot{T},\ddot{T})dl \qquad (9)\end{aligned}$$

with $T(L) = 0$, $\dot{T}(L) = -P_{\text{rec}}$, and $\ddot{T}(0) = -P_{\text{in}}$. In this formulation, σ^2_{tot} is a functional of $T(l)$. Therefore, finding the optimal $T^*(l)$, and hence $P^*(l)$, that minimizes σ^2_{tot} is a standard variational calculus problem. To find $T^*(l)$, one can either numerically solve the characteristic Euler [10]

$$\frac{\partial F}{\partial T} - \frac{d}{dl}\frac{\partial F}{\partial \dot{T}} + \frac{d^2}{dl^2}\frac{\partial F}{\partial \ddot{T}} = 0 \qquad (10)$$

or use a polynomial approximation of $T(l) = \sum_{p=0} t_p l^p$ and optimize the set of coefficients $\{t_1, t_2, t_3, \ldots\}$. Another method is to use discrete approximation of distributed amplification and optimize the gains of the inline lumped amplifiers to find the optimal power profile. Using the optimal values of P_{in} and P_{rec} found from the linear power profiles, we have found the optimal power profile, which is shown in Fig. 2. A variance reduction of 75% is obtained using the optimal profile, compared to 65% obtained using the linear profile (as stated previously), indicating that the linear profile is close to optimal. The path-averaged powers are -1.5 and -3.5 dBm for the optimal profile and the linear profile, respectively. The higher power of the optimal profile may excite NL effects not considered in our analysis.

IV. Discussion

We have shown that in a phase modulated system, the signal power profile from transmitter to receiver can be effectively designed to minimize the variance of total linear, NL, and receiver phase noise. For the set of *linear power profiles* in a terrestrial link with typical system parameters, it is shown that a transmit power of about 0 dBm and received power of -8 dBm results in a 65% reduction in total phase noise variance compared to typical operating power levels. In addition, for a given transmitted and received power, the problem of finding the optimal power profile for total phase noise variance minimization has been formulated and numerically solved. The interplay of the Kerr effect and chromatic dispersion introduces additional phase noise through "intrachannel cross-phase modulation" and "intrachannel four-wave mixing" [6]. The impact of these effects, and of a higher path-averaged power in the optimal profile, will be investigated in future work.

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