Time Domain Lapped Transform and its Applications

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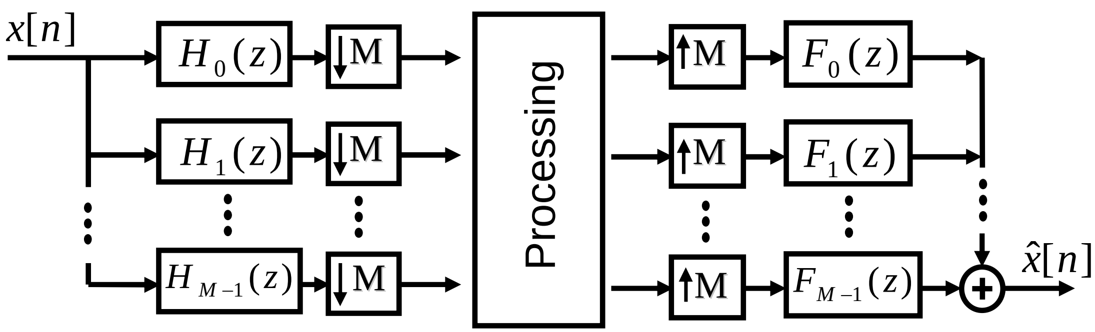
Outline

- Introduction
  - From filter bank to time-domain lapped transform (TDLT)
- Fast TDLT
- Pre/post-filtering for 2D and 3D Wavelet transform
- Error Resilient TDLT
- Current Works
- Summary

Filter Bank Fundamental

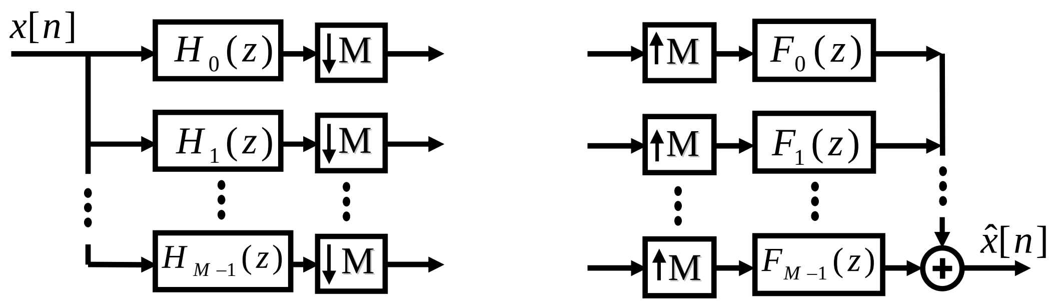


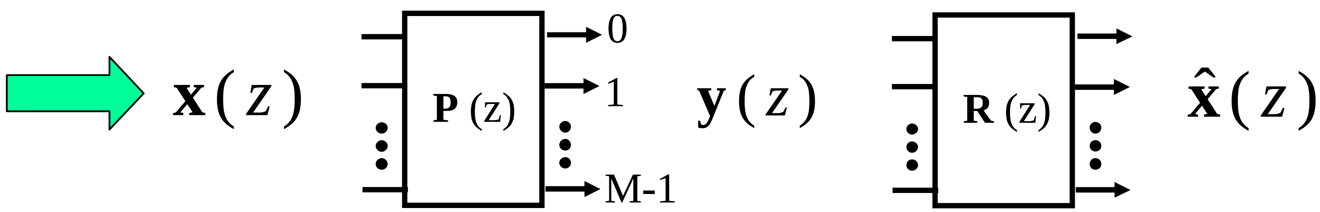
$$\begin{bmatrix} \vdots \\ \mathbf{y}_m \\ \mathbf{y}_{m+1} \\ \mathbf{y}_{m+2} \\ \vdots \end{bmatrix} = \begin{bmatrix} \ddots & \ddots & & & & & \\ 0 & \mathbf{P}_{K-1} & \cdots & \mathbf{P}_1 & \mathbf{P}_0 & 0 & \\ & 0 & \mathbf{P}_{K-1} & \cdots & \mathbf{P}_1 & \mathbf{P}_0 & 0 \\ & & 0 & \mathbf{P}_{K-1} & \cdots & \mathbf{P}_1 & \mathbf{P}_0 \\ & & & & \ddots & \ddots & \ddots \end{bmatrix} \begin{bmatrix} \vdots \\ \mathbf{x}_m \\ \mathbf{x}_{m+1} \\ \mathbf{x}_{m+2} \\ \vdots \end{bmatrix}$$

$$\mathbf{y}_m = \mathbf{P}_0\,\mathbf{x}_m + \mathbf{P}_1\,\mathbf{x}_{m-1} + \ldots + \mathbf{P}_{K-1}\,\mathbf{x}_{m-K+1}$$

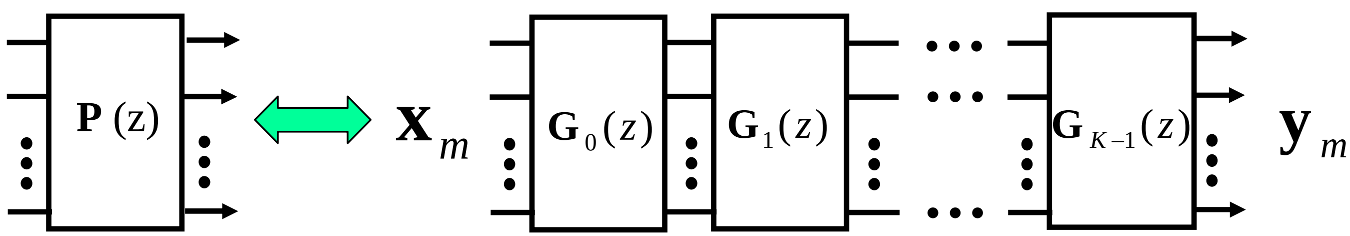
$$\mathbf{Y}(z) = \left(\mathbf{P}_0 + \mathbf{P}_1\,z^{-1} + \ldots + \mathbf{P}_{K-1}\,z^{-K+1}\right)\mathbf{X}(z) = \mathbf{P}(z)\,\mathbf{X}(z)$$

Filter Bank Fundamental



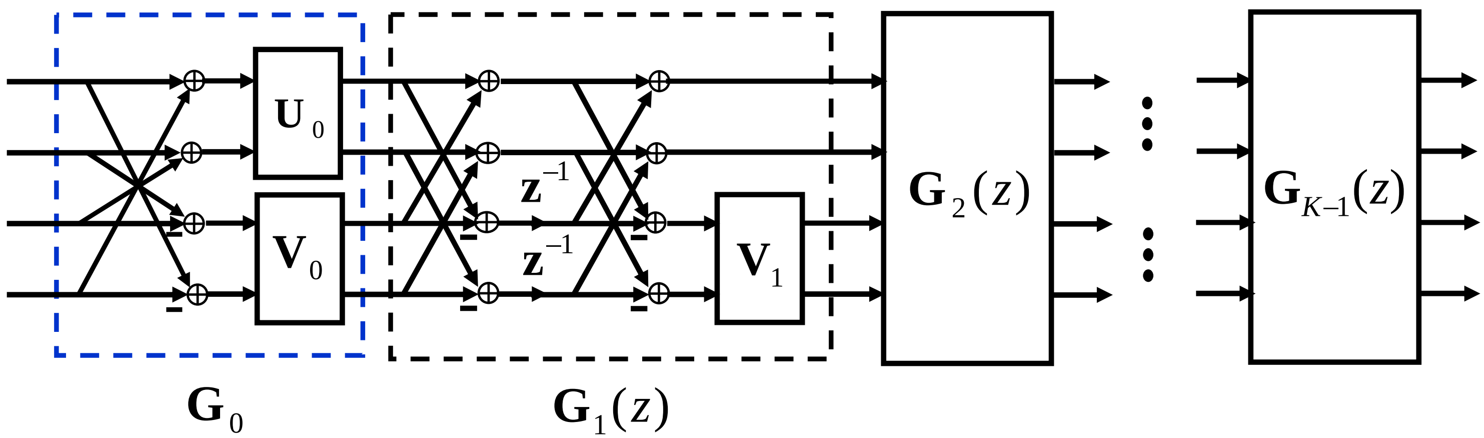


- Perfect Reconstruction: $\mathbf{R}(z)\,\mathbf{P}(z) = \mathbf{I}$, or $\mathbf{R}(z) = \mathbf{P}^{-1}(z)$.
- Fast Implementation: Factorization of $\mathbf{P}(z)$



Linear Phase Filter Banks

- Linear phase:
  - Desired property for image/video coding
- General structure [Vaidyanathan93, Gao01, Gan01]

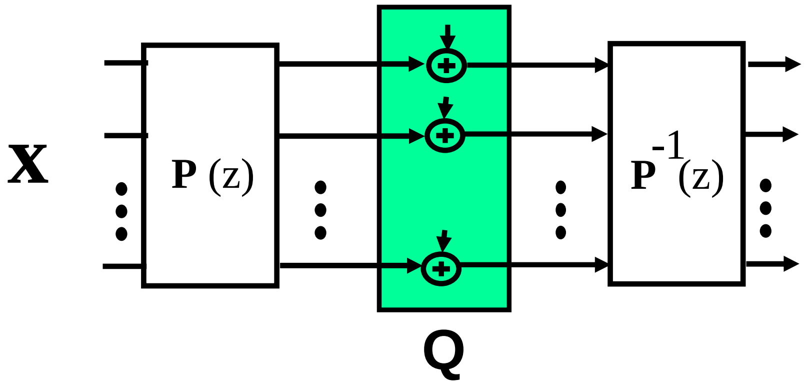


$\mathbf{U}_i, \mathbf{V}_i$: Invertible matrices.

Can be optimized for different applications.

Rate-Distortion Optimization

- Objective: Design the filter bank to minimize the MSE for a given bit rate.



$$\hat{\mathbf{x}} = \mathbf{x} + \mathbf{e}$$

$$\delta_{\mathbf{P}}^2 = \text{trace}\left\{\mathbf{R}_{\mathbf{ee}}\right\}$$

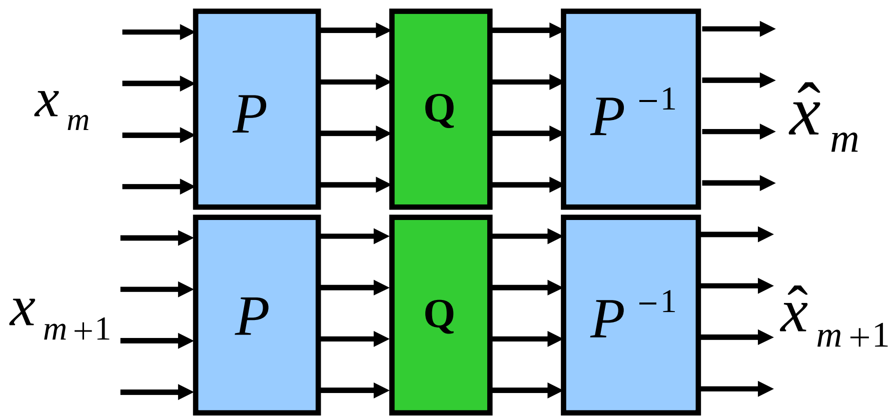
- Design Criterion: Coding Gain

MSE reduction of transform coding w.r.t. PCM

$$\gamma = 10\log_{10}\left(\frac{\sigma^2_{PCM}}{\sigma^2_{\mathrm{P}}}\right)$$

A Special Case: Block Transform

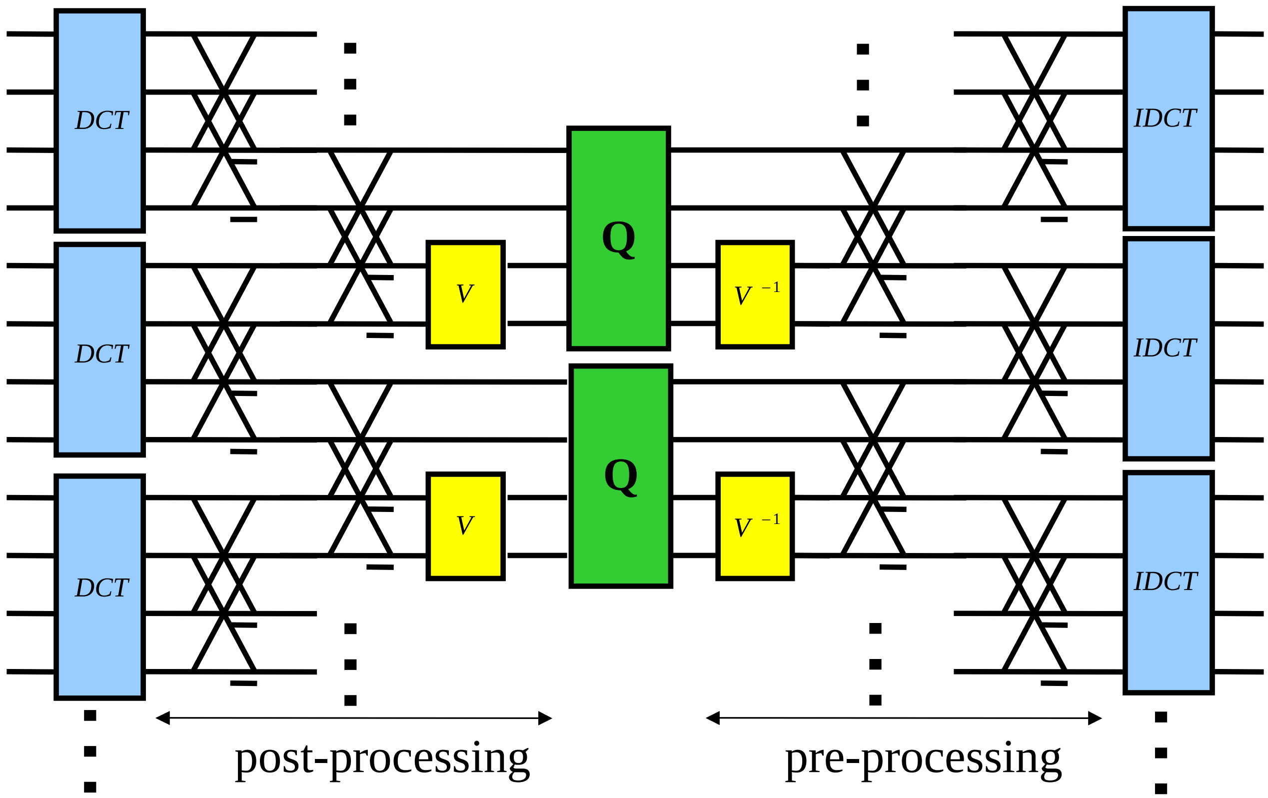
$$\mathbf{P}(z) \Rightarrow \mathbf{P}$$



- Karehunen-Loeve Transform (KLT):
  - Optimal block transform
  - Signal dependent, No fast algorithm
- Discrete Cosine Transform (DCT):
  - Fast approx. of the KLT for AR(1) signals.
- Drawbacks of block transform:
  - Blocking artifact
  - Limited compression capability

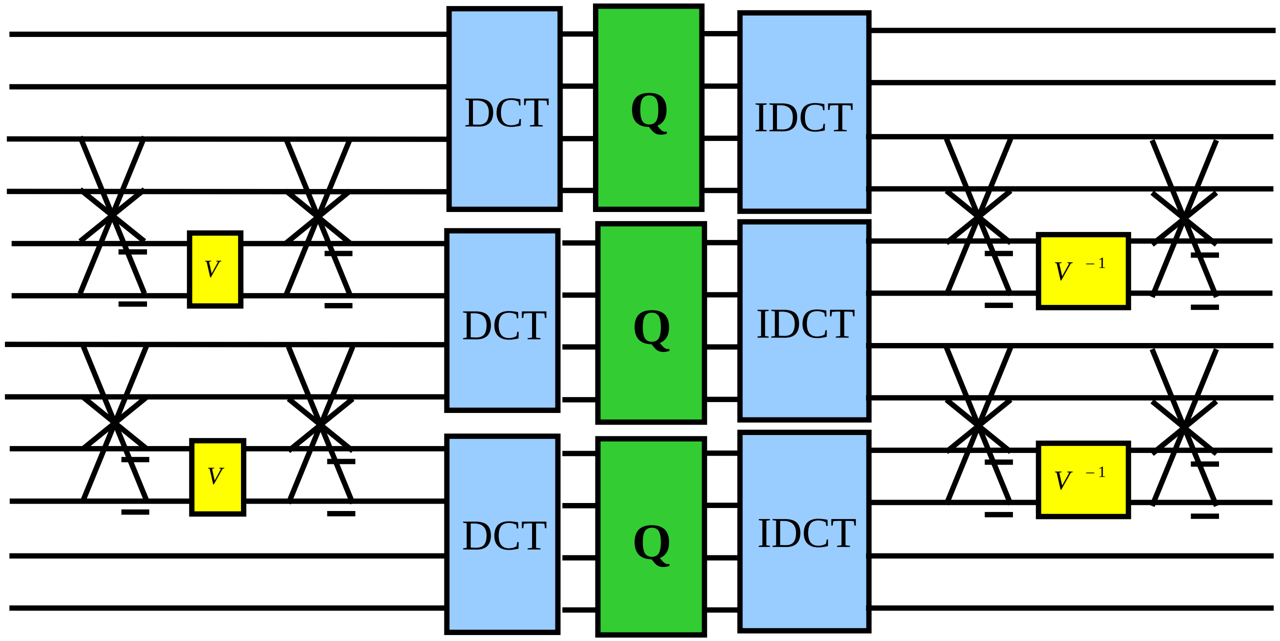
Lapped Transform [Malvar *et al*. 1985]

- Apply post-processing of the DCT to
  - Improve compression efficiency and reduce blocking artifact
- A special case of linear phase filter banks



Time-Domain Lapped Transform

- [Tran-2001]



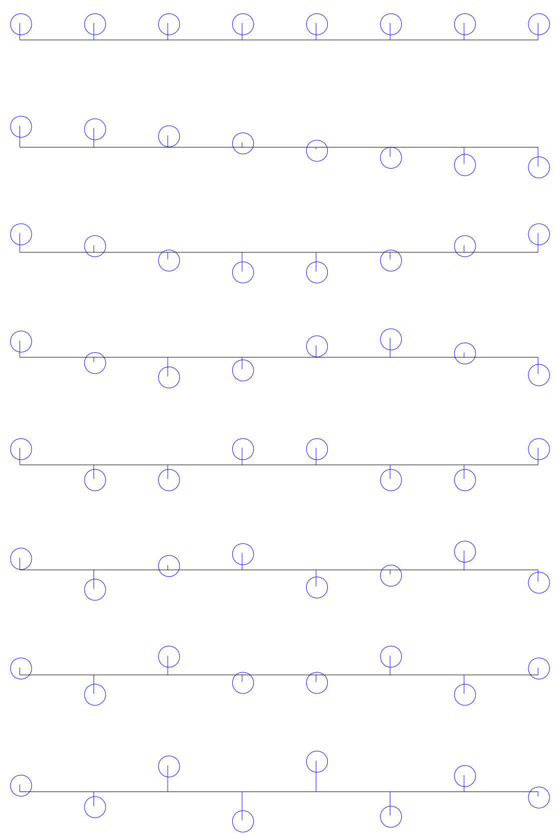
- More compatible to DCT-based schemes
- Also a special case of linear phase filter banks
- Adopted by MS WMV-9, SMPTE VC-1, HD-DVD.

Effect of Prefiltering

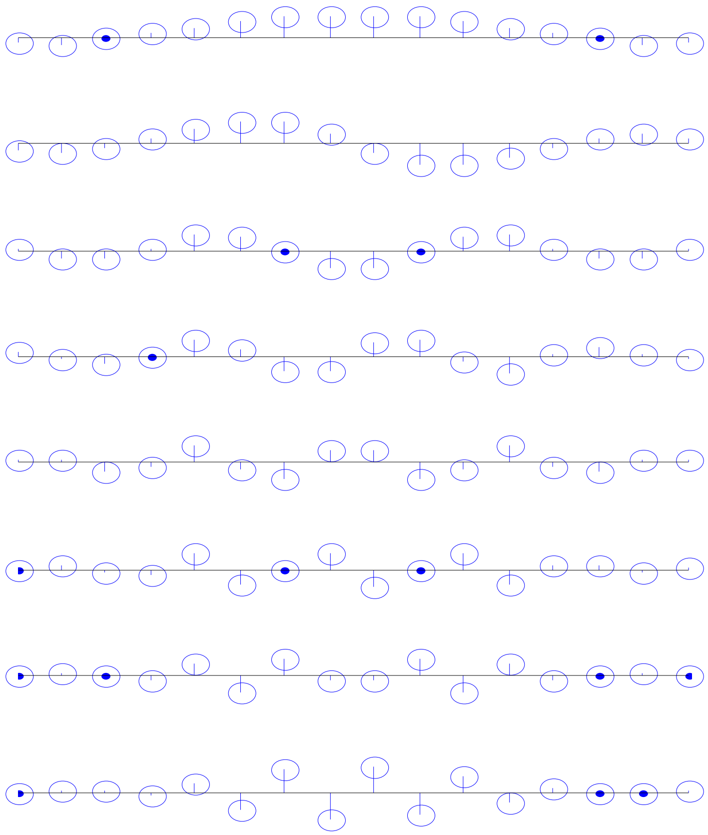
- A flattened image

DCT vs LT: Basis Functions

8-point DCT



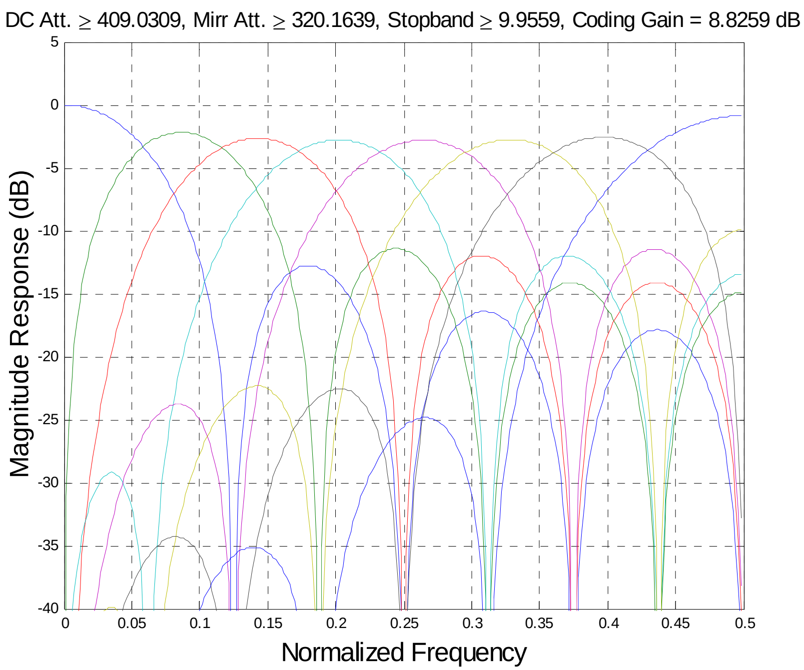
8 x 16 TDLT



Frequency Responses

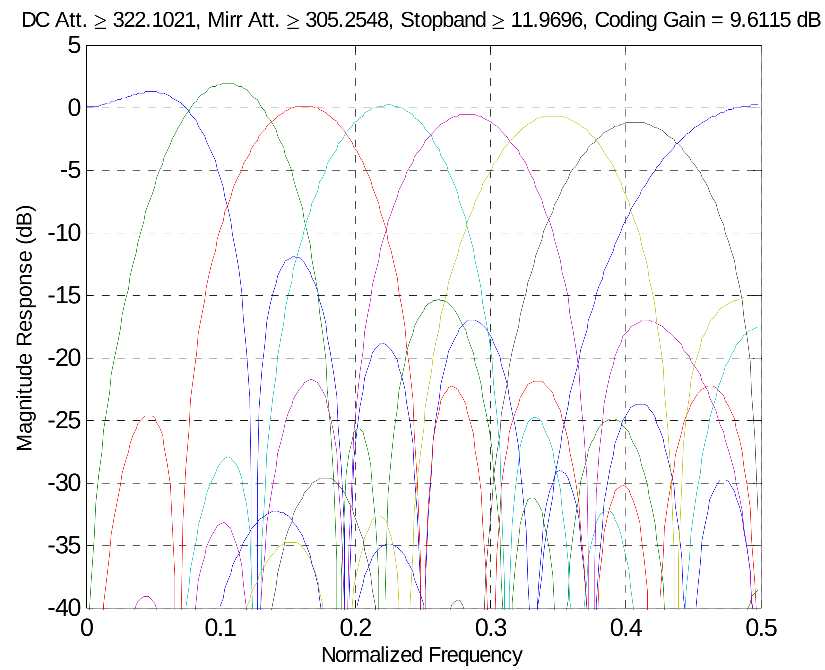
8-point DCT

8.83 dB



8 x 16 TDLT

9.61 dB



Applications & Generalizations

- Fast TDLT [Liang et al. 2001]
- Pre/Post-filtering for Wavelet [Liang et al. 2003]
- Error Resilient TDLT [Tu et al. 2002, Liang *et al.* 2005]
- Generalized Lapped Transform [Liang et al. 2002]
- Adaptive Entropy Coding for TDLT [Tu et al. 2001]
- Oversampled TDLT [Gan-Ma-2002]
- Undersampled TDLT [Tu et al. 2004]
- Regularity Constrained TDLT [Dai et al. 2001]
- Adaptive TDLT [Dai et al. 2005]

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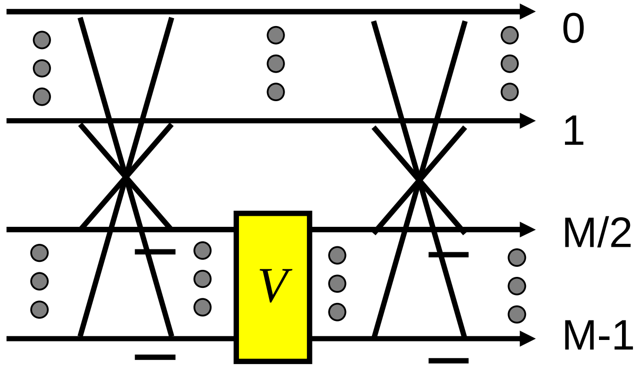
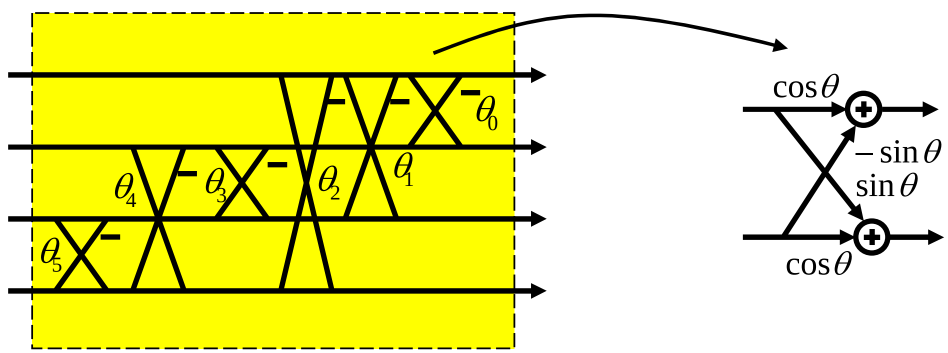
Fast Orthogonal TDLT

M x M Prefilter:

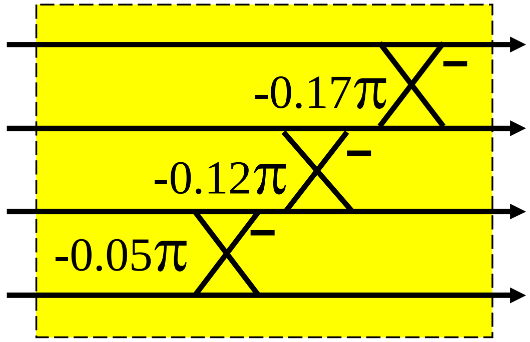


- $\mathbf{V}^{-1} = \mathbf{V}^{T}$
- General structure of **V**:



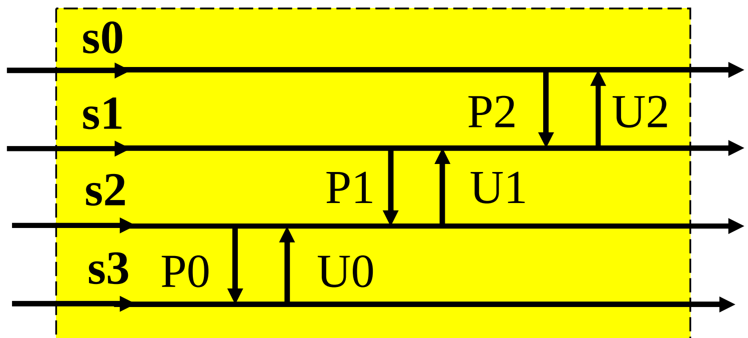
Fast Orthogonal TDLT

- Quasi-optimal coding gain [Liang-Tran-Tu-01]:
  - 9.26 dB for M = 8
  - Close to optimal filter bank
- Can be generalized to large block size (e.g., 128)



Fast Biorthogonal TDLT

- $\mathbf{V}^{-1} \neq \mathbf{V}^{T}$: More freedoms, better performance
- Fast approximation (lifting steps, LU factorization):



- Integer Solutions: > 0.3 dB higher than orthogonal TDLT

S0	S1	S2	S3	P0	U0	P1	U1	P2	U2	**Gain**
4/3	8/7	8/7	8/7	-1/16	1 / 4	-1/4	1 / 2	-3/8	3 / 4	9.59
3/2	9/8	9/8	9/8	-1/16	1 / 4	-1/4	1 / 2	-3/8	3 / 4	9.58
1	1	1	1	0	1 / 4	-1/4	1 / 2	-1 / 2	3 / 4	9.37

Image Coding Performance

- TDLT vs Wavelet
  - Both coded by SPIHT [Said, Pearlman, 1996]
  - (Improved entropy coding in [Tu, Tran, 2001])

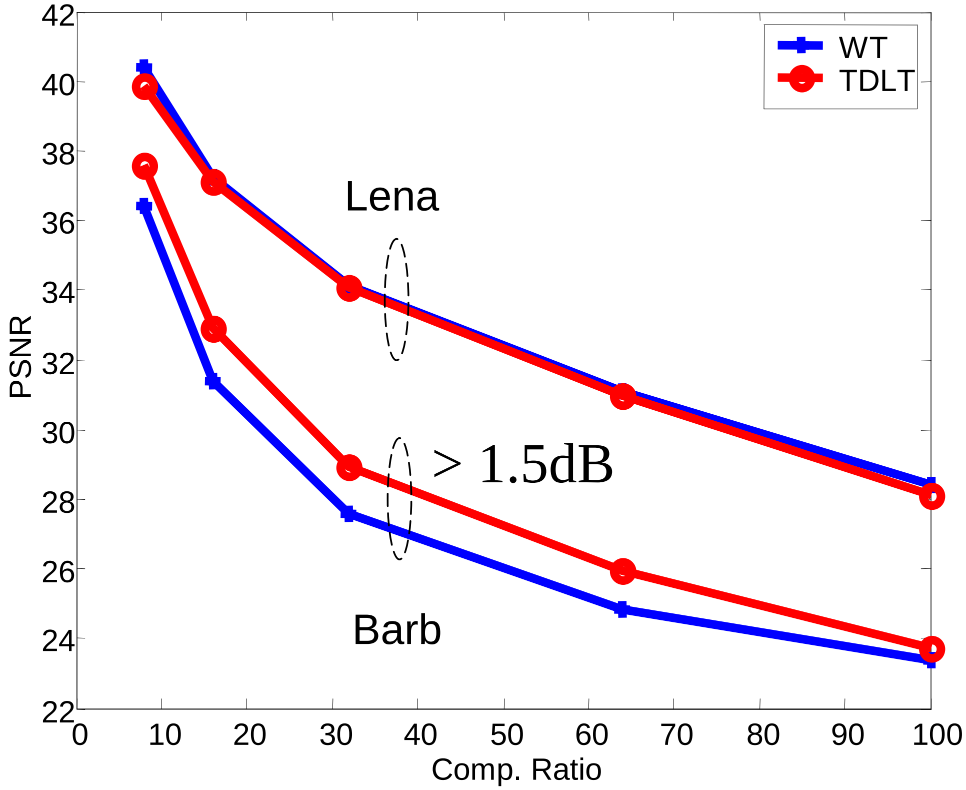


Image Coding Performance

- WT 32:1: 27.58 dB
- TDLT 32:1: 28.95 dB

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JPEG 2000 & Tiling Artifact

- No blocking artifact if WT is applied to the entire image
- Used by JPEG 2000

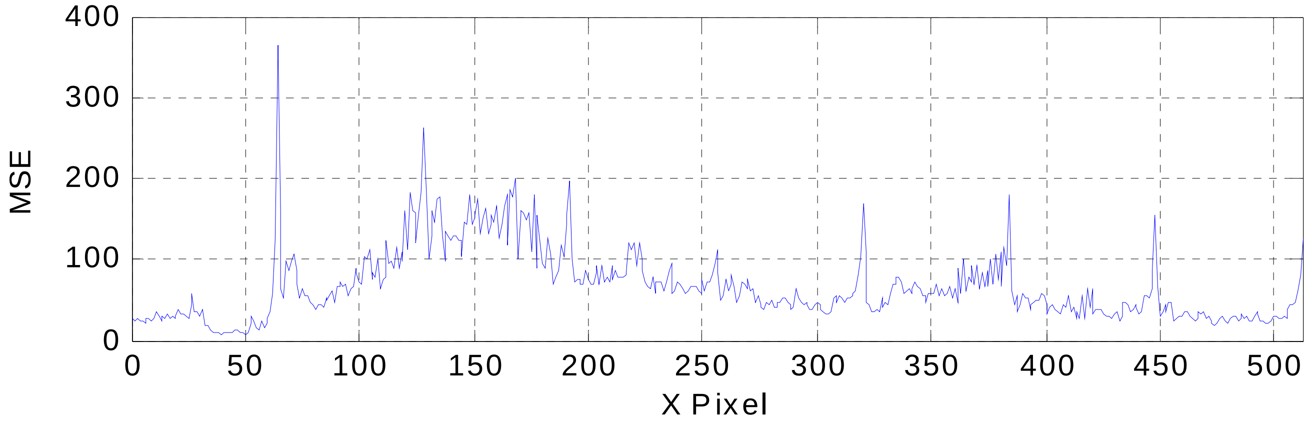
- Problem:
  Memory requirement
- Tradeoff:
  Tiling approach

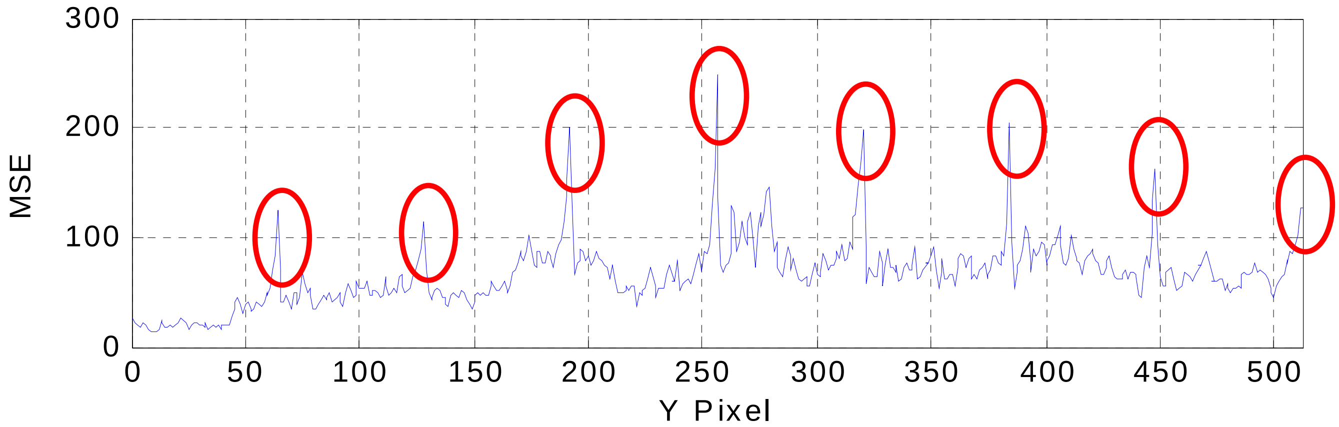
→ Tiling Artifact

Tile size: 64 x 64, 0.2bpp

Average MSE of All Rows & Columns

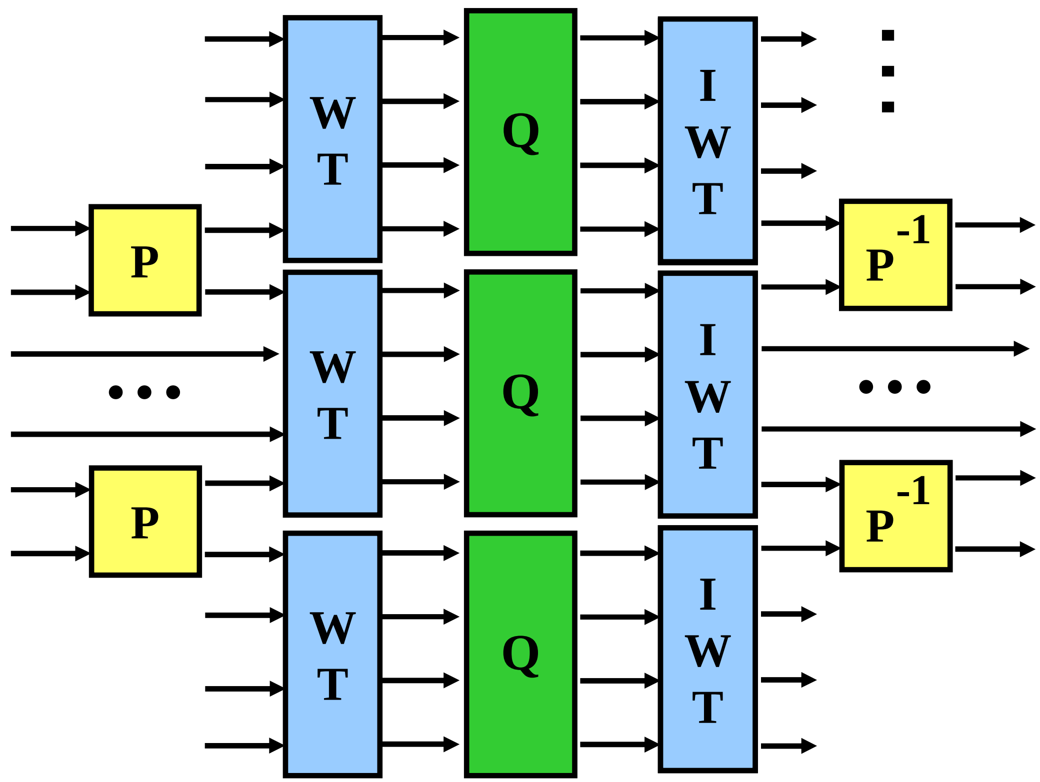
- MSE is more than doubled at tile boundaries





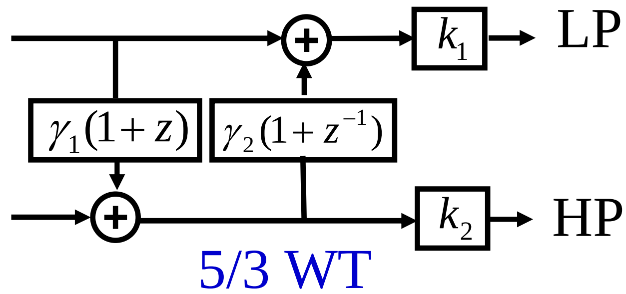
Pre/post-filtering for WT

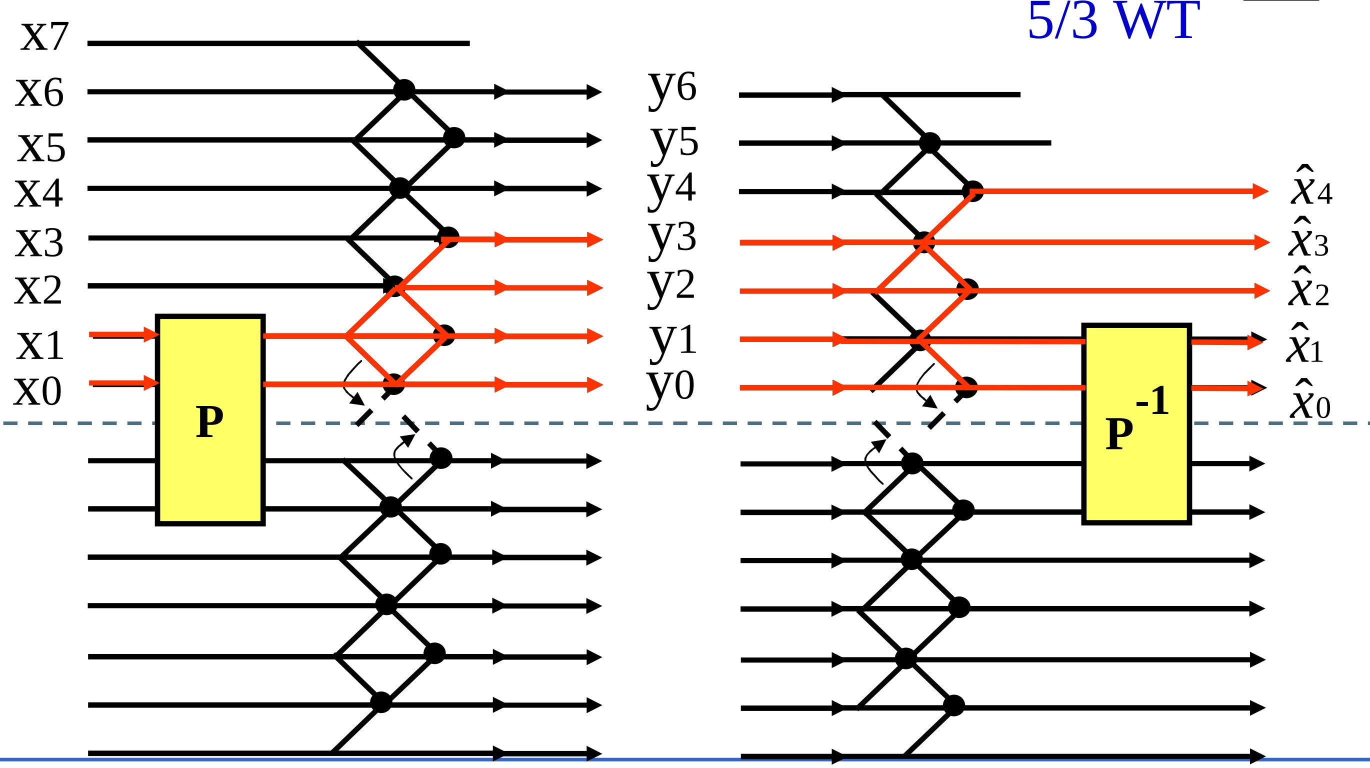
- Apply small pre/post filters at tile boundaries



Problem Formulation

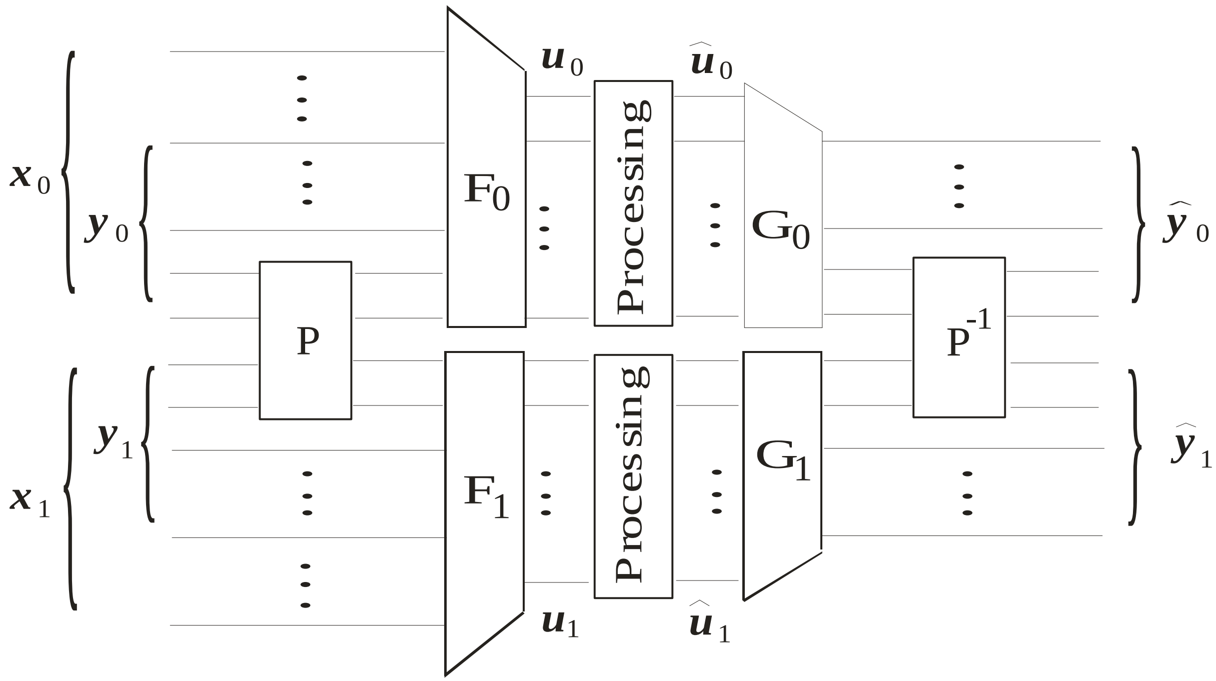
- Effect of pre/post-filters:
  - In LT: affect all subbands
  - In WT: affect some subbands





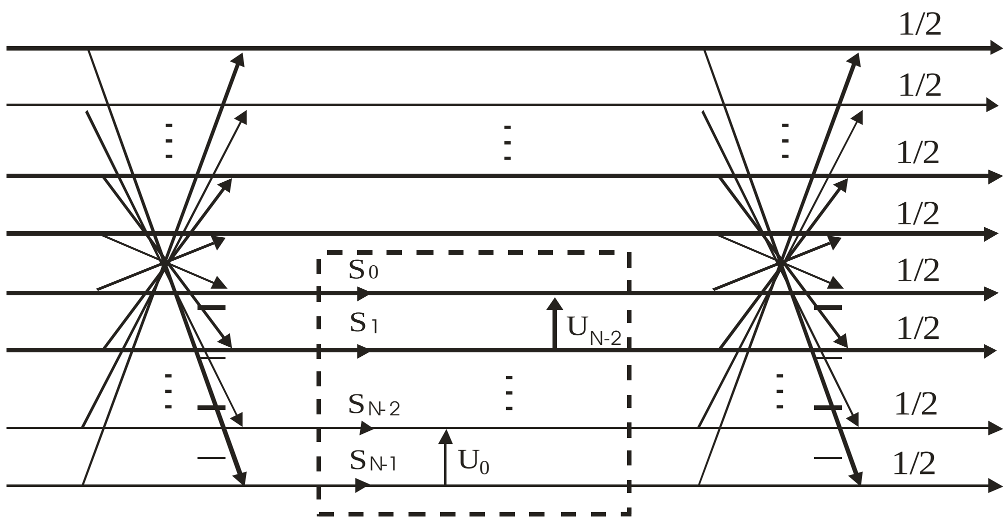
Problem Formulation

- Boundary Filter Bank
- Optimization can be performed similar to LT



Fast Structure

- Optimal pre/post-filters for WT and DCT are similar



- Examples:

S = [5/3, 4/3, 6/5, 9/8], U = [1/8, 1/4, 5/8],

S = [2, 1, 1, 1], U = [1/8, 1/4, 1/2], (lossless)

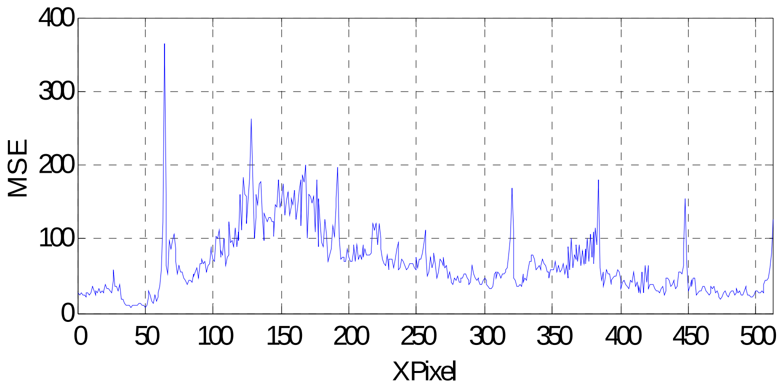
Examples

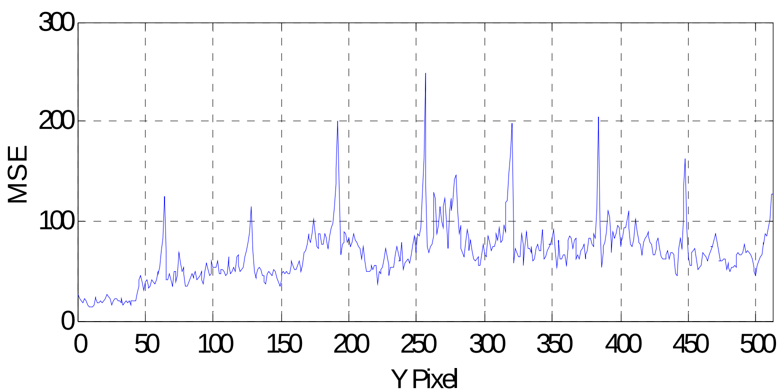
- 9/7 WT, 8 x 8 Pre/Post Filters: 0.2bits/pixel
- JPEG 2000 (Kakadu) JPEG 2000 & Pre/Post

29.87 dB 29.97 dB

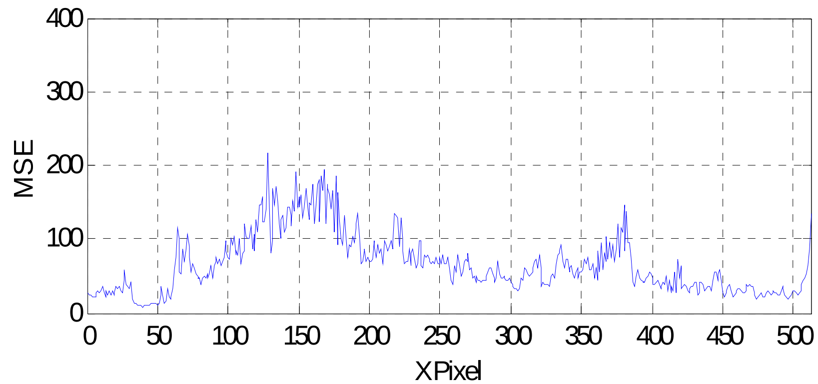
Average MSE of All Rows and Columns

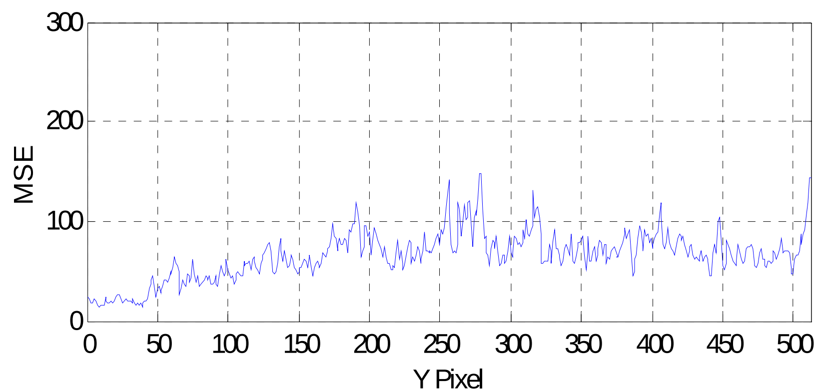
JPEG 2000 (Kakadu)





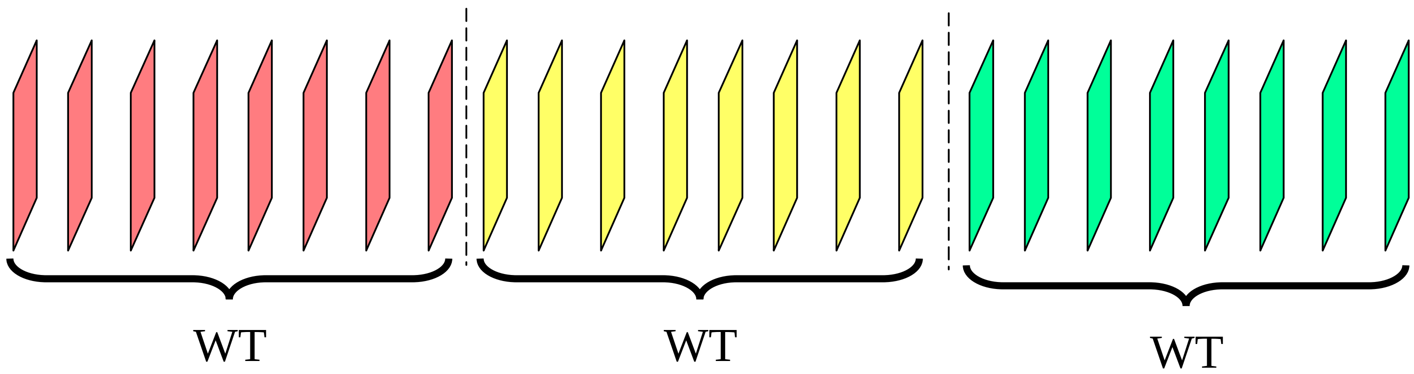
JPEG 2000 & Pre/Post





Applications in 3D WT Video Coding

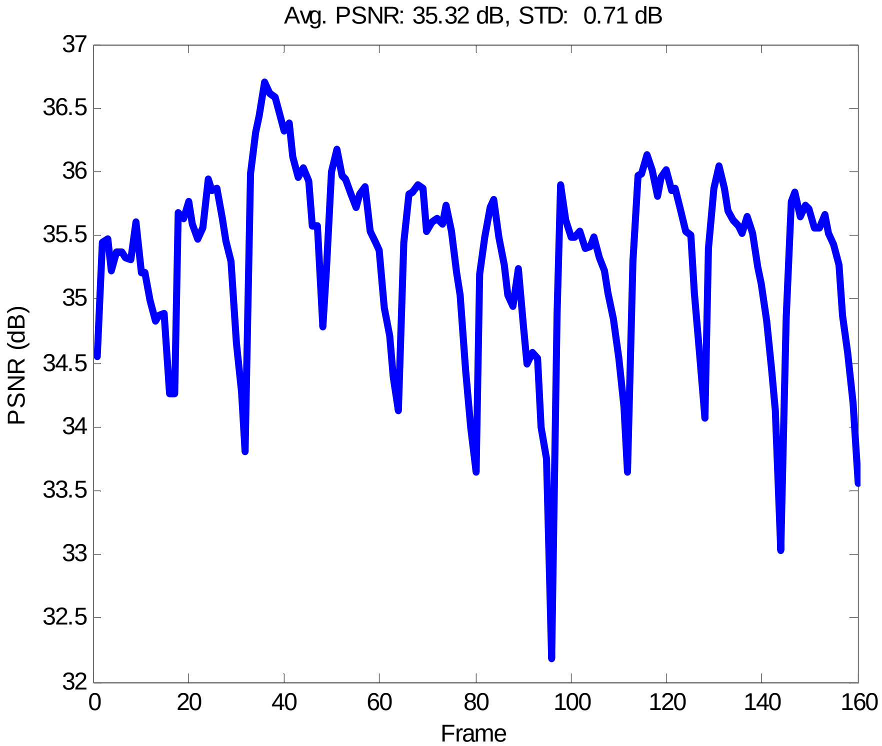
- Divide video sequence into groups of frames
- Apply 2D WT within each frame
- Apply another 1D WT in temporal direction



- Advantages:
  - ☑ Lower complexity (no motion estimation)
  - ☑ Full scalabilities: SNR, resolution, and frame rate.

Jittering Artifact

- Performance degradation at group boundaries



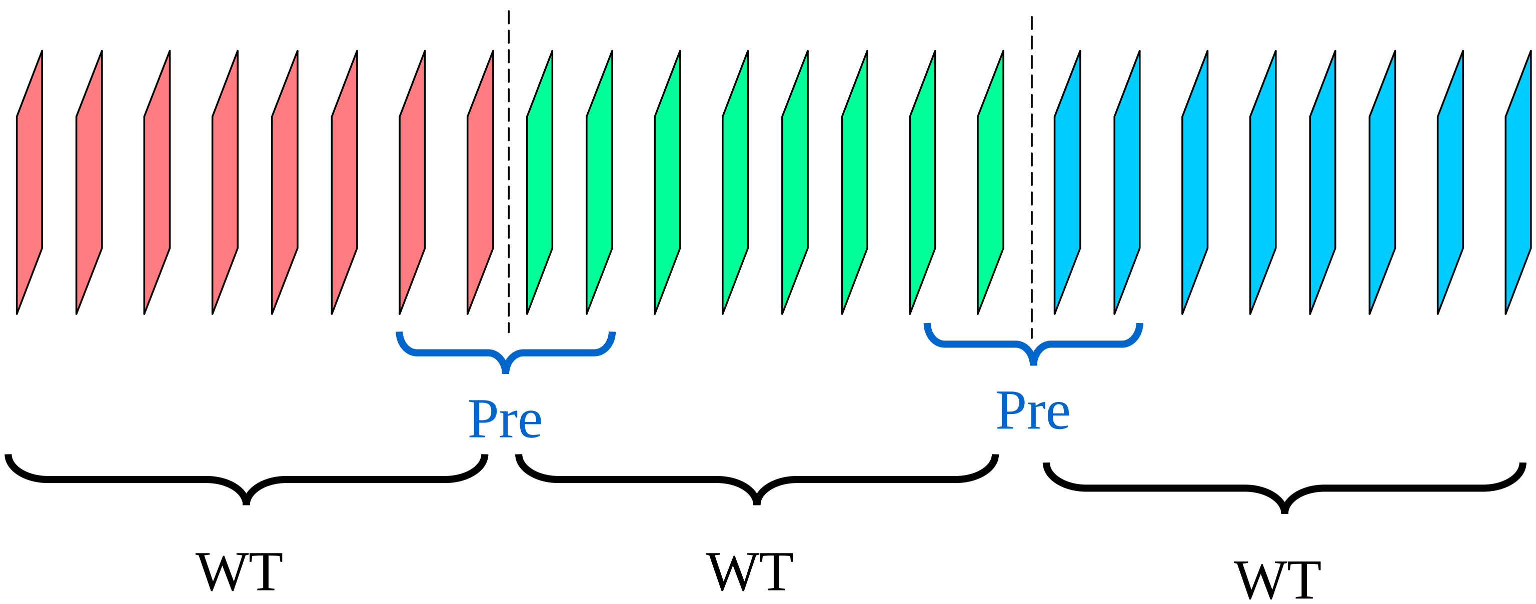
- 144 frames
- 16 frames per GOP
- 120 : 1

Solution 1: Global WT [Xu et al. 2002]

Problems: Memory, random access, error resilience …

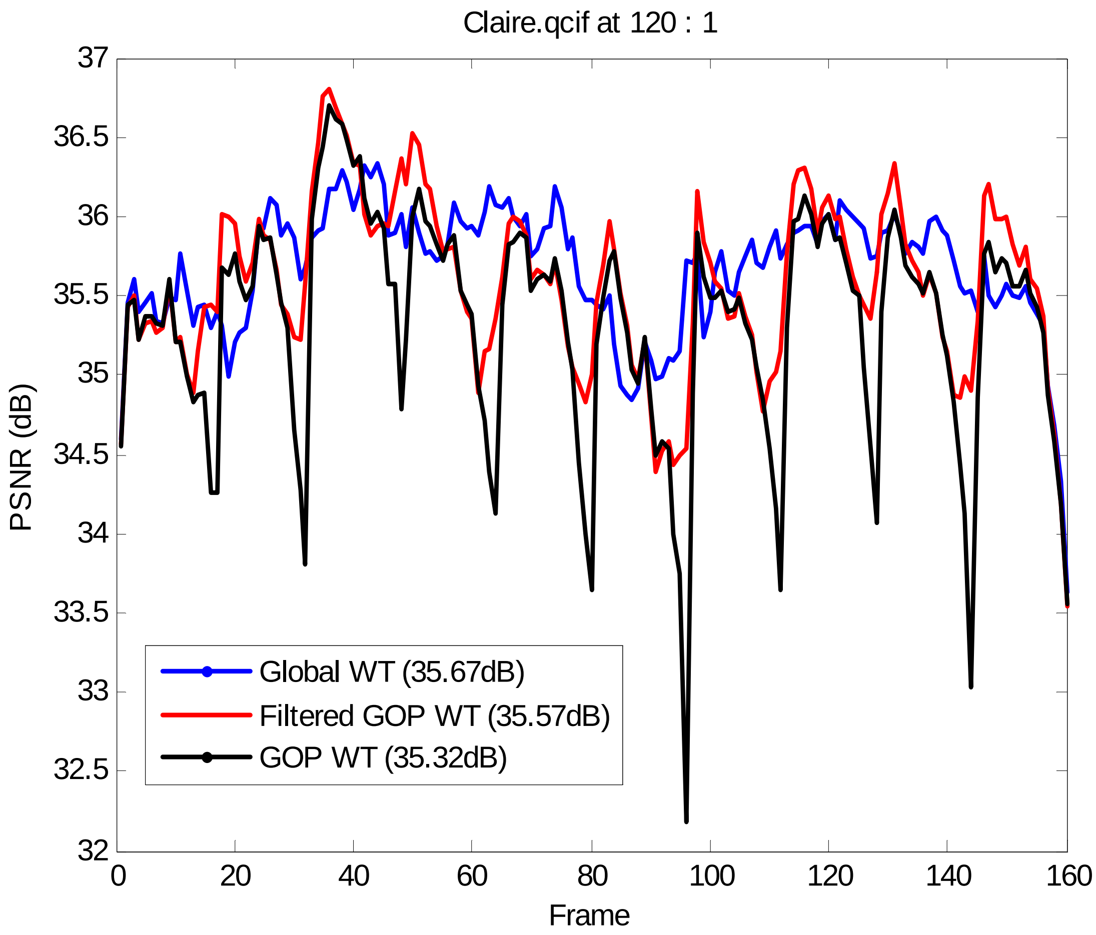
Pre/Post-filtering for 3-D WT

- Apply pre-filter before WT, and post-filter after IWT
- Previous pre/post design can be directly applied.



Comparison with GOP and Global WT

- Group: 16 frames
- 6-tap pre/post filter
- 9/7 WT
- Up to 2.5 dB gain at boundaries



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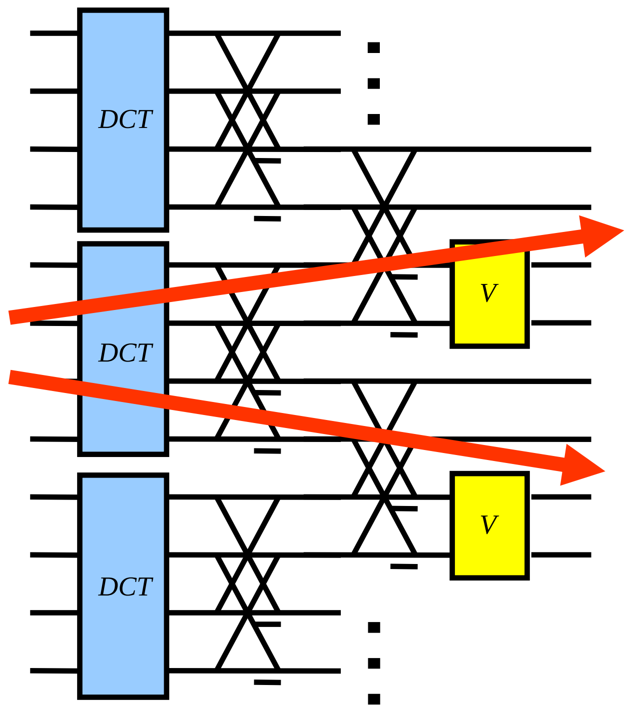
Error Concealment and Error Resilience

- Compressed bit stream is sensitive to transmission error/loss
- Traditional solutions:
  - Channel coding, Retransmission
  - Not always acceptable
- Human visual system can tolerate some errors:
  - Error concealment at the decoder is preferred.
- Error resilient encoder:
  - Encoder introduces some redundancies to facilitate concealment at the decoder.
  - Lapped transform is a good candidate…

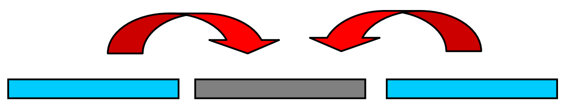
Motivation for Error Resilient LT



- Encoder:
  - Can spread the information of each block into two blocks
- Decoder:
  - Prediction of the lost block is easier
- Conflicting requirements:
  - Compression
  - Error resilience
- Trade-off required

Error Resilient Lapped Transform

- [Hemami1996]
  - First error resilient LT design.
  - Encoder: Trading compression for error resilience.
  - Decoder:
    - 1. Estimate lost blocks by mean reconstruction method:



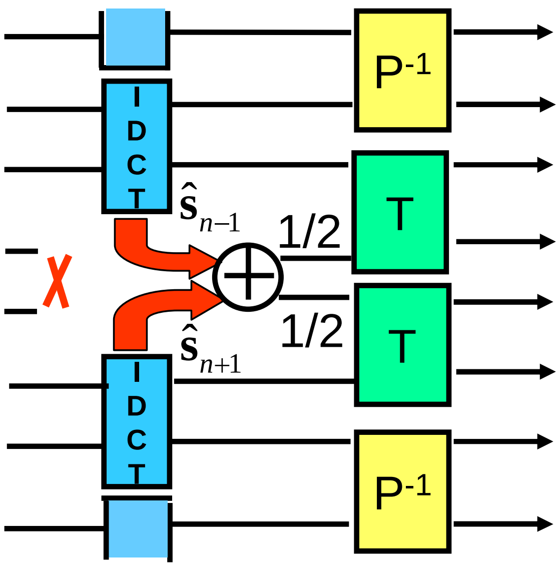
$$\hat{\mathbf{y}}_n \leftarrow \left(\hat{\mathbf{y}}_{n-1} + \hat{\mathbf{y}}_{n+1}\right)/2$$

    - 2. Apply inverse lapped transform.

- [Chung-Wang1999, 2002]
  - Multiple description coding
  - Maximal smoothness reconstruction
    - Improved visual quality

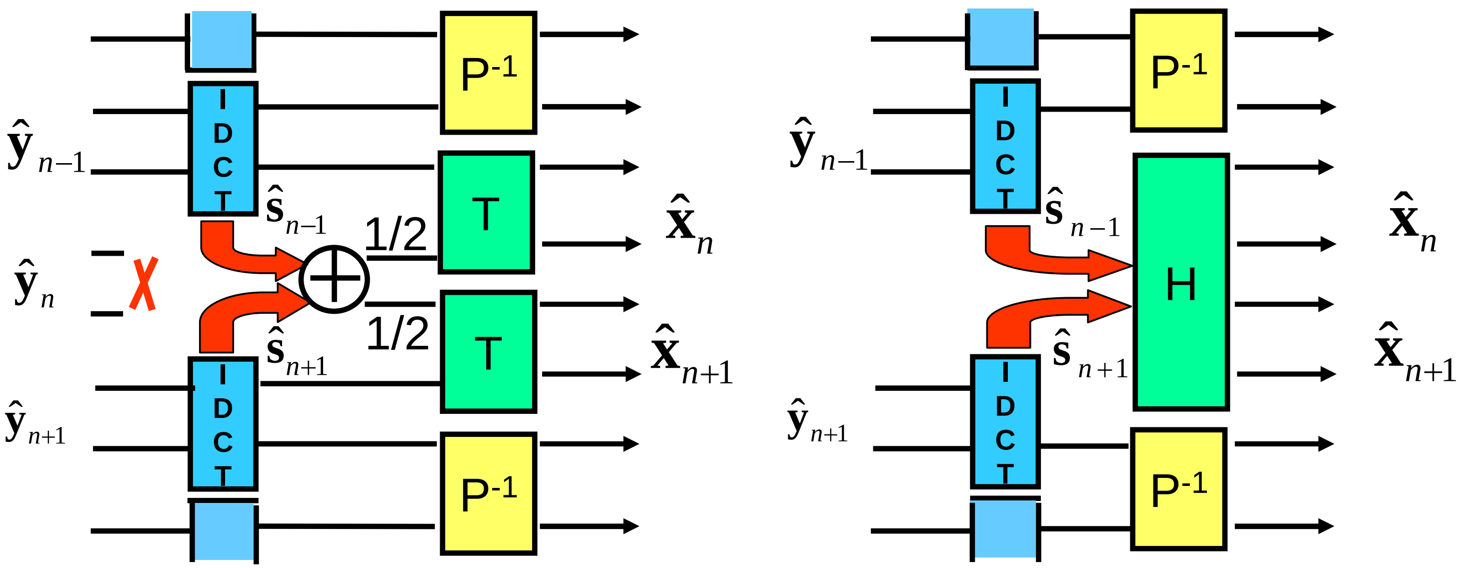
Error Resilient TDLT

- [Tu-Tran-Liang-2003]
- Only need to design the pre/post-filters
  - Old approach: M x 2M unknowns
  - Pre/post-filter: M/2 x M/2
- More flexibilities:
  - Biorthogonal filter
  - Non-perfect-construction design
- Limitation:
  - Mean reconstruction is still used



General Wiener Filter Solution

Existing Method General Framework [Liang et al. 05]



- General framework: Estimate $\hat{\mathbf{x}}_n$ and $\hat{\mathbf{x}}_{n+1}$ from $\hat{\mathbf{s}}_{n-1}$ and $\hat{\mathbf{s}}_{n+1}$

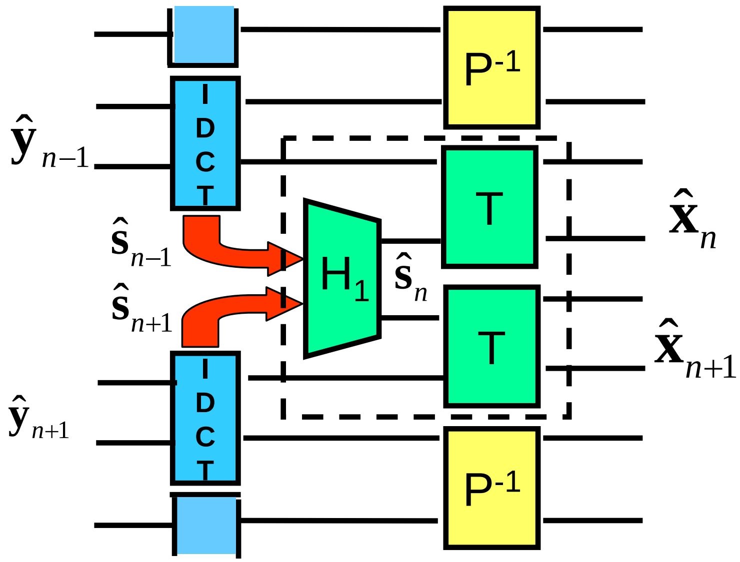
$$\mathbf{R}_{\mathbf{ee}} = E\left\{\left(\mathbf{H}\hat{\mathbf{s}}_2 - \mathbf{x}_2\right)\left(\mathbf{H}\hat{\mathbf{s}}_2 - \mathbf{x}_2\right)^T\right\}$$

$$\min \quad \text{MSE}=\text{tr}\left\{\mathbf{R}_{\mathbf{ee}}\right\} \longrightarrow \boxed{H_{2M\times 2M} = R_{x_2\hat{s}_2} R_{\hat{s}_2\hat{s}_2}^{-1}}$$

Expression can be found if **Rxx** is known (e.g., AR(1)).

Special Cases

Divide H into two stages:



H_1: M x 2M Wiener filter (Near optimal performance)

Link to previous approach:

H_1 = [I I] / 2 ➔ Mean reconstruction method.

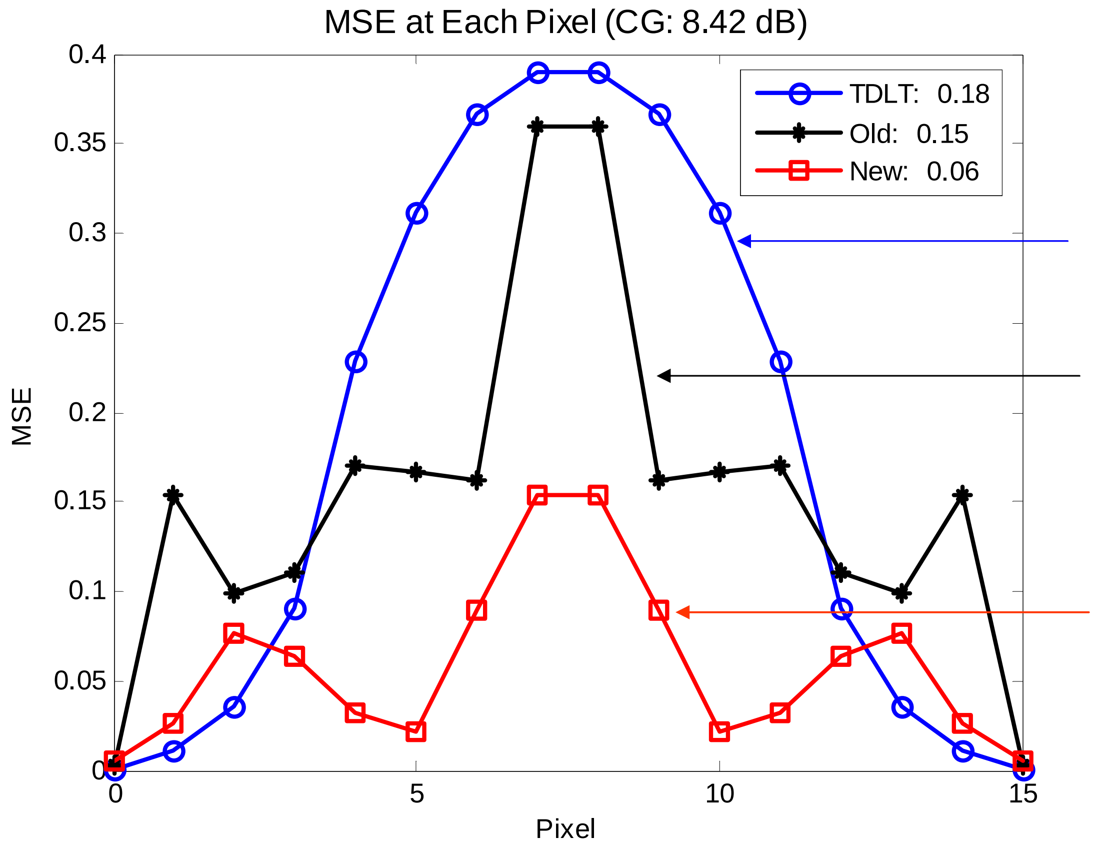
Design Criteria for Optimization

- Use Matlab to find optimal pre-filter and post-filter
- Trade coding performance for error concealment.
- Objective function for optimization:

$$\text{maximize } \varepsilon = (\text{CG}) - \alpha\ (\text{CR}) + \beta\ (\text{RG})$$

- Coding Gain (CG):
  - MSE when there is no transmission error
- Concealment Residual (CR):
  - The MSE after transmission error and error concealment
- Reconstruction Gain (RG)
  - [Hemami96]
  - Control the distribution of error to improve visual quality

Design Example



Cfg 1:
- Coding gain optimized filters
- M x 2M Wiener filter

Cfg 2:
- Joint optimized filters
- Mean reconstruction
- [Tu et al 03]

Cfg 3:
- Joint optimized filters,
- M x 2M Wiener:

60% less than the mean reconstruction method.

Simulation Results (512 x 512 Lena, 1bpp, 50% loss)

Loss Pattern

Cfg 1: 24.3 / 40.1 dB

Cfg 2: 26.0 / 38.3 dB

Cfg 3: 30.5 / 39. 2 dB

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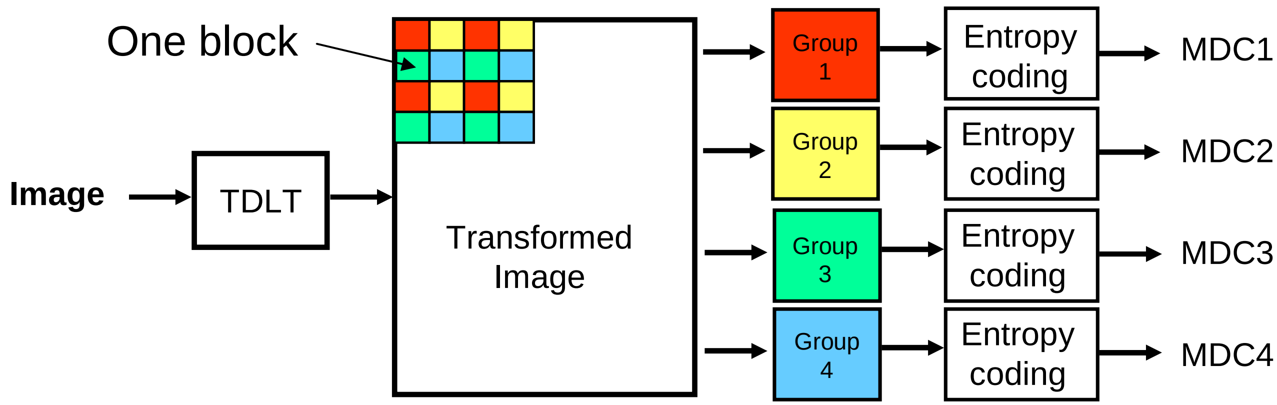
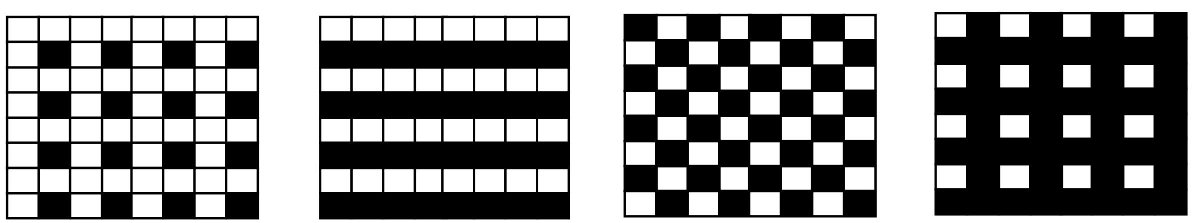
Multiple Description Coding

- An alternative to improve the robustness to transmission error:
  - Create multiple (equally important) output bit streams.
  - Each stream alone can give a coarse reconstruction.
  - Quality can be improved if more descriptions are received.

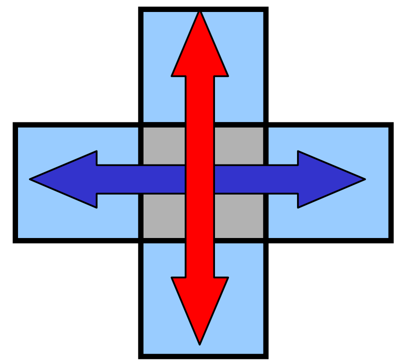


- Error scenarios (15 cases):

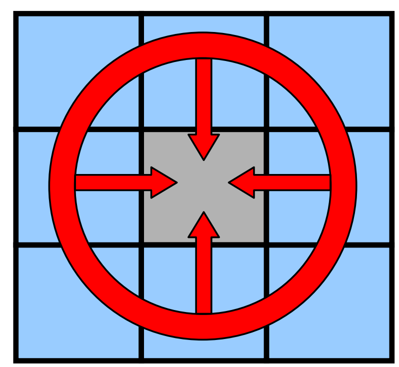


2-D Error Concealment

- Current method:
  - 1-D prediction
  - Average of row and column results



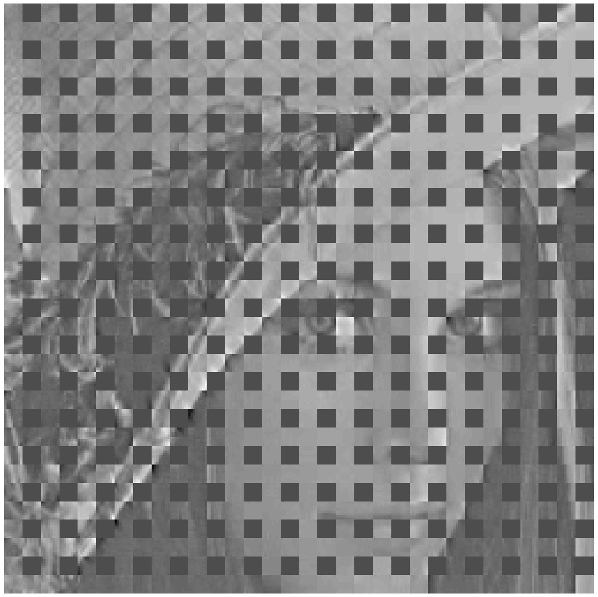
- Ideal method:
  - Joint prediction from 2-D neighbors
  - How to predict?
  - How to design the pre/postfilters?



Difficulties

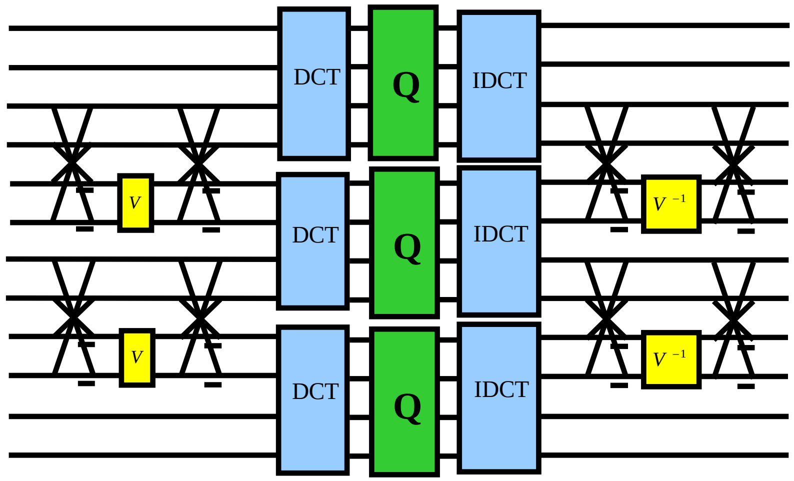
- Related work:
  - Edge directed prediction [Li-Orchard-2002]
- But geometrical structure is disturbed by prefiltering
  - Pixel-by-pixel approach may not work

Prefiltered image



After IDCT (with loss)

Summary



- Time domain lapped transform
  - Fast algorithms
  - Applications in 2D and 3D WT
  - Error resilient design
- Comparison to JPEG 2000:
  - Lower complexity
  - Competitive performance
  - Promising for handheld devices

References

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