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Performance limit for optical components

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Summary

We show a general limit to performance of dispersive optical components based on a limit to the number of orthogonal waves that can be generated by an object of given composition. The limit applies generally to linear systems, including nanophotonic structures, and is illustrated with a one-dimensional example of dispersing pulses.

With nanophotonic fabrication techniques, we can controllably structure optical materials on a subwavelength scale, and can in principle design new and very compact components for a broad range of complex optical functions. Examples could include wavelength splitters, mode splitters, spectral filters, dispersion compensators, and customized dispersion devices such as “slow light” structures. But we do not apparently have a limit tells us whether a given desired device can be made in a given size with a given material. Here we derive a very general result for evaluating just such a limit. This general result applies to a very broad range linear systems, in 1D, 2D, and 3-D, and is not restricted to optics. We illustrate the limit with a one-dimensional example of a limit to the minimum size of device that would split pulses of 32 different wavelengths in the telecommunications C-band.

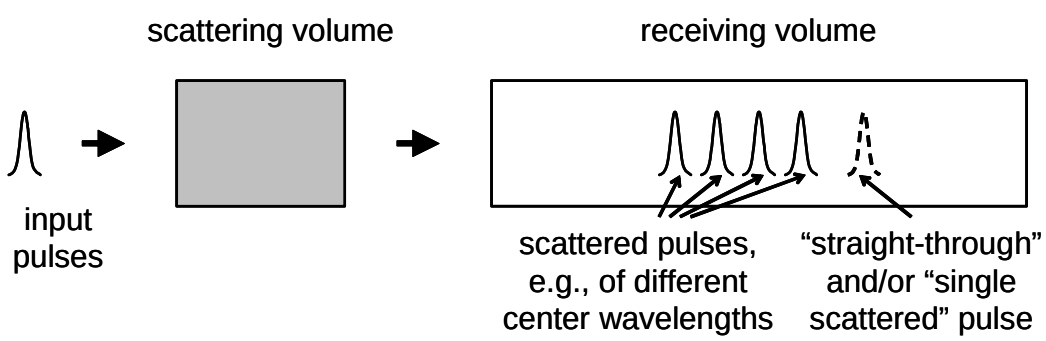

Fig. 1. Illustration of scattered pulses for a temporal disperser.

We imagine we have some scatterer as shown in Fig. 1, here separating pulses in time dependent on their center wavelength. Formally, we ask first how many orthogonal outputs we can generate in some receiving space with this scatterer when those outputs are also orthogonal to “straight-through” and “single-scattered” pulses. We find that there is only a finite number M of these, limited by a quantity S we can calculate, based on the strength of the scatterer, its volume, and the receiving space in which we look, i.e., $M \leq S$. This approach is related to previous work establishing the orthogonal “communications modes” between volumes [1,2].

We can think of the scattering as consisting of two processes, each described by its own linear operator. The first operator, C, tells us the strength of the effective source for waves that results from the interaction of a wave with the scatterer. This operator is essentially just the local dielectric constant; any variation of this dielectric constant from its uniform background value leads to a source term in the wave equation. The second operator, G_S, is essentially the Green’s function of the wave equation in the

uniform background material. If we knew the strength of all sources in the scattering volume (where those sources are actually the result of the wave interacting with the scatterer), this operator would give us the total scattered wave inside the volume.

Our central general result is that we are able to show that

$$M \le \sqrt{N_{GS} N_C}$$

where N_{GS} and N_C are given by

$$N_{GS} = Tr\left(\mathsf{G}_S^{\dagger}\mathsf{G}_S\right); \quad N_C = Tr\left(\mathsf{C}^{\dagger}\mathsf{C}\right)$$

where the traces should be taken over the space of source functions in the scattering volume that can give rise to non-zero waves in receiving volume.

Explicitly, we can consider a one-dimensional dielectric stack example, for a relatively narrow bandwidth around some center (angular) frequency. The length of the scattering volume is Δz_S. The background relative dielectric constant throughout is ε_{ro}, and the wave propagation velocity in the background medium is $v_o = c/(\varepsilon_{ro})^{1/2}$. With a variation in dielectric constant of $\Delta\varepsilon$, the relative variation of the dielectric constant compared to the background is $\eta = \Delta\varepsilon / \varepsilon_{ro}$. We choose the receiving volume as long as we want, so we can capture all possible scatterings. We find that the number of pulses of equally spaced center frequencies that we can separate in a reflective device is

$$N_B \le \left(\pi / \sqrt{3}\right) N_{S\lambda} \eta_{rms}$$

where $N_{S\lambda}$ is the length of the scattering volume in wavelengths, and η_{rms} is the average root mean square relative variation in dielectric constant.

Hence, for example, suppose we wanted to be able to separate pulses to $M = 32$ distinct positions in time, depending on their wavelength, and we vary the dielectric equally between being air and being glass (dielectric constant ~ 2.25) in equal amounts overall, so that η_{rms} = 0.385, with an average background dielectric constant of 1.625. For light in some narrow enough band round 1.55 micron wavelength, we therefore calculate that the thickness, Δz_S, of this structure must be at least ~ 54 microns. This limit holds no matter how we try to design the structure, with resonators, dielectric mirrors, chirped structures or any other approach.

Time delays of pulses can correspond to spatial shifts of c.w. beams if the structure is illuminated at an angle [3,4] (both phenomena result from group delay), and so we expect similar results if we were to try to use this one-dimensional structure to separate beams in space. Indeed, the result we obtain here is similar to the semi-empirical result [3,4] derived through consideration of 623 designs for a linear dispersive device based on beam shifting in dielectric stacks.

In conclusion, we have shown quite generally that, if we want to make some linear element to separate multiple beams or pulses, then there is a limit to the number that can be separated. We expect this result to be broadly applicable to nanophotonic structures. We acknowledge support from the DARPA/ARO CSWDM Program.

References

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