Equivalence of diffusive conduction and giant ambipolar diffusion

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Two high speed diffusion mechanisms in semiconductor heterostructures, diffusive conduction, and giant ambipolar diffusion, are compared and shown to be nearly equivalent descriptions of the same physical process. Fundamental limits of this process are discussed. © 2002 American Institute of Physics. [DOI: 10.1063/1.1453508]

I. INTRODUCTION

The response of semiconductor devices to photogenerated carriers has been a critical area of research over the past several decades due to the wide variety of uses for light-sensitive devices, such as photodetectors, optical switches, and lasers. Carrier dynamics due to diffusion often play an important role in the behavior of these types of devices and have been extensively investigated. For example, it has been shown that in bulk material, ambipolar diffusion is the primary diffusion mechanism for photogenerated carriers. In the mid-to-late 1980’s, it was discovered, however, that diffusion in semiconductor p-i-n diode and n-i-p-i structures could exhibit a response several orders of magnitude faster than ambipolar diffusion. Two mechanisms were separately proposed: diffusive conduction and giant ambipolar diffusion. Since that time, work based on these phenomena has progressed.

Unlike in a bulk semiconductor material, in reverse-biased diodes, n-i-p-i’s, and biased n-i-n or p-i-p devices electrons and holes separate, building up a carrier density-dependent screening potential between them. As will be shown, it is this difference which accounts for the dramatically enhanced diffusion in diodes versus bulk material. These very large diffusion mechanisms are strong enough to play a central role in high-speed electronic devices and optoelectronic switches; it is therefore critical to clarify these mechanisms. Moreover, studying this behavior also provides a greater understanding of semiconductor carrier dynamics.

To the best of our knowledge, however, a direct comparison has not been made between giant ambipolar diffusion and diffusive conduction. In this article, qualitative descriptions of enhanced diffusion are provided first. Next, a general approach to modeling charge transport dynamics is given, and the branching point in the assumptions made between the phenomenological approaches of diffusive conduction and giant ambipolar diffusion is identified. Quantitative descriptions of these approaches and their resulting conclusions are reviewed and then compared, showing their equivalence. Finally, fundamental limits of this enhanced diffusion are discussed.

II. QUALITATIVE DESCRIPTIONS OF ENHANCED DIFFUSION

Enhanced diffusion may be described from two different perspectives: microscopic, focusing on charge motion, or macroscopic, observing the voltage dynamics of the system. Diffusion in semiconductors is often approached microscopically. In bulk semiconductor material in the absence of electric fields, photogenerated carrier dynamics are well described by regular ambipolar diffusion: when a neutral distribution of excess carriers is created in bulk semiconductor material, e.g., via photogeneration, the electrons and holes predominantly move together. Local charge neutrality is approximately maintained, in spite of the different mobilities of the charge carriers, because the Coulomb attraction between an electron and hole is much stronger than the dispersive effects of diffusion alone. As a result, electrons and holes diffuse together with a single diffusion coefficient that equals a weighted average of the (isolated) electron and hole diffusion coefficients.

The material composition of p-i-n diodes, n-i-p-i’s, and other similar semiconductor structures is direction dependent; carrier motion in the direction perpendicular to the layers, hereafter referred to as either z or “vertical,” may be quite different from that of motion parallel to the planes, here defined as “lateral” or p. In these types of devices, photogenerated electrons and holes in the intrinsic region separate in the vertical direction due to the built-in and/or applied voltage across the layers of the device. Note that this article only addresses lateral carrier dynamics, not vertical. Vertical carrier transport has been the subject of extensive research; see, for example, Refs. 16–20. Understanding the effects that vertical charge separation has on lateral carrier movement is, however, critical and can be subtle. Briefly, the vertical separation of a localized group of photogenerated carriers creates a lateral voltage gradient that pushes both electrons and holes away much faster than does ambipolar diffusion alone.

How and why does this happen? A schematic view of a p-i-n device is presented in Fig. 1 (top). The bias voltage across the intrinsic region is linearly related to $\Phi_{np}$, the separation between the quasi-Fermi levels of the electron, $\Phi_n$, and hole, $\Phi_p$. After a pulse of light is absorbed, the photogenerated carriers vertically separate and the electric field in the intrinsic region is screened, although only in the vicinity of the incident pulse light beam. The results are il-
Illustrated in Fig. 1 (middle). As more carriers are injected, separate, and screen the field, the built-in and/or reverse bias voltage decreases in the vicinity of the absorbed light beam pulse, and $\Phi_{np}$ changes. The magnitude of this shift in the quasi-Fermi levels is strongly dependent on the magnitude of the photogenerated charge that has separated. This dependence is linear, at least for small voltage changes and/or large reverse biases, because it results primarily from the reduction in voltage from the vertical charge separation. An equivalent statement is that the derivative of $\Phi_{np}$ with respect to the density of the separated charge density is large or "giant." This shift is much larger than the typically logarithmic shift of quasi-Fermi levels found in a bulk semiconductor which results from the shift of quasi-Fermi levels solely due to the statistical mechanics of the change in carrier density. It is this fundamental difference—due to charge separation—that is responsible for enhanced diffusion.
Continuing with a microscopic perspective, if a pulse of light with a lateral spatially varied profile, e.g., Gaussian, is absorbed in the intrinsic region, and the photogenerated electrons and holes quickly separate vertically, \( \Phi_n \) and \( \Phi_p \) are forced initially to have corresponding, though opposite, lateral Gaussian spatial dependence, and consequently so does \( \Phi_{np} \). Figure 1 (bottom) illustrates this situation. A gradient of a quasi-Fermi level defines an effective electric field along that gradient (e.g., a gradient in \( \Phi_n \) creates a field in the \( n \) layer). The lateral spatial variation of the input pulse—when combined with vertical charge separation—thus creates a lateral electric field in both the \( n \) and \( p \) layers. These fields help the carriers in both doped regions to disperse laterally. The relatively large magnitudes of these extra fields are what account for the enhanced diffusion effects of giant ambipolar diffusion.

Arguably, this process is not a diffusion process in the conventional sense. The motion of the carriers can be viewed as a consequence of the electric fields, corresponding to normal resistive transport. Note, too, that all of the carriers in the \( n \) and \( p \) regions move in response to the lateral fields, not merely the additional photocarriers. The mathematical equation describing the resulting movement of the carrier density of the voltage pulse does have the form of a diffusion equation. The diffusion constant of this equation, though, depends on the conductivity of the layers and the gradient of \( \phi \), where \( \phi = \Phi_n + \Phi_p \) is the electric field in both the \( n \) and \( p \) layers. These fields help the carriers in both doped regions to disperse laterally. The relatively large magnitudes of these extra fields are what account for the enhanced diffusion effects of giant ambipolar diffusion.

From a macroscopic perspective, diffusive conduction is essentially an extension of the voltage dynamics of a one-dimensional dissipative transmission line. A voltage pulse in a transmission line can travel at a speed much faster than that of the individual electrons as is well known in conventional inductive-capactive transmission lines (e.g., a coaxial cable carrying signals at speeds near the velocity of light). The structures of interest here are dissipative transmission lines, in which the series resistive impedance of the \( p \) and \( n \) layers dominates over the inductive impedance leading to dissipative wave propagation, but it is still true that the dissipative wave can move faster than the individual electrons and holes. This is possible in part because the particles in the medium exert strong forces on one another and because the medium of particles extends throughout the length of the line. A \( p-i-n \) structure can be viewed as a two-dimensional (lateral) version of a dissipative line, as illustrated in Fig. 2. The doped \( p \) and \( n \) regions each have a resistance per square and there is also a capacitance per unit area between them across the intrinsic region. When a spatially localized pulse of light is absorbed (e.g., a light beam with a small spot size is absorbed in the center of a mesa structure), the photogenerated electrons and holes in the intrinsic region will separate, shielding the voltage. This results in a spatially localized voltage pulse. The behavior of the pulse in this dissipative structure may be modeled by a diffusion equation. The result is voltage diffusion that dissipates the voltage build up across the entire device. As in a dissipative transmission line, this response is not limited by individual carrier motion. Instead, this diffusion depends only on the capacitance per unit area, the spot size, and the resistance per square and, consequently, may be very fast.

### III. GENERAL MODELING APPROACHES

With an understanding of the qualitative behavior of enhanced diffusion, a compelling question becomes: can this behavior be modeled from first principles? The general response of \( p-i-n \) diodes and related structures to photogenerated carriers can be determined from three relationships: (1) the forces present, including those due to the photogenerated carriers, (2) the motion of all the carriers due to the forces present, and (3) overall charge neutrality (an equal number of electrons and holes are created by photogeneration). Combining these relationships along with the initial and boundary conditions allows a self-consistent description of the carrier dynamics to be found. Some of the issues involving this process are discussed next.

The primary forces involved in semiconductor carrier dynamics are the Coulomb attraction and/or repulsion due to the electric fields of space charges. These can be well modeled by using, for example, Poisson’s equation.

Determining what should be the degree of accuracy of the equations governing carrier motion is also an essential task in order to solve for the system dynamics. One of the most fundamental approaches that may be considered is the use of the Boltzmann transport equation (BTE) to express charge motion via the evolution of a charge distribution function,

\[
\frac{\partial f}{\partial t} + \frac{\partial f}{\partial p} \cdot \mathbf{v} + \frac{\partial f}{\partial \mathbf{r}} = \frac{\partial f}{\partial t} \quad \text{coll},
\]

where the last term is the change in \( f \) due to collisions and \( \mathbf{p} \) and \( \mathbf{r} \) are the momentum and position vectors, respectively. Only two assumptions need to be made: that carriers may be treated semiclassically (i.e., they have a well-defined position and momentum), and that there is a sufficient number of carriers to meaningfully use a distribution function. These are reasonable assumptions for many devices. One of the more dramatic simplifications results in a drift–diffusion equation:

\[
\mathbf{j}_n = \mathbf{a} \mu_n n \mathbf{E} + q D_n \nabla n.
\]

which expresses the current density of electrons, \( n \), as a function of \( \text{ensemble} \) values (mobility, \( \mu_n \), and diffusion coefficient, \( D_n \), each of which is dependent on the distribution function, electric field, and temperature) combined with the electric field, \( \mathbf{E} \), and the gradient of the charge density. \( q \) is the unit of charge. This equation is the basis for describing carrier transport that will be used in this article.

In writing the drift–diffusion equation, several assumptions have been made: magnetic fields have been assumed.
negligible; current due to carrier temperature gradients (the thermoelectric effect) is small.\textsuperscript{,25} Finally, mobility and diffusion coefficients are not dependent on the detailed structure of the device (spatial variations are large compared to scattering lengths). These last two assumptions are valid if the electric fields are small or, if large, uniform. The enhanced diffusion transport discussed in this article only involves the carrier transport of carriers in the lateral, doped planes. Thus, even though the vertical dimensions of some layers may be small, since the transport is not in that direction such variation is not critical. There are electric fields generated in the lateral direction as will be described next that play an important role in these transport mechanisms. However, for spot sizes with radii of a few microns or larger, the variation in electric field occurs over a distance large compared to the scattering length. As a consequence, the validity of the assumptions remains uncompromised.

In order to use the drift–diffusion equation, the mobility and diffusion coefficient must also be known \textit{a priori}.\textsuperscript{,26} Using these ensemble quantities implicitly removes information regarding the statistical variances of these quantities. This is a safe simplification if device behavior is not sensitive to such statistical fluctuations.\textsuperscript{,27} These quantities are also determined by using an expected value for (momentum) scattering rate. Hence, only dynamics that occur on a time scale that is large compared to the inverse of this scattering rate (typically on the order of a picosecond at room temperature) are well defined.\textsuperscript{,24} It is worth noting that subsequent behavior based on these assumptions describes the behavior of the ensemble of particles, not of individual particles themselves.

Having discussed how to describe carrier motion, next we look more carefully at the assumption of charge neutrality. Overall charge neutrality arises because photogenerated carriers are always produced in electron and hole pairs. In bulk semiconductors, as previously mentioned, local charge neutrality is also maintained in the absence of external fields even if electrons and holes have different mobilities. The Coulomb attraction between the carriers is significantly stronger than other prevailing forces (e.g., such as diffusion, which could separate electrons and holes) and acts to keep electrons and holes close together on the time scales of interest. Local charge neutrality is a key assumption of ambipolar diffusion and accounts for electrons and holes diffusing together in spite of differing mobilities.\textsuperscript{,5,28} In \textit{p-i-n} and \textit{n-i-p-i} structures, however, there are built-in and/or applied fields along the \textit{z} direction that separate the charge species. Clearly, local charge neutrality no longer applies since the electrons and holes are separated, typically on the order of one micron or less in many devices. However, the separation is small enough (i.e., the Coulomb attraction is still sufficiently large) that an effective local charge neutrality \textit{does} continue to hold in the \textit{lateral} directions. (Note that in studying enhanced diffusion, we assume that no external fields are present in the lateral directions.)

Even with a simplified expression for carrier transport, Eq. (2), and the assumption of local charge neutrality in the lateral directions, solving for the carrier dynamics is not easy. Each of the two approaches described next make additional assumptions to make the math tractable and provide an analytic solution. Diffusive conduction drops the explicit diffusion term of the transport equation. Giant ambipolar diffusion, on the other hand, simplifies the carrier density description by assuming a Maxwell–Boltzmann (MB) distribution function in the doped regions.

\section*{IV. GIANT AMBIPOLAR DIFFUSION}

Modeling the effects of charge separation using the drift–diffusion equation is clearly presented in the paper of Dohler and Gulden \textit{et al.}\textsuperscript{,15} In the following equations, subscripts \textit{n} and \textit{p} refer to electrons and holes, respectively; \textit{j} is current density, \textit{\mu} is mobility, \textit{n} and \textit{p} are the carrier densities, \textit{\phi} is the quasi-Fermi level, and \textit{r}=(\textit{z},\textit{\rho}, and \textit{\theta}). Note that in this article, \textit{\rho} represents in-plane radial distance, not resistivity. A MB distribution is assumed and, therefore, the Einstein relations relate diffusion and mobility through a simple expression:

\begin{equation}
\mu_n = \frac{qD_n}{kT}, \quad \mu_p = \frac{qD_p}{kT},
\end{equation}

where \(D_n\) and \(D_p\) are the MB electron and hole diffusion constants. Consequently, current density may be expressed as a function of the quasi-Fermi level gradient:

\begin{equation}
\begin{aligned}
\mathbf{j}_n &= \mu_n n \nabla \phi_n (\mathbf{r}) \\
\mathbf{j}_p &= \mu_p p \nabla \phi_p (\mathbf{r}).
\end{aligned}
\end{equation}

If the electron and hole current densities could be expressed in terms of gradients of \textit{n} and \textit{p}, respectively, we could write the relationships as diffusion equations:

\begin{equation}
\begin{aligned}
\frac{dn}{dt} &= qD_n \nabla^n \mathbf{j}_n \\
-\frac{dp}{dt} &= qD_p \nabla^p \mathbf{j}_p.
\end{aligned}
\end{equation}

And if, as will be shown, these current densities were equal, a single diffusion coefficient would describe the dynamics. We next describe how such an equivalence arises and how to express the current densities in terms of carrier gradients and in the process derive an expression for the giant ambipolar diffusion constant, \(D_{\text{amb. diff.}}\).
In this article, we are concerned only with lateral carrier motion. Equation (2) is a separable equation and so treating the lateral components of the gradients alone, as in, e.g., Eq. (4), is a valid approach. In the equations presented next, \( \nabla_p = \partial \phi_p / \partial p + (1/p) \partial \phi / \partial \theta \). Furthermore, in this derivation of giant ambipolar diffusion, it is assumed that the photogenerated electrons and holes have already separated in the vertical direction across the intrinsic region of the device. The assumption of vertical photogenerated carrier separation is not required and does not affect the calculation of the diffusion coefficient; it does, however, allow for a simplification of some of the equations. Therefore, if \( n_0 \) and \( p_0 \) are the concentrations of electron and hole ionized dopants,

\[
n = n_0 + \Delta n \quad p = p_0 + \Delta p,
\]

where \( n, p, n_0, p_0, \Delta n, \) and \( \Delta p \) are no longer per unit volume but rather per unit area: \( n \) is the electron density integrated across the thickness of the \( n \) region; \( p \) is the hole density integrated across the \( p \) region. Similarly, \( \Delta n \) and \( \Delta p \) are the photogenerated carrier densities integrated across the doped layers assuming they have vertically separated. Thus, \( n, p, \Delta n, \) and \( \Delta p \) are functions of just the lateral dimensions \( \rho, \theta \).

It is assumed that at any given time, there is “local” charge neutrality at each lateral “point” in space. Consequently, there is no net lateral flow of charge. In other words, because

\[
\Delta p = \Delta n,
\]

which, when combined with Eq. (6), leads directly to

\[
\nabla_p n = \nabla_p p,
\]

we may write

\[
j_n = -j_p,
\]

in which \( j_n \) and \( j_p \) are the integrated lateral current densities in the \( n \) and \( p \) layers.

The relationship between the quasi-Fermi levels and the carrier densities are examined next. Changes in the number of available states result in the logarithmic dependence of the electron quasi-Fermi level on the MB distribution of the carrier density:

\[
\phi_n^{\text{mech}} = E_i + kT \ln \left( \frac{n}{n_i} \right),
\]

where \( E_i \) is the Fermi level in the intrinsic region, \( T \) is the temperature, \( n_i \) is the intrinsic carrier concentration, and \( \phi_n^{\text{mech}} \) is the quasi-Fermi level due to this statistical mechanics-based effect. Changes in the electron density therefore result in logarithmic changes to the electron quasi-Fermi level. A similar relationship exists for changes in hole carrier density and \( \phi_p^{\text{mech}} \). Because the changes in electron and hole carrier densities are equal [Eq. (7)], the change in both quasi-Fermi levels may be well described by referring only to changes in electron or changes in hole density. For example, the hole quasi-Fermi level can be expressed as a function just of changes in electron density. Starting with

\[
\phi_p^{\text{mech}} \propto \ln(p),
\]

we can write

\[
\Delta \phi_p^{\text{mech}} \propto \Delta p.
\]

But because \( \Delta n = \Delta p \), the hole quasi-Fermi level may be expressed as

\[
\Delta \phi_p^{\text{mech}} \propto \Delta n
\]

With some foresight, we examine the difference between the quasi-Fermi levels:

\[
\phi_{np} = \phi_n - \phi_p
\]

Changes in \( \phi_{np} \) can thus be expressed either as a function of changes in \( n \) or as a function of changes in \( p \). It is also critical to account for the effect that carrier separation has on the quasi-Fermi levels. The quasi-Fermi levels separation is directly proportional to the voltage across the intrinsic region which itself is strongly determined by the electric field due to carrier separation and thus depends (linearly) on the changes in separated photogenerated carrier density. Hence,

\[
\phi_{np}^{\text{separation}} \propto \Delta n,
\]

where \( \Delta n \) is the separated carrier density and \( \phi_{np}^{\text{charge}} \) is the quasi-Fermi level difference due to the resulting voltage change. Combining both the electrostatic and statistical mechanical dependencies of the photogenerated carrier densities,

\[
\phi_{np} = \phi_{np}^{\text{charge}} + (\phi_{np}^{\text{mech}} - \phi_p^{\text{mech}}).
\]

All the terms on the right-hand side of Eq. (14) are one-to-one functions of the changes in either carrier density, e.g., \( \Delta n \), as shown by Eqs. (10), (11c), and (13).

Because the relationship between carrier density and quasi-Fermi level is monotonic (one to one), it may be inverted. Thus, the changes or the gradient in carrier density are a function of \( \phi_{np} \)

\[
\nabla_p n = \frac{\partial n(\rho)}{\partial \phi_{np}} \frac{\partial \phi_{np}}{\partial \rho}.
\]

It is assumed that \( \phi_{np} \) has no \( z \) (vertical) dependence; this is equivalent to assuming well-defined, constant quasi-Fermi levels across the \( n \) and \( p \) regions.

Combining Eq. (15) with Eq. (8), we find that

\[
\nabla_p n = \frac{\partial n(\rho)}{\partial \phi_{np}} (\nabla_p \phi_n - \nabla_p \phi_p) = \nabla_p \rho.
\]

Note the dependence on \( \rho \). Finally, by combining Eq. (16) with Eqs. (9) and (4) and recalling that conductivity and mobility are related by the expression \( \sigma = q \mu n \), where \( q \) is the unit charge and \( n \) is the charge density, the relationship between the gradients of quasi-Fermi level and current density may be found:

\[
\nabla_p \phi_n = \frac{1}{1 + \sigma_n / \sigma_p} \frac{\partial \phi_{np}}{\partial n} \nabla_p n.
\]
For electrons $\nabla p \phi \propto \nabla p$; a similar expression exists for holes. Using Eq. (9), Eq. (6) may now be written as
\[
-j_p = qD\nabla p = qD\nabla n = j_n, \tag{18}
\]
with $D$ an as-of-yet undefined proportionality constant—the diffusion coefficient. Therefore, by substituting Eq. (17) into Eq. (18) the diffusion coefficient is found to be
\[
D = D_{\text{amb.diff.}} = \frac{1}{q} \left( \frac{\sigma_n}{\sigma_n + \sigma_p} \right) \frac{\partial \phi}{\partial n}. \tag{19}
\]
As can be seen, this diffusion coefficient is directly related to the separation of the quasi-Fermi levels and how quickly this energy difference changes with the carrier density of separated photogenerated carriers, as had been suggested by the aforementioned qualitative description. Investigations of giant ambipolar diffusion have measured diffusion coefficients on the order of $10^4$ V/cm$^2$.\textsuperscript{31}

The assumption of MB statistics is usually appropriate in silicon devices. In AlGaAs material systems, however, degenerate electron populations are reached at lower doping levels. This is not correctly modeled when a MB distribution is assumed. Degeneracy increases the diffusion coefficient for ambipolar diffusion: in GaAs at room temperature, the diffusion coefficient rises from about 40 to 130 cm$^2$/s as the electron density grows from $10^{16}$ to $5 \times 10^{19}$ cm$^{-3}$ while in silicon it rises from approximately 20 to 60 cm$^2$/s as the carrier density grows from $10^{19}$ to $10^{21}$ cm$^{-3}$.\textsuperscript{6} This effect is likely due to a higher effective temperature in degenerate systems, particularly of those carriers that contribute to diffusion at the edge of the Fermi sphere. This increase principally effects the diffusion term, not the drift term, of a drift–diffusion equation. Since enhanced diffusion is principally effects the diffusion term, not the drift term, of a drift–diffusion equation. Similarly, Eq. (24) does not describe the diffusion of a particular particle but rather an effective diffusion for the system of particles.

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V. DIFFUSIVE CONDUCTION

Equations governing the voltage behavior may be derived using a discrete element approach to the problem. From the perspective of this article, the simplification in the diffusive conduction approach (i.e., $qD\nabla n \approx 0$) compared to the standard drift–diffusion equation results in the following charge transport equation
\[
j = q \mu n E, \tag{20}
\]
which is simply Ohm’s law. For a $p$-$i$-$n$-like structure, this expression may be integrated to show
\[
-\frac{dV}{dp} = \frac{R_{\text{sq}}}{2 \pi \rho}, \tag{21}
\]
where $R_{\text{sq}}$ is the sum of the resistance per square of the $n$ and $p$ layers. By using this particular derivative in the expression, another assumption has been implicitly made: current does not flow in the vertical direction. This is a reasonable approximation in a reverse-biased diode. From $Q = CV$, the relationship used to provide the dependence of the field on carrier density is
\[
I = \frac{d}{dt} \int VCA2\pi \rho dp, \tag{22}
\]
where $C_A$ is the capacitance per unit area. Note that the local charge neutrality relationship is implicitly assumed by using $Q = CV$.

Together, Eqs. (21) and (22) provide a complete description of the voltage dynamics:
\[
\frac{dV}{dt} = D\nabla^2 n, \tag{24}
\]
This suggests that there will be only a small underestimation of the diffusion coefficient in degenerate systems due to the assumption of a MB distribution in giant ambipolar diffusion. In fact, the size of this error should be similar in magnitude to that due to the assumption of a negligible diffusion term in diffusive conduction.

The Laplacian is two-dimensional and the (voltage) diffusion coefficient is $D = 1/R_{\text{sq}}C_A$ where $R_{\text{sq}} = R_{\text{sq}}^{\text{layer}} + R_{\text{sq}}^{\text{layer}}$ and $C_A$ is the capacitance per unit area. This effective diffusion coefficient depends on parameters (resistance and capacitance) that can be controlled by proper device design. For the purpose of comparison, it is worth noting that Eq. (23) may be equivalently written as
\[
\frac{dV}{dt} = \frac{1}{2 \pi \rho} \nabla^2 n, \tag{24}
\]
This assumes a MB distribution in giant ambipolar diffusion. In fact, the size of this error should be similar in magnitude to that due to the assumption of a negligible diffusion term in diffusive conduction.

In such cases, the voltage at the center of the spot arises away with a predominantly hyperbolic time dependence, $\tau/(t + \tau)$. For a properly designed device, the decay coefficient, $\tau$, may be on the order of picoseconds. To reiterate, this fast diffusion is not the movement of a single carrier from the center outwards. Rather, it is the ensemble of electrons and holes across the whole extent of the $p$ and $n$ regions each moving a small amount—just the same as when in a transmission line the voltage signal is able to propagate much faster than an individual electron can. Such voltage behavior has been verified.\textsuperscript{9,10}
VI. COMPARISON BETWEEN APPROACHES

What is the relationship between the diffusion coefficients of diffusive conduction, Eq. (23), and giant ambipolar diffusion, Eq. (19)? The answer is: they are nearly the same. For these types of structures, the relationship between carrier concentration and voltage is primarily linear

$$q\Delta n = Q = CV.$$  

Clearly, if voltage diffuses away at a given rate, the carrier concentration at that location must change by a proportional amount. In this sense, the similarity between the microscopic and macroscopic approaches could be expected. This may be shown more explicitly. Recalling that

$$\frac{1}{\sigma} = R_{sq} l,$$  

where \( l \) is the thickness of the doped layers, we can write

$$\frac{\sigma_n \sigma_p}{\sigma_n + \sigma_p} = \frac{1}{\frac{1}{\sigma_n} + \frac{1}{\sigma_p}} = \frac{1}{l(R_{sq}^n + R_{sq}^p)},$$  

Using

$$\frac{\partial n}{\partial V} = \frac{\partial n}{\partial \phi_{np}},$$  

and

$$C = \frac{\Delta Q}{\Delta V} = \frac{q\Delta n(Al)}{\Delta V},$$  

the capacitance per unit area may be related to the quasi-Fermi level difference:

$$\frac{\partial \phi_{np}}{\partial n} = q \frac{\partial V}{\partial n} = \frac{q^2 (Al)}{C} = \frac{q^2 l}{C_A}.$$  

Combining this key relationship with Eq. (28) can then be used to show that, indeed,

$$D_{\text{giant amb. diff.}} = \frac{1}{q^2 \frac{\sigma_n \sigma_p}{\sigma_n + \sigma_p} \frac{\partial \phi_{np}}{\partial n}} = \frac{1}{(R_{sq}^n + R_{sq}^p)} \frac{1}{C_A} \frac{1}{D_{\text{diffusive conduction}}}.$$  

Diffusive conduction is anchored around \( V = IR \), focusing on the voltage dynamics of the system. If this is rewritten as \( I = V/R \), the perspective changes, emphasizing carrier dynamics. Giant ambipolar diffusion, on the other hand, starts from the expression for current density, \( j_n = \mu_n q \frac{\partial \phi_p}{\partial r} \). When this relationship is now compared to \( I = V/R \), the similarities become easy to see.

VII. USES, LIMITS, AND LIMITATIONS OF ENHANCED DIFFUSION

The different approaches have different advantages aside from whether one is more interested in carrier or voltage dynamics. The diffusive conduction approach lends itself well, for example, to modeling the coupled behavior in multiple-layer devices, such as for a \( p-i-n-i-p \) or more general \( n-i-p-i \) structures. An interesting feature that diffusive conduction brings to light is that enhanced diffusion may occur before the photogenerated carriers finish vertically separating. As the photogenerated carriers begin to separate, they alter the voltage across the intrinsic region. This voltage change immediately starts to diffuse away as the free carriers in the doped regions respond to this localized voltage change. In fact, in such a case the photogenerated carriers themselves are not directly involved in any lateral diffusion in the system! This effect may be used, for example, in heterostructures where the photogenerated carriers may find it difficult to cross over the barriers at the interfaces between the intrinsic and doped regions. Nevertheless, the change in voltage their vertical motion engenders may quickly diffuse away. This behavior is not clearly reflected in giant ambipolar diffusion due to the assumption, in the derivation presented here [see Eq. (19)], of the photogenerated carriers already being vertically separated and present in the doped regions (recall we ignored the vertical transport responsible for that). If this assumption was not made, the diffusion coefficient would not change, but the initial free carriers in the doped regions would not drop out of the equations; it is those free carriers that would then be responsible for the enhanced diffusion.

Having identified the assumptions made in deriving diffusive conduction and giant ambipolar diffusion, we are now in a position to consider the effects on diffusive conduction when some of the assumptions made in using the drift–diffusion equation fail. When large (lateral) electric fields are present, the current density ceases to be a linear function of field (e.g., mobility becomes field dependent); as the saturated drift velocity is reached carrier transport is no longer well described by Ohm’s law. Diffusion, consequently, tends not be as large as expected. This situation may arise when a large number of photogenerated carriers are created within a small spot size and separate, creating a large shielding voltage.

If the lateral device dimensions are small (e.g., on the order of 0.1 \( \mu \text{m} \) in GaAs), scattering events may not occur as the carriers move from one side of the device to the other. Instead, carrier transport begins to approach that ballistic transport. Similarly, if the diffusive conduction coefficient is very fast—with a time constant \( \tau \) less than 1 ps—so that it is smaller than the expected scattering time, it becomes important to account for ballistic transport. Both of these effects tend to limit the carrier motion.

For both large electric fields and fast diffusion times, a magnetic field proportional to \( dE/dt \) is created. The result is the slowing of the diffusion. In the diffusive conduction approach, this would be modeled by including the inductance per unit area; giant ambipolar diffusion would add a \( q\mu(n \times B) \) term to the drift–diffusion equation with careful attention now having to be paid to the different directions of current flow. The ultimate limiting case is a device with negligible resistance per unit area (e.g., conductor-like doped regions). This situation is analogous to a two-dimensional lossless transmission line; the voltage decay is limited by the speed of light in the medium.
VIII. CONCLUSION

In summary, we have provided a general approach to understanding enhanced diffusion in biased semiconductor devices. Giant ambipolar diffusion and diffusive conduction both make use of the drift–diffusion equation but with different added assumptions. Giant ambipolar diffusion assumes a MB distribution; diffusive conduction does not specify a particular distribution but assumes the regular diffusion term is negligible. These assumptions and their limitations were examined in detail and the resulting diffusion coefficients of the two approaches were shown to be nearly equivalent.

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1 W. van Roosbroeck, Phys. Rev. 91, 282 (1953).
14 Footnote 8 of Ref. 13 (giant ambipolar diffusion) mentions that a description in terms of capacitance provides a link to Ref. 8 (diffusive conduction). This relationship, however, is not elaborated upon any further. As Eq. (31) here shows, this is indeed a key connection between the two approaches.
23 For diodes with quantum wells in their intrinsic regions, the hot-generated electrons and holes often cannot be treated semiclassically. However, once the carriers have escaped from the quantum wells and moved to the p or n regions, semiclassical treatment is appropriate for those carriers. For other carriers already in those doped regions the semiclassical model holds throughout. As previously mentioned, the dynamics discussed in this article do not treat vertical transport, legitimizing this assumption.
25 Even if the thermoelectric effect was not ignored, the magnitude of the resulting diffusion is within an order of magnitude of the low-field regular carrier diffusion values and often significantly smaller. (See Ref. 24). As will be shown, the enhanced diffusion coefficients are two to three orders of magnitude larger, and thus ignoring the temperature gradient seems reasonable. The actual diffusion coefficient will be slightly larger than what is predicted.
26 The generalized Einstein relations provide the relationship (See Ref. 6)

\[ \mu_r = \frac{eD_r}{k_B T n} \]

where \( \mu_r \) is the Fermi–Dirac integral of order \( r \), \( k_B \) is Boltzmann's constant, and \( D_r \) is the reduced chemical potential in which \( \phi_0 \) is the quasi-Fermi level, and \( D_r \) is the MB diffusion coefficient. Here, \( n \) denotes electrons. A similar expression may be written for holes.
27 Even at low energies, tens of thousands to hundreds of millions of electrons and holes may be photogenerated in the intrinsic region. Therefore, if the device response is not sensitive to individual electrons or holes, ignoring statistical fluctuations is often reasonable.
30 As the combination of Eqs. (10), (11c), and (13) in Eq. (14) suggests, \( \partial \phi_0 / \partial n \) is a well-defined quantity.
32 Recall, the gradient of the difference between the quasi-Fermi levels is similar to an induced electric field and, therefore, behaves as a “drift” term in a drift–diffusion equation. The magnitude of this term is orders of magnitude larger than regular diffusion coefficients. Behavior similar to diffusive conduction, in which the regular diffusion terms are simply dropped, is therefore not surprising.
33 The values for the diffusion coefficient of giant ambipolar diffusion and diffusive conduction presented in this article are measured results from test devices. Their values are different from each other simply due to the differences in their material structure and doping compositions.
34 In principle, the lateral voltage distribution could also cause the carriers in the intrinsic region to move horizontally. We neglect such movement on the assumption that the spot radius, \( a_{sp} \), is much larger than the thickness of the intrinsic region. Consequently, the lateral carrier gradient (and hence transport due to regular carrier diffusion) is small even when compared to the vertical carrier gradient.
35 This ignores the logarithmic dependence of the quasi-Fermi levels on electron and hole density. This is a reasonable approximation as the magnitude of the linear effects in practical examples are larger by two or more orders of magnitude.
36 Although the conductivity is related to the resistance, Eq. (24) disregards the distinction between the assumed MB distribution of giant ambipolar diffusion and the unrestricted distribution of diffusive conduction.
37 The charge density–voltage relationship of Eq. (29) implicitly describes a functional relationship of a change in both the electron and hole density that are vertically separated.
38 M. B. Yairi, H. V. Demir, and D. A. B. Miller, Optical and Quantum Electronics 33, 1035 (2001).