Sequential Optimization of Adaptive Arrays in Coherent Laser Communications

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***Abstract*—In optical wireless communications, a channel-matched adaptive coherent receiver may be implemented using an array of receive apertures. After atmospheric channel fading estimation, several replicas of a message received through the atmosphere are combined. As an alternative to training-based channel estimation, we analyze the performance of sequential techniques for direct optimization of multi-aperture array receivers in free-space coherent laser communications.**

***Index Terms*—Adaptive coherent array receivers, coherent communications, free-space optical communications, sequential channel matching.**

I. INTRODUCTION

RECENT advances in coherent optical communications in fiber transmission systems [1] have stimulated interest in applying coherent detection in optical wireless communications (OWC) [2]–[9]. In a coherent OWC system, transmitted information can be encoded in the complex electric field, including amplitude, phase and polarization. Coherent OWC can provide excellent background noise rejection and offer improved spatial and frequency selectivity.

A coherent receiver can measure all the degrees of freedom in the complex electric field by interfering the received signal with a local oscillator. In a free-space coherent system, atmospheric turbulence can reduce the spatial coherence of the received signal that is to be mixed with the local oscillator. The downconverted coherent power is maximized when the spatial field of the received signal matches that of the local oscillator [2]–[5]. A single-aperture phase-compensated coherent receiver based on adaptive optics can overcome atmospheric limitations by adaptive tracking of the beam wavefront and correction of atmospherically induced aberrations [4].

As an alternative to single-aperture receivers, there is at present significant interest in coherent OWC systems based on linear combining techniques, where signals detected by multiple receive apertures in an array are combined to reduce the probability of deep fades and improve detection efficiency (see Fig. 1) [6]–[9]. In effect, after propagation through atmospheric turbulence, the optical signal to each receiver aperture varies randomly with time. If atmospheric fading information is known for each aperture, the corresponding output signals can be adaptively processed, co-phased, and scaled before they are summed, mitigating signal fading caused by atmospheric turbulence. The result of this signal processing and linear combining is an optimal, channel-matched adaptive coherent receiver implemented using multiple receive apertures.

Since the atmospheric channel characteristics are unknown and change over time, the preferred expression of the combiner performing channel-fading estimation is a structure that is adaptive in nature. Transmitter-assisted techniques employ a pre-assigned time slot, which is periodic for the time-varying atmospheric channel, during which a training signal known in advance by the receiver is transmitted. In the receiver, weight coefficients are then changed and adapted to the fading conditions by using adaptive algorithms and linear estimation so that the output of the receiver carefully matches the training sequence. However, presence of this training sequence with the transmitted information adds an overhead and thus reduces the throughput of the system. In an array system, this overhead is more intense because a dedicated training sequence for each array element has to be inserted in the transmitted information so that each element can estimate its useful fading adaptation coefficients. In high-data-rate coherent optical systems, the transmission of a training sequence would be very costly in terms of data throughput and would grow linearly with the symbol data rate and the number of elements in the array. Therefore, to reduce the system overhead, adaptation schemes may be preferred that do not require training.

To improve bandwidth utilization, we have investigated adaptation techniques where simpler receiver architectures can be considered. We develop an efficient method for fading parameter adaptation of signal components based on an iterative optimization technique. The adaptation approach is based on sequential optimization of signal components in each subaperture, and works by iteratively updating individual adaptation parameters to maximize coherent received power.

In Section II we describe the adaptation technique as an optimization problem, presenting the basic concepts incorporated in our approach and introducing the implementation of subaperture piston phase. In Section III we estimate the efficiency of the adaptation and draw relevant conclusions.

II. CHANNEL ADAPTATION AS AN OPTIMIZATION PROBLEM

Adaptive arrays may be implemented using two options. In electrical combining, optical signals are downconverted separately, and the electrical signals are combined to improve

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Manuscript received December 10, 2012; revised February 25, 2013; accepted February 26, 2013. Date of current version March 12, 2013. The work of A. Belmonte was supported in part by the Spanish Department of Science and Innovation MICINN under Grants TEC 2009-10025 and TEC 2012-34799.

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Digital Object Identifier 10.1109/JLT.2013.2250484

detection statistics. In principle, optical combining is possible (Fig. 1). In this approach, a set of phase shifters is used to co-phase the optical signals from the subapertures prior to optical linear combining and downconversion of the combined optical signal. When using either combining technique, system performance should improve with an increasing number of receivers. Although the main conclusions of the analysis are equally applicable to electrical combining, we focus here on channel-matched adaptive receivers using optical combining. We consider a fiber array where each subaperture feeds a single-mode fiber and where the fields of each fiber are properly phased using fiber optic phase shifters (performing as piston actuators) and coherently added in a fiber combiner. The array output is then available in a single fiber and superimposed with the local oscillator field in a directional coupler. The receiver uses a balanced detector, and a digital sampler connected directly to the output port element of the receiver, so the instantaneous downconverted electrical signal can be measured coherently.

For a coherent array receiver system that is not phased to the optical input field, the signals received by the different subapertures in the array exhibit random fluctuations of both envelope and phase over time, and destructive interference occurs in the optical combiner. Acting as a multiplicative noise affecting the received signals, optical fading on the L receive subapertures can be aggregated into a complex channel column vector $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \ldots, \alpha_L)^{\boldsymbol{T}} \in \mathbb{C}^{\boldsymbol{L}}$, where the superscript ^{T} denotes transposition. A general entry of the atmospheric fading vector is denoted by

$$\alpha_l = |\alpha_l| \exp(j\phi_l) \tag{1}$$

where $|\alpha_l|$ represents the fading envelope and ϕ_l the corresponding random phase of the optical signal at subaperture $l \in \{1, 2, \ldots, L\}$.

Here, an adaptive linear combiner $\boldsymbol{w}$ is considered to compensate for fading effects and match the coherent array to the optical input field. In this case, the fading vector becomes $\boldsymbol{w}^*\boldsymbol{\alpha}$, where the asterisk denotes complex conjugate transposition and $\boldsymbol{w} = (w_1, w_2, \ldots, w_L)^{\boldsymbol{T}} \in \mathbb{C}^{\boldsymbol{L}}$ are the (as yet unknown) weights of the linear combiner. The complex weight applied to the lth subaperture output can be characterized at large as

$$w_l = |w_l| \exp(j\theta_l) \tag{2}$$

where $|w_l|$ and θ_l are the amplitude and phase controls, respectively, provided by the linear combiner. For an optical combiner set with phase shifters, actuators provide phase (piston) θ_l control only and $|w_l|$ is unity. In any case, both phase and amplitude control could be achieved by introducing an additional amplifier stage with gain $|w_l|$ after each optical phase shifter in the combiner.

For a coherent optical receiver whose dominant noise source is local oscillator shot noise, the signal SNR γ can be taken as the number of signal photons collected on the array aperture γ_0 multiplied by the fading envelope $|\boldsymbol{w}^*\boldsymbol{\alpha}|^2 = \boldsymbol{w}^*\boldsymbol{\alpha}\boldsymbol{\alpha}^*\boldsymbol{w}$

$$\gamma = \gamma_0 |\boldsymbol{w}^*\boldsymbol{\alpha}|^2 \tag{3}$$

Note that, conditional on a realization of the atmospheric channel described by $\boldsymbol{\alpha}$, this is an additive white Gaussian noise (AWGN) receiver where the signal SNR is a function of the random channel through the fading envelope $|\boldsymbol{w}^*\boldsymbol{\alpha}|^2$. Here, $|\boldsymbol{w}^*\boldsymbol{\alpha}|^2 = \gamma/\gamma_0$ acts as a signal combining efficiency. In ideal combining systems, where $\boldsymbol{w} = 1/\boldsymbol{\alpha}^*$ produces perfect mixing of the array signals, the efficiency is unity and $\gamma = \gamma_0$. When the signals are not properly combined, the contributions to the total power from different subapertures of the array can interfere destructively, reducing the instantaneous combining efficiency $|\boldsymbol{w}^*\boldsymbol{\alpha}|^2$ and causing diminished SNR $\gamma < \gamma_0$.

The problem is now to determine an optimal coefficient vector $\boldsymbol{w}$ so as to maximize the instantaneous output SNR for the current observed channel vector $\boldsymbol{\alpha}$. To find the optimum weight vector solution, in training-based channel estimation, we assume the availability of an input sequence of training signals and its corresponding observation sequence γ of receiver SNR outputs. Then, a simple maximum-likelihood method using a linear least square estimator can be implemented to obtain the atmospheric fading components $\alpha_l \{1 \leq l \leq L\}$ and match the receiver using the ideal combiner weights $w_l = 1/\alpha_l^*$. However, training-based operation is not effective when the channel changes rapidly with time. In atmospheric OWC we need to consider the performance of techniques that do not rely on training symbols for adaptation.

We present a new channel-matched array receiver based on a search of the optimum weight vector solution $\boldsymbol{w}$ by successive selecting the lth branch (subaperture) in the receiver, trying different settings of the corresponding lth combiner actuator (piston phase settings if phase shifters actuators are consider), selecting the optimal weights w_l to maximize the combined SNR, and repeating the procedure for all branches $l \in \{1, 2, \ldots, L\}$ in the receiver. The method has the attractive property that the combiner weight estimates can be obtained in a closed form from optimizing a quadratic cost function based on the measurement SNR in (3).

Having said that, we re-formulate the optimization of (3) as in terms of the quadratic objective function

$$\gamma = \gamma_0 \sum_{l=1}^{L} \left\{ |\alpha_l|^2 |w_l|^2 + \left[\sum_{m \neq l} w_m^* \left(\alpha_m \alpha_l^*\right) \right] w_l \right\} \tag{4}$$

Because quadratic optimization problems are always convex, standard multivariable quadratic programming algorithm would always find a global, although not necessarily unique, solution [10]. However, we consider coordinate-wise ascent algorithms for our convex optimization problem [11]. Underlying coordinate ascent approaches is the decomposition of the overall detection problem into a set of smaller subproblems of decreasing dimension. Coordinate ascent is attractive because scalar maximization is simpler than multivariate maximization.

For our multivariable maximization problem, the coordinate-ascend approach is based on the fact that the total optical field is a linear superposition of the contributions from all subapertures. This means that we can construct the optimal linear combiner by optimizing each of the weights w_l individually. The method maximizes the objective function (4) by solving a sequence of scalar maximization subproblems, where each subproblem improves the estimate of the solution by maximizing w_l along a

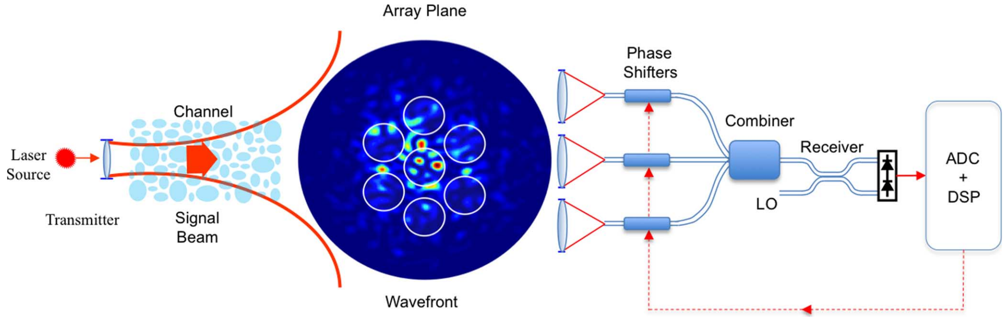

Fig. 1. In optical wireless communications, channel-matched coherent diversity receivers are implemented using multiple optical apertures. As an alternative to electrical combining, we consider optical combining where a set of phase shifters co-phases the optical signals prior to optical combining and electrical downconversion. Phase shifters actuators provide phase control only. Phase and amplitude control could be achieved by introducing an additional amplifier stage after each optical phase shifter in the combiner. An analog-to-digital converter (ADC) is used in conjunction with digital signal processing (DSP) manipulation.

selected coordinate l with all other coordinates $j \neq l$ fixed. The objective function (4) becomes:

$$\gamma = \gamma_0|\alpha_l|^2|w_l|^2 + \left[\gamma_0 \sum_{m\neq l} w_m^*\left(\alpha_m\alpha_l{}^*\right)\right] w_l + C_l \tag{5}$$

Here, the constant term $C_l \in \mathbb{R}$ combines instantaneous SNR for coordinates $j \neq l$:

$$C_l = \gamma_0 \sum_{j\neq l}\left\{|\alpha_j|^2|w_j|^2 + \left[\sum_{m\neq j} w_m^*\left(\alpha_m\alpha_j{}^*\right) w_j\right]\right\} \tag{6}$$

Equation (5) is just a simple quadratic function on w_l

$$\gamma = A_l|w_l|^2 + B_l w_l + C_l \tag{7}$$

and its maximum, being a vertex point, accept the closed solution $w_l = -B_l/2A_l$. Constants $A_l \in \mathbb{R}$ and $B_l \in \mathbb{C}$ are easily identifiable in (5) as

$$\begin{aligned} A_l &= \gamma_0|\alpha_l|^2 \\ B_l &= \gamma_0 \sum_{m\neq l} w_m^*\left(\alpha_m\alpha_l{}^*\right) \end{aligned} \tag{8}$$

As the solutions to the optimization problem in (7) are expressed in closed form, they can be solved quickly. The coordinate-wise ascent procedure must find the values of the constants $A_l, C_l \in \mathbb{R}$ and $B_l \in \mathbb{C}$ varying sequentially the parameter w_l along a specific path. It requires four independent measurements of the instantaneous coherent SNR γ obtained when the settings w_l of the corresponding lth combiner is adjusted sequentially to four different specific values. These settings define the matrix of coefficients relating the constants A_l, B_l, and C_l to the SNR measurements. An inverse matrix method finds the quadratic function and its optimal solution $w_l = -B_l/2A_l$.

The relevant equations can be updated sequentially as we cycle through the variables w_l for $l = 1, \ldots, L$. The idea is to apply a coordinate-wise ascent procedure for each value of the parameter w_l and use each solution as a start for the next estimate w_{l+1}. The weight of each branch is set to its optimal value $w_l = -B_l/2A_l$ directly after each estimation. With this approach the optimization process runs continuously and dynamically follows changes in the fading signals behavior. Furthermore, the combined signal starts to increase directly, which increases the signal to noise ratio of successive measurements.

The physical interpretation of (5) is easy when the optical combiner is be set with phase shifters (see Fig. 1). Phase shifters actuators provide phase (piston) control only, $|w_l|$ is unity in (2), and the quadratic function (5) becomes a circular function of the piston phase shift θ_l at the lth array branch

$$\gamma = B_l exp\left(j\theta_l\right) + \left(A_l + C_l\right) \tag{9}$$

In this case, the argument of the optimal combiner coefficient w_l is equal to the argument of B_l, i.e., the angle difference

$$\theta_l = \chi_l - \phi_l \tag{10}$$

Here, ϕ_l is the argument of the optical signal (1) at the array branch with index l, and χ_l is the argument of B_l, i.e., the argument of the optical signal aggregated from the apertures with index $m \neq l$

$$\chi_l = \arg\left(\sum_{m\neq l} w_m^*\alpha_m\right) \tag{11}$$

As expected, phase-compensated receivers maximizes the potential for overcoming atmospheric fading with the piston angle θ_l by adaptive tracking of the beam wave-front ϕ_l of each array branch and phasing the optical signal to the aggregated angle χ_l. At this optimal phase, the contribution of the single lth subaperture is in phase with the background contribution coming from all other subapertures. For this phase-only combiner, the procedure needs just three measurements to estimate the constants $B_l \in \mathbb{C}$ **and** $(A_l + C_l) \in \mathbb{R}$.

III. PERFORMANCE EVALUATION AND CONCLUSIONS

We illustrate the performance profiles offered by the proposed array adaptation approach when a practical optical combiner, set with phase shifters, confronts typical atmospheric channel conditions. We have conducted a numerical analysis and the following set of experiments is performed on synthetically built array signals. It considers L-element arrays and assumes that each subaperture contains a lens that couples the received light into a single-mode fiber. For comparison of the receiver performance between a L-element array and a single large aperture, we force each subaperture in the array to have a pupil area equal to $1/L$ times the pupil area of the single receiver system. The adaptation algorithm changes the signal phase of one subaperture at a time. When one subaperture contributes only a small fraction of the optical power, the change in signal to noise ratio is low.

In order to synthesize the array signals, we consider that the statistical properties of the atmospheric channel fade α_l are well described by a Rice distribution [7]. Moderate-to-strong turbulence levels are used in this analysis. Atmospheric turbulence is quantified by a normalized aperture diameter $D/r_0 = 10$. Here, D is the aperture diameter of the single receiver system and the wavefront coherence diameter r_0 describes the spatial correlation of phase fluctuations in the receiver plane. For a fixed coherent diameter r_0, as aperture diameter D is increased, turbulence reduces the photoelectric downconversion efficiency. In order to assess the impact of turbulence, both wavefront phase and amplitude fluctuations should be considered. The value of the scintillation index set to $\sigma_\beta^2 = 0.3$ corresponds to scintillation below the saturation regime.

Fig. 2 considers (a) co-phasing error and (b) combining efficiency of the adaptation technique as a function of the number of subapertures L of the array ($L = 2, 4, 8, 12$) and the number of signal photons collected on the array aperture γ_0. Any practical coherent OWC system can easily provide a large number of photons per measurement. In effect, note that the number of signal photons detected per measurement γ_0 can be related to the number of photons-per-bit γ_b collected by the array aperture

$$\gamma_b = \frac{3L\gamma_0}{\tau R_b} \tag{12}$$

where $3L$ is the number of measurements required to adapt the L-branch array, R_b is the link bit rate in bits/s, and τ is the atmospheric coherent time. The rate at which phases must be adjusted will be dictated by the rate $1/\tau$ at which the atmospheric turbulence fluctuates, generally no higher than 1 kHz. For example, a moderate-rate 10-Mbits/s link needs to provide less than 4 photons-per-bit to have 1000 photons per measurement (30 dB measurement SNR) and guarantee array adaptation within an atmospheric coherent time of 1 ms. Interestingly, the number of photons-per-bit required by the adaptation algorithm is smaller than the number of photons-per-bit required by the sensitivity of any ideal communication receiver with coherent detection and AWGN [12].

In this array scheme, the receiver co-phases the intermediate signals and sums them to obtain an improved composite signal. The adaptation algorithm needs to find the optimal phase setups for the combiner where the contribution of the single lth subaperture is in phase with the background contribution coming from all other subapertures (inset, Fig. 2(a)) so all signal array contributions end up with almost identical phase (inset, Fig. 2(b)). Fig. 2(a) shows that the adaptation requires approximately 10000 photons per measurement (40 dB measurement SNR) to co-phase all the channels in the arrays within a very narrow error margin of just 3°. On the analysis we have considered phase shifters with continuous control over the phase prior to the optical combiner. With a digital phase shifter, which only approximates linear phase shift with discrete phase steps, quantization error would be negligible for phase resolution greater than 7 bits (2.8 degrees).

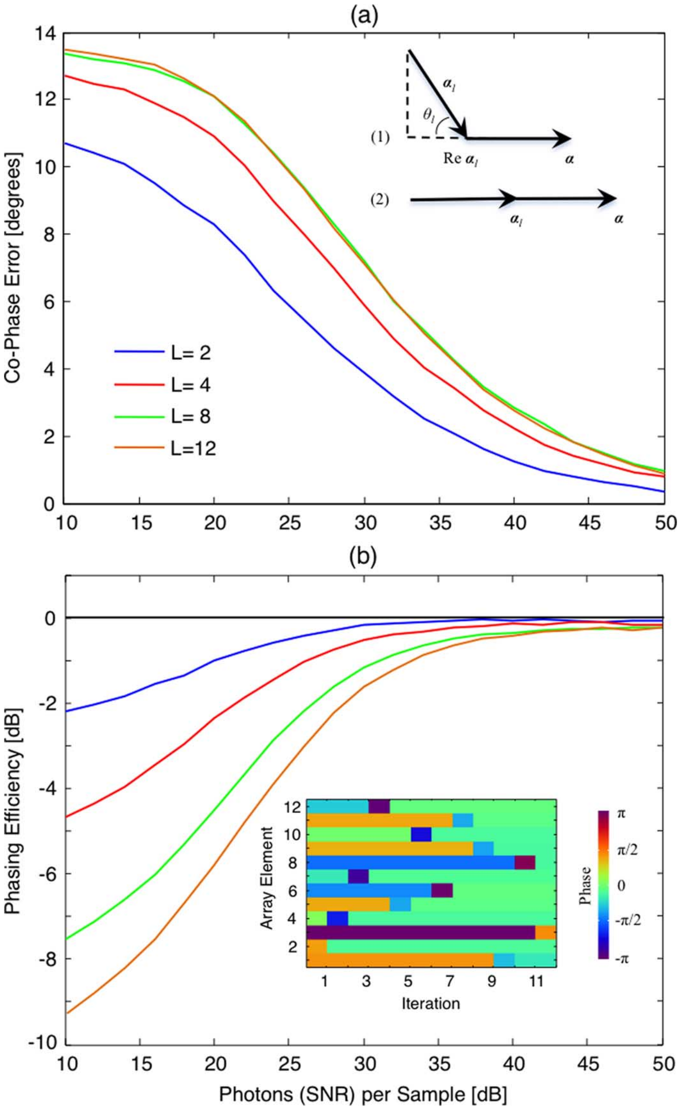

Fig. 2. Co-phasing error (a) and combining efficiency (b) of the adaptation technique as a function of the number of subapertures L in the array and the number of signal photons (SNR) collected on the array aperture γ_0. In (a) (inset), the fading contribution α_l of the single lth subaperture is in phase, after adaptation, with the background contribution coming from all other subapertures. In (b) (inset), the adaptation algorithm needs to find the optimal phase setups for the combiner so the array signal contributions end up with almost identical phase after each coordinate $l = 1, \ldots, L$ is visited (step) just one time. In the example, we have considered $L = 12$.

Fig. 2(b) shows the corresponding combining efficiency, defined as the ratio of the actual SNR γ to the ideal SNR γ_0 obtainable with perfect phasing of the subfields. For a 40-dB measurement SNR, combining efficiency is better than -1 dB in all the cases considered in this analysis.

In conclusion, to improve bandwidth utilization, we have presented a new channel-matched receiver based on a search of the optimum phase vector solution by successive selecting the received signal in the lth subaperture, trying different settings of the corresponding lth phase shifter, selecting the optimal lth phase to maximize the fading SNR, and repeating the procedure for all branches in the receiver.

The fading estimation is linear in phase for each phase shifter l, it can be done with just 3 measurements and, as satisfies conditions of convex problem, it is guaranteed to converge to a global optimum. This direct optimization approach, which is quite tractable and where no training sequences are used, allows maximizing convergence speed to achieve efficient signal coherent downconversion.

Each coordinate l maximization can be done quickly, so the iterations are very fast. Generally, each coordinate $l = 1, \ldots, L$ is visited just one time to reach a maximum and nearly ideal detection can be achieved in a successive manner with a single-pass receiver, without cycling over the combiner many times.

As the combining algorithm outputs match the phases of the optical signals in the array aperture, the system can be used to find the phase distribution of the incoming optical field adaptively. Interestingly, other than providing a means of spatially integrating the signal in coherent detection, this simple optimal combining scheme may find several applications in image synthesis and remote sensing.

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