

# Queueing models of optical delay lines in synchronous and asynchronous optical packet-switching networks

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**Abstract.** Buffering with optical delay lines (ODLs) in optical packet-switching networks is a way to mitigate network contention. We study queueing models of ODLs in synchronous and asynchronous optical packet-switching networks under uniform Bernoulli traffic. We first introduce a Markov chain model for a finite-length ODL forward-buffering system in synchronized networks and calculate the stationary distribution of its queueing length as well as two important queueing parameters: packet loss rate (PLR) and average queueing delay (AQD). We then introduce an asymptotic analysis based on the generating function of an infinite buffering system and present approximate expressions for PLR and AQD. Numerical calculations demonstrate that these asymptotic estimates of PLR and AQD are quite accurate. We then extend the queueing analysis to feedback-buffering ODLs in synchronized networks. We present numerical calculations of PLRs and AQDs for feedback-buffering ODLs. Finally, we introduce queueing models in asynchronous networks for forward buffering or for feedback buffering without multiple recirculations. We carry out the asymptotic analysis to characterize the performance degradation of ODL buffering in asynchronous switching when the traffic load is high. © 2003 Society of Photo-Optical Instrumentation Engineers. [DOI: 10.1117/1.1572500]

Subject terms: optical packet switching; optical delay line; Markov chain; packet loss rate; average queueing delay.

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## 1 Introduction

Telecommunication networks are experiencing a dramatic increase in demand for capacity. The wide range of future services will require networks to handle diverse forms of traffic. Optical packet switching has attracted considerable research interest because of its potential to achieve higher capacity and increased flexibility.<sup>1-8</sup> Migrating switching functionality from electronics to optics can resolve the electrical-optical-electrical conversion bottleneck in optical networks. Optical packet switching also offers a finer bandwidth granularity than circuit switching, enabling more efficient bandwidth sharing among different applications. Optical packet-switching networks can be divided into two categories: synchronous (slotted) and asynchronous (unslotted) networks. In a synchronous network, all the input packets have the same size and are aligned in phase before entering the switching matrix. In an asynchronous network, the packets may or may not have the same size and they arrive and enter the switch without being aligned. Therefore, the packet-by-packet switch action can occur at any point in time.<sup>8</sup>

In all-optical packet-switching networks, high-speed optical cross-connects (OXC) are required at each switching node to route packets. However, when two or more packets try to exit an OXC from the same output port and they are on the same channel, i.e., the same wavelength, contention

occurs. In electrical packet-switching networks, contention is usually resolved using the store-and-forward technique, where packets in contention are stored in a queueing memory and sent out when the desired output port is free. Because optical random-access memory (RAM) is not available currently, it is difficult to implement an equivalent approach in OXCs. Many other contention resolution schemes have been proposed, including schemes exploiting three dimensions: wavelength,<sup>4,5</sup> time,<sup>1,6</sup> and space.<sup>7</sup>

Time-domain contention resolution schemes typically make use<sup>6</sup> of optical delay lines (ODLs) as first-in-first-out (FIFO) buffers in OXCs. ODLs are fixed-length optical fibers. An optical packet propagates through an ODL after a fixed amount of time, which is chosen to be an integer multiple of the duration of a packet. In this paper, we assume the input packets in asynchronous networks also have the same size, which is the case for most proposed asynchronous optical packet-switching networks.<sup>8,9</sup> Various optical buffering schemes based on ODLs have been proposed. They can be simply divided into two categories<sup>8</sup>: forward buffers and feedback buffers. The difference is that the optical packets stored in feedback buffers reenter the switching matrix before exiting the desired output port, while optical packets in forward buffers do not. In this paper, we study the queueing properties in both types of ODLs.

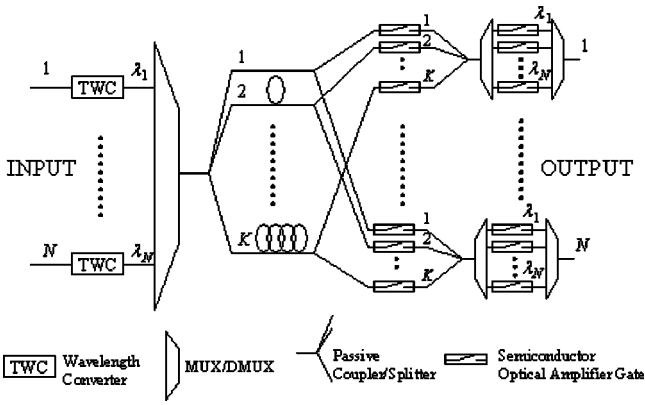


Fig. 1 Broadcast and select switch proposed in the KEOPS project.

In this paper, we assume uniform Bernoulli traffic, in which case, the arrival of incoming packets at each input port is memoryless and independent of the arrivals of other input ports. There is a constant probability  $Q$  (equal to the traffic load) that a packet will be received within a time slot. The traffic is uniformly distributed over the output ports except the one corresponding to its input. Such a simple model has some limitations, since real traffic is often bursty and is not necessarily uniformly distributed on all output ports. Nonetheless, our simple traffic models enable us to achieve an essential characterization of optical buffer performance.<sup>6</sup>

The remainder of this paper is organized as follows. Section 2 presents the ODL queueing models for synchronous optical packet-switching networks. In Sec. 2.1, we first introduce the Markov chain (MC) analysis for forward-buffering queues under uniform Bernoulli traffic and the calculations of two useful parameters to evaluate the performance of OXC: packet loss rate (PLR) and average queueing delay (AQD). We introduce an asymptotic analysis based on the generating function of the infinite buffering system. We use it to derive a simpler algorithm to approximate PLR and AQD. Then we present numerical calculations to demonstrate the accuracy of such approximations. Section 2.2 further extends the queueing analysis to study the queueing properties of feedback-buffering ODLs. We also present numerical calculations and discussion to compare the performance of these two buffering schemes and their control algorithms. Section 3 introduces queueing models for asynchronous optical packet-switching networks and derive approximations for their PLR and AQD. We again introduce an asymptotic analysis of the infinite buffering system to study the performance degradation of asynchronous optical packet-switching networks under high traffic load. We conclude in Sec. 4.

**2 Queueing in Synchronous Optical Packet-Switched Networks**

In this section we present the queueing of ODLs in synchronous optical packet-switched networks. We will study the forward-buffering and feedback-buffering ODLs in Secs 2.1 and 2.2, respectively.

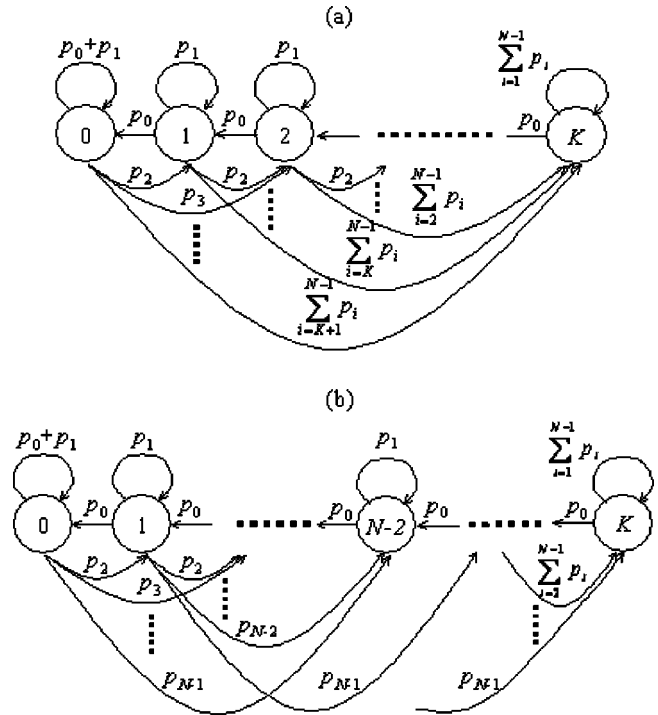


Fig. 2 MC transition diagram in forward-buffering ODLs with  $N$  pairs of input/outputs and  $K$  ODLs: (a)  $K \leq N-2$  and (b)  $K > N-2$ . The state of the MC is the queue length in units of the number of optical packets in the ODLs counted at the instant just before new packets enter the ODLs. Here,  $p_i$  is the probability distribution of a Binomial counting process, i.e., the sum of  $N-1$  independent Bernoulli random variable with probability  $p(1) = Q/(N-1)$ .

**2.1 Queueing in Forward-Buffering ODLs**

For forward buffering, the optical packets enter the switching fabric only once. An example is the broadcast and select space switch proposed<sup>1</sup> in European Advanced-Communication Technologies and Services (ACTS) KEOPS (keys to optical packet switching) (Fig. 1). The queueing model of one-stage forward-buffering ODLs under uniform Bernoulli traffic is simply an MC. Assume the OXC has  $N$  pairs of input/outputs and  $K$  ODLs with propagation delays  $1, 2, \dots, K$ , respectively. The corresponding MC transition diagrams of queues in optical delay lines for  $K \leq N-2$  and  $K > N-2$  are shown in Figs. 2(a) and 2(b), respectively. The state of the MC is the number of optical packets buffered in ODLs counted at the instant just before new packets entering ODLs, and  $p_i$  is the probability distribution of a Binomial counting process, i.e., the sum of  $N-1$  independent Bernoulli random variables with probability  $p(1) = Q/(N-1)$ :

$$p_i = \binom{N-1}{i} \left(\frac{Q}{N-1}\right)^i \left(1 - \frac{Q}{N-1}\right)^{N-1-i} \tag{1}$$

The balance equation of the MC in Figs. 2(a) and 2(b) can be written as a general expression for all  $K$  and  $N$ :

$$\pi_i = \begin{cases} \pi_0(p_0 + p_1) + \pi_1 p_0 & i=0 \\ \min(i+1, N-1) \sum_{j=0}^{i-1} p_j \pi_{i+1-j} & 1 \leq i \leq K-1 \\ \sum_{j=\max(0, K+2-N)}^K \pi_j \left( \sum_{l=K+1-j}^{N-1} p_l \right) & i=K, \end{cases} \quad (2)$$

where  $\pi_i$  = probability(queue length =  $i$ ) is the stationary distribution of the MC. Therefore,

$$\sum_{i=0}^K \pi_i = 1. \quad (3)$$

Using Eqs. (2) and (3), we can calculate the stationary distribution. The PLR is

$$PLR = \sum_{j=\max(0, K+3-N)}^K \pi_j \left[ \sum_{l=K+2-j}^{N-1} (l+j-K-1)p_l \right]. \quad (4)$$

The AQD is

$$AQD = \sum_{j=0}^K j \pi_j. \quad (5)$$

To further investigate the properties of this queueing system, we introduce an asymptotic analysis for  $K \rightarrow \infty$ . The generating function of such queueing system is

$$\Pi(s) = \sum_{i=0}^{\infty} \pi_i^{\infty} s^i. \quad (6)$$

From Eq. (2) and let  $K \rightarrow \infty$ , we can derive  $\Pi(s)$ :

$$\Pi(s) = \frac{(s-1)\pi_0^{\infty} p_0}{s-P(s)}, \quad (7)$$

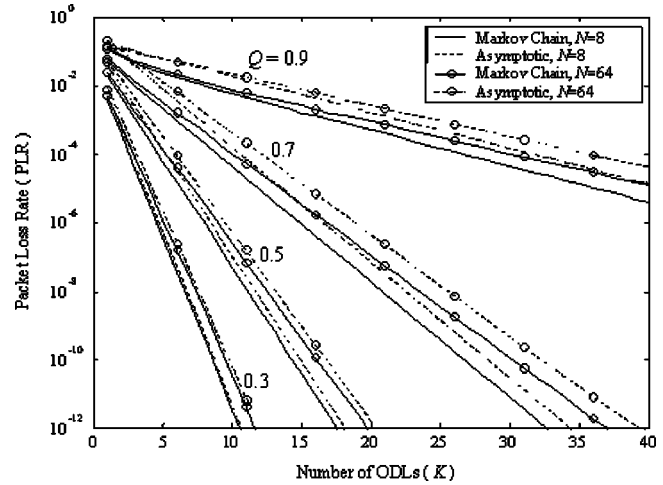
where  $P(s) = \sum_{i=0}^{N-1} p_i s^i = [1 - (Q/N-1) + (Qs/N-1)]^{N-1}$ . To calculate  $\pi_0^{\infty}$ , multiply  $s-P(s)$  on both sides of Eq. (7) and differentiate it. Since  $\Pi(s)|_{s=1} = 1$ ,  $P(s)|_{s=1} = 1$  and  $P'(s)|_{s=1} = Q$ , we have

$$\pi_0^{\infty} = \frac{1-Q}{p_0}. \quad (8)$$

From Eqs. (7) and (8), we have<sup>10,11</sup>

$$\Pi(s) = \frac{(1-Q)(s-1)}{s-P(s)}. \quad (9)$$

With Eq. (9), we can derive a simple algorithm to calculate the stationary distribution of queueing length with an infinite number of ODLs ( $K \rightarrow \infty$ ):



**Fig. 3** PLR versus the number of ODLs  $K$  with traffic loads  $Q = 0.3, 0.5, 0.7,$  and  $0.9$ , respectively, for forward-buffering ODLs in synchronous optical packet-switching networks. The number of input/output ports  $N$  is chosen to be 8 (lines without circles) and 64 (lines with circles), respectively. The PLRs calculated with the MC model analysis using Eqs. (2) to (4) are plotted using solid lines. The PLRs calculated with asymptotic generating function method using Eqs. (10) and (11) are plotted using dashed lines.

$$\pi_i^{\infty} = \begin{cases} \frac{1-Q}{p_0} & i=0 \\ \frac{1-Q}{p_0} \frac{1-p_0-p_1}{p_0} & i=1 \\ \frac{\pi_{i-1}^{\infty} - \sum_{j=1}^{\min(i, N-1)} p_j \pi_{i-j}^{\infty}}{p_0} & i > 1. \end{cases} \quad (10)$$

It is well known that for finite length buffer ( $K < \infty$ ) the PLR in a wide variety of queueing models can be approximated by<sup>12</sup>

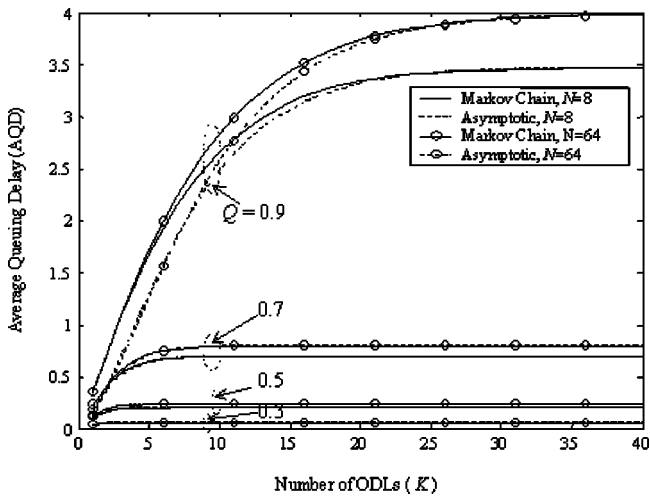
$$PLR \approx \text{prob}(\text{queue length} > K) = \sum_{i=K+1}^{\infty} \pi_i^{\infty} = 1 - \sum_{i=0}^K \pi_i^{\infty}, \quad (11)$$

while the AQD can be approximated by

$$AQD \approx \sum_{i=0}^K i \pi_i^{\infty}. \quad (12)$$

To verify that the approximations in Eqs. (11) and (12) are valid for the queueing model proposed here for forward-buffering ODLs with uniform Bernoulli traffic, we present some numerical calculation results for the queueing models of forward-buffering ODLs already proposed.

We first calculate the PLR versus the number of ODLs  $K$  with traffic load  $Q = 0.3, 0.5, 0.7,$  and  $0.9$ , respectively. We assume the number of input/output ports  $N$  to be 8 and 64. The results are indicated in Fig. 3 by lines without and with circles, respectively. The PLRs are calculated with the MC model analysis using Eqs. (2) to (4) (solid lines) and the asymptotic generating function method using Eqs. (10) and (11) (dashed lines), respectively, in Fig. 3. The corre-



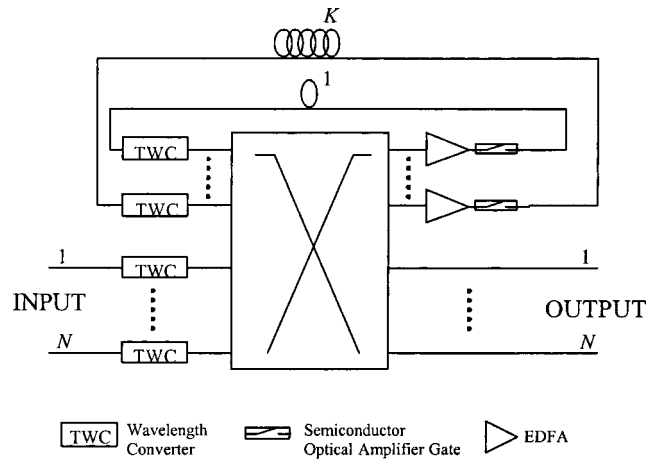
**Fig. 4** AQD versus the number of ODLs  $K$  with traffic loads  $Q = 0.3, 0.5, 0.7,$  and  $0.9,$  respectively, for forward-buffering ODLs in synchronous optical packet-switching networks. The number of input/output ports  $N$  is chosen to be 8 (lines without circles) and 64 (lines with circles). The AQDs are calculated with the MC model analysis using Eq. (5) (solid lines) and the asymptotic generating function method using Eq. (12) (dashed lines), respectively.

sponding results of AQD versus  $K$  are shown in Fig. 4. The AQDs are calculated with Eq (5) (solid lines) and Eq. (12) (dashed lines), respectively.

From Figs. 3 and 4, we see that the results calculated by asymptotic analysis and their approximations by Eqs. (10) and (12) are in very close agreement to those calculated by the MC model for finite-length ODLs. The asymptotic analysis method therefore offers a much simpler algorithm to study forward-buffering ODLs and the performance of optical packet-switching networks implementing such contention resolution methods. We extend the asymptotic analysis and their approximation to model the feedback-buffering ODLs in Sec. 2.2.

From the numerical results in Figs. 3 and 4, we can gain insight into the applicability and limitations of the contention resolution scheme using forward buffering. We observe that when the traffic load is not very high, even several optical buffering ODLs can greatly help to decrease the PLR and will not introduce significant delay in the OXC. When the traffic load is very high, however, we require a very large number of ODLs to achieve a low PLR. In this case, the AQD will increase, introducing significant delay. Also, longer ODL propagation will more severely degrade the signal-to-noise ratio (SNR) of the optical signals, imposing limits on the number of ODLs that can be used in heavily loaded optical packet-switching networks. In this case, other contention resolution schemes, such as wavelength conversion<sup>4,5</sup> and deflection routing,<sup>7</sup> should be considered. Most recent optical buffering designs also employ wavelength conversion.<sup>4</sup>

Another observation based on our results is that PLR and AQD are not sensitive to the number of input/output ports  $N$  in an OXC. This is an artifact of the uniform Bernoulli traffic assumption made in this paper.<sup>10</sup> In real OXCs, traffic might not be uniformly distributed over all



**Fig. 5** SMOP switch.

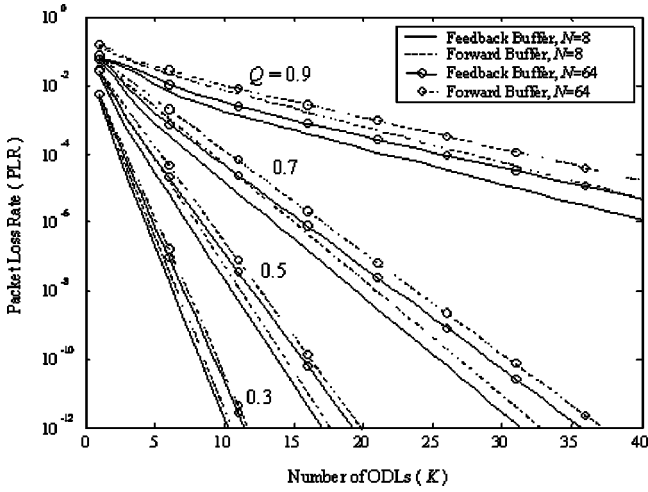
input/output ports, and some busy input/output ports will inevitably experience a more severe degradation in PLR if  $N$  increases.<sup>13</sup>

## 2.2 Queueing in Feedback-Buffering ODLs

In feedback-buffering OXCs, the congested packets are stored in the recirculating ODLs and will reenter the switching matrix to be routed to the desired output port, recirculated again, or discarded, depending on the current queueing state and the choice of control algorithm. Therefore, control algorithms in feedback-buffering OXCs are more complex than in forward-buffering OXCs. In feedback-buffering OXCs, buffered packets must be properly allocated in ODLs to avoid having more than one packet destined for the same output port reentering the switching matrix after recirculation, and to avoid unnecessary delays. Also, extra control mechanisms are required to recirculate or preempt buffered packets. However, in feedback-buffering systems, it is possible for higher priority packets to preempt the lower priority packets that are already in ODLs, enabling implementation of quality of service (QoS) in networks. An example is the shared-memory optical packet (SMOP) switch, as shown<sup>5</sup> in Fig. 5.

In this paper, we do not consider QoS, so that the buffering will follow the FIFO queueing discipline. We assume that the feedback-buffering OXC also has  $N$  pairs of input/outputs and  $K$  ODLs with propagation delays  $1, 2, \dots, K,$  respectively. Since we can recirculate the packets in feedback ODLs, we may be able to make full use of the  $K(K + 1)/2$  available buffering slots. However, the detailed MC transition diagram of queueing in ODLs depends on the corresponding control algorithm. In this section, we simply assume that all  $K(K + 1)/2$  buffering slots are available.

From Sec. 2.2, we observe that the asymptotic analysis of an infinite FIFO buffering system offers an accurate way to evaluate the forward-buffering system. In this section, we will extend this asymptotic analysis to study the feedback-buffering systems operating under a FIFO queueing discipline. We can approximate the stationary distribution of  $\pi_i [0 \leq i \leq K(K + 1)/2]$  with  $\pi_i^\infty$  using Eq. (10). This



**Fig. 6** PLR versus the number of ODLs  $K$  with traffic loads  $Q = 0.3, 0.5, 0.7,$  and  $0.9,$  respectively, for feedback buffering (solid lines) and forward buffering (dashed lines) in synchronous optical packet-switching networks. The number of input/output ports  $N$  is chosen to be 8 (lines without circles) and 64 (lines with circles). The PLRs of feedback-buffering ODLs are calculated using Eq. (13) and the PLRs of forward-buffering ODLs are calculated using Eqs. (2) to (4).

is a good approximation in the region of interest, where the PLR is low. In this case, the PLR can be approximated by

$$\text{PLR} = \sum_{i=0}^K \left\{ \sum_{j=\max(0, i+3-N)}^{\max(0, i-1)} \frac{\pi_{K(K+1)}^\infty}{2} - \frac{i(i+1)}{2} + j \left[ \sum_{l=i+2-j}^{N-1} (l+j-i-1)p_l \right] \right\}, \quad (13)$$

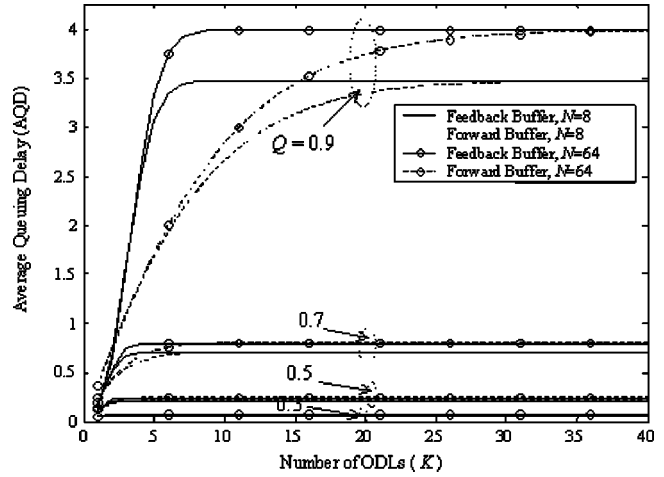
and the AQD can be approximated by

$$\text{AQD} = \sum_{i=0}^{K(K+1)/2} i \pi_i^\infty. \quad (14)$$

Here we present some numerical calculations for the queuing models of feedback-buffering ODLs in the proposed synchronized networks and compare it with the forward-buffering case.

We first calculate the PLR of feedback-buffering ODLs versus the number of ODLs  $K$  with traffic loads  $Q=0.3, 0.5, 0.7,$  and  $0.9,$  respectively. We assume the number of input/output ports  $N$  to be 8 and 64. The results are indicated in Fig. 6 by solid lines without and with circles, respectively. The PLRs are calculated using Eq. (13). To compare these with the results of forward-buffering schemes, we also plot the PLRs calculated using Eqs. (2) to (4) (dashed lines) in Fig. 6. The corresponding results of AQD versus  $K$  for both feedback buffering and forward buffering in synchronous optical packet-switching networks are shown in Fig. 7. The AQDs are calculated with Eq. (14) (solid lines) and Eq. (5) (dashed lines), respectively.

From Fig. 6 we observe that although feedback buffering has the potential to make full use of the  $K(K+1)/2$



**Fig. 7** AQD versus the number of ODLs  $K$  with traffic loads  $Q = 0.3, 0.5, 0.7,$  and  $0.9,$  respectively, for feedback buffering (solid lines) and forward buffering (dashed lines) in synchronous optical packet-switching networks. The number of input/output ports  $N$  is chosen to be 8 (lines without circles) and 64 (lines with circles). The AQDs of feedback-buffering ODLs are calculated using Eq. (14) and the AQDs of forward-buffering ODLs are calculated using Eq. (5).

available buffering slots provided by ODLs, the PLR does not improve much compared to that of forward buffering. This is because the traffic is memoryless and during each time slot the feedback-buffering ODLs can still accept at most  $K$  packets, and perhaps fewer if the queue is full. Therefore, the PLRs for feedback-buffering OXC's are just slightly better than forward-buffering OXC's. In Fig. 7, we can see the AQDs of feedback-buffered packets reach the saturation AQD much faster than forward-buffered packets, especially when the load is high. This is because the queues in feedback buffers are longer, and therefore introduce more delay for delivered packets. To serve real-time applications, we must implement packet priorities to enable QoS management.

From the numerical results in Figs. 6 and 7, we see that although feedback-buffering ODLs can recirculate packets and accommodate longer queues than their forward-buffering counterparts, the actual PLR performance will not improve much, because of the limited input access to buffering ODLs. Feedback buffering will also introduce extra AQD to delivered packets, which is deleterious for many real-time applications. Considering the system complexity associated with multiple recirculation of packets and the SNR degradation caused by recirculation, an admission control algorithm that avoids multiple recirculation will help reduce the complexity of feedback-buffering algorithms and improve system reliability.

### 3 Queuing in Asynchronous Optical Packet-Switched Networks

In Sec. 2, we presented queuing models of ODLs in synchronous optical packet-switching networks, in which all the packets are aligned in phase before entering the switching matrix. Since packets enter a switching node from different links, this requires implementation of a synchronization stage, which greatly increases the complexity of OXC's. An alternative is provided by asynchronous optical

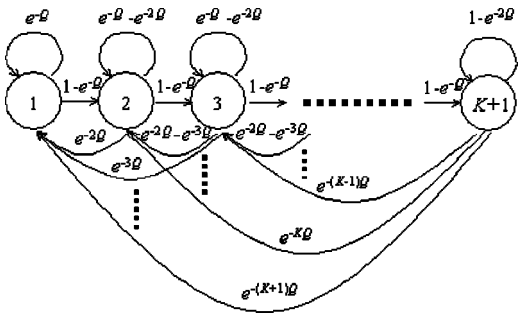


Fig. 8 MC transition diagram in asynchronous forward-buffering ODLs with  $K$  ODLs.

packet-switching networks, in which there is no packet alignment stage, and a packet may enter the switching matrix at any time, independent of the arrival times of the packets from other ports. Obviously, asynchronous networks are less complex and expensive, but are subject to a higher probability of contention, because packet arrivals are less well-controlled.

In this section, we study the queueing properties of ODLs in asynchronous optical packet-switching networks. As we observed in Sec. 2.2, recirculating packets more than once in feedback-buffering ODLs does not significantly improve PLR, while it greatly increases control complexity. Therefore, in this section we consider only feedback buffers operated under admission control so that the queued packets are circulated in feedback ODLs only once. The queueing model for such feedback-buffering ODLs will be the same as that of the forward-buffering ODLs.

Again, we assume uniform Bernoulli traffic at all input ports. However since these packets will arrive at the output port at any time, we must still model the probability distribution of packet arrival at a particular output port. It is well known that for  $N \rightarrow \infty$ , the probability distribution of  $p_i$  in Eq. (1) becomes<sup>10</sup>

$$p_i = \frac{Q^i e^{-Q}}{i!}, \quad i=0,1,2, \dots \quad (15)$$

Equation (15) shows that when  $N \rightarrow \infty$ , the number of packets arriving within a packet time slot is Poisson distributed. We assume that the packet arrival at each output port is a Poisson process with rate  $Q$ . Since PLRs and AQDs are not sensitive to  $N$  under uniform traffic, as we observed in Sec. 2.1. We can accurately approximate real finite input/output port systems by results derived using Eq. (15), which assumes  $N \rightarrow \infty$ .

Assume the OXC has the same set of ODLs as described in Sec. 2.1. In this section, we define the state of each output port to be the residual time (the waiting time in the queue and the serving time of the output port) of the last packet in the corresponding queue counted right after the time when it enters the queue. This enables us to set up a finite-state MC model whose transition diagram is shown as in Fig. 8. The balance equation is

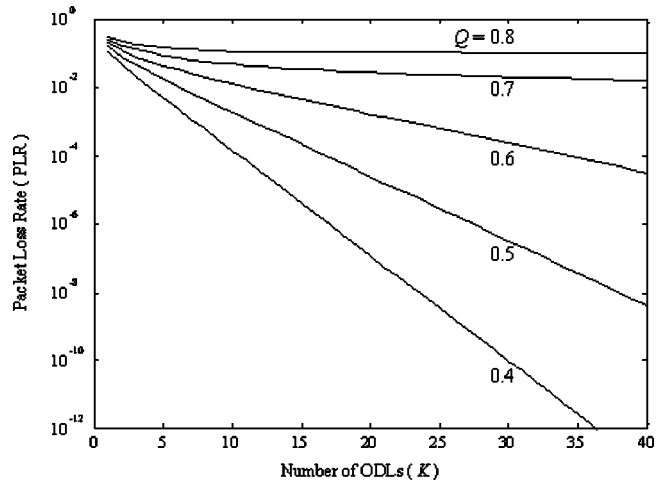


Fig. 9 PLR versus the number of ODLs  $K$  with traffic loads  $Q = 0.4, 0.5, 0.6, 0.7,$  and  $0.8$ , respectively, for in asynchronous optical packet-switching networks. The number of input/output ports  $N$  is chosen to be  $\infty$ . The PLRs are calculated using Eq. (18).

$$\pi_i = \begin{cases} \sum_{j=1}^{K+1} \exp(-jQ) \pi_j & i=1 \\ \sum_{j=i-1}^{K+1} \{\exp[-(j-1)Q] - \exp(-jQ)\} \pi_j & 2 \leq i \leq K+1. \end{cases} \quad (16)$$

The solution to Eq. (16) is simply

$$\pi_i = \frac{q^{i-1}(1-q)}{1-q^{K+1}} \quad 1 \leq i \leq K+1, \quad (17)$$

where  $q = e^Q - 1$ .

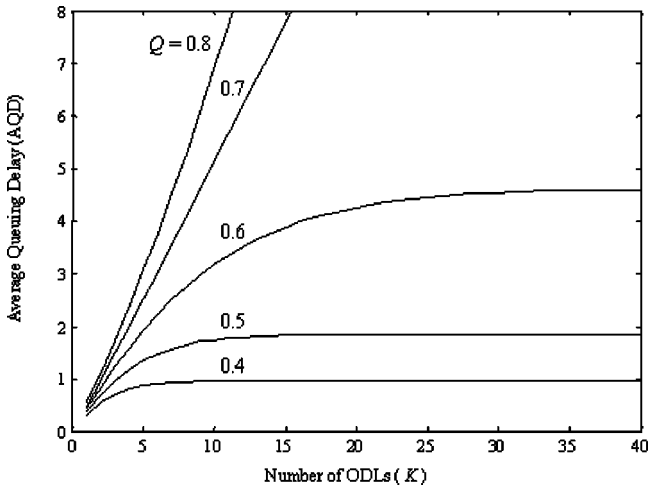
The PLR can be calculated by

$$\text{PLR} = \pi_{K+1}(1 - e^{-Q}) = \frac{(1-q)q^{K+1}}{(1-q^{K+1})(1+q)}. \quad (18)$$

The AQD is

$$\text{AQD} = \sum_{i=1}^{K+1} \pi_i(i-1) = \frac{q - (K+1)q^{K+1} + Kq^{K+2}}{(1-q^{K+1})(1-q)}. \quad (19)$$

We can use Eqs. (18) and (19) to calculate PLRs and AQDs for asynchronous OXCs. In Figs. 9 and 10, we plot the PLR and the AQD versus the number of ODLs  $K$  with traffic loads  $Q = 0.4, 0.5, 0.6, 0.7,$  and  $0.8$ , respectively. In Fig. 9, we see that the PLRs of asynchronous networks are much larger than those of synchronous networks, for which results are shown in Figs. 3 and 6. Moreover, for  $Q \geq 0.7$ , the PLRs tend to not to decrease as  $K$  is increased beyond a certain value. In Fig. 10, we observe that when  $Q \leq 0.6$ , the AQDs do not increase as  $K$  is increased beyond a certain value. However, when  $Q \geq 0.7$ , the AQDs of asynchronous networks increases linearly with  $K$ . These observations imply that when the traffic load is high, the buffering queues in asynchronous OXCs have a very high probability of being full no matter how large the buffer is. To verify



**Fig. 10** AQR versus the number of ODLs  $K$  with traffic loads  $Q = 0.4, 0.5, 0.6, 0.7,$  and  $0.8,$  respectively, for asynchronous optical packet-switching networks. The number of input/output ports  $N$  is chosen to be  $\infty$ . The PLRs are calculated using Eq. (19).

this, we apply an asymptotic analysis for  $K \rightarrow \infty$ . The generating function of such a queueing system is

$$\begin{aligned} \Pi(s) &= \sum_{i=0}^{\infty} \pi_i^{\infty} s^i = \sum_{i=0}^{\infty} \{ \exp(-iQ) - \exp[-(i+1)Q] \} \\ &\quad \times \left( \sum_{j=0}^i \pi_j^{\infty} + \sum_{j=i+1}^{\infty} \pi_j^{\infty} s^{j-i} \right) s, \end{aligned} \quad (20)$$

where  $\pi_0^{\infty} = 0$ . From Eq. (20), we have

$$\pi_i^{\infty} = \pi_1^{\infty} q^{i-1} \quad i \geq 1. \quad (21)$$

It is well known that only for  $q = e^Q - 1 < 1$ , i.e.,  $Q < \ln 2 \approx 0.693$ , this countable state space MC is positive recurrent and will have a stationary distribution.<sup>14</sup> For  $Q > \ln 2$ , this MC is transient, and there is no stationary distribution,  $\pi_i^{\infty} = 0$  for any  $i < \infty$ . This implies that no matter how large the buffer is, this queueing system will eventually use up all the buffering space if  $Q > \ln 2$ .

These observations help explain the results shown in Figs. 9 and 10. When  $Q > \ln 2$  the stationary distribution of the finite-buffering systems will concentrate on the full-buffer state for arbitrary buffer size. Therefore, the PLR is not sensitive to the number of ODLs  $K$ , and the AQR is proportional to  $K$ . By contrast, for  $Q < \ln 2$ , the stationary probability distribution will decrease exponentially with the queueing length, as indicated in Eq. (21). Therefore the decrease of the PLRs with  $K$  is approximately exponential, and the AQRs saturate even for larger values of  $K$ , as shown in Figs. 9 and 10, respectively.

The preceding discussion shows that although asynchronous optical packet switching offer a much simpler structure for OXCs, it greatly increases the probability of contention. Therefore, we may require ODLs for asynchronous networks. Moreover, when the traffic load exceeds a certain threshold, i.e.,  $Q > \ln 2$  in our model, increasing the

buffer size  $K$  will not help relieve contention, and actually increases the delay of the system. Under such high traffic loads, asynchronous optical packet switching with only ODL buffering is not appropriate. We must either switch back to synchronous network design or implement other contention resolution methods exploiting wavelength and/or space dimensions.

## 4 Conclusions

We introduced queueing models of ODLs in synchronous and asynchronous optical packet-switching networks under uniform Bernoulli traffic.

We first introduced forward-buffering queueing models in synchronous networks. We describe an MC model for finite-length ODL forward-buffering systems and calculated the stationary distribution of queueing length in ODLs as well as two useful parameters: PLR and AQR. We then carried out an asymptotic analysis based on the generating function of an infinite buffering system and obtained approximations for PLR and AQR. Numerical results demonstrated that the asymptotic analyses yield accurate estimates of these parameters. We then extended the queueing model to feedback-buffering ODLs in synchronized networks and numerically calculated the corresponding PLR and AQR. Comparing the results of forward buffering and feedback buffering, we found multiple recirculation of packets in feedback ODLs will not significantly improve PLR. Moreover it causes optical signal SNR degradation and increases control algorithm complexity, significantly offsetting the design simplicity of asynchronous optical packet-switching schemes. Therefore, simpler control algorithms and admission control techniques, which avoid complicated multiple recirculations, are more suitable for feedback buffering, and enable prioritized switching of optical packets.

We then introduced queueing models for asynchronous networks. We assumed either forward buffering or the subset of feedback buffering that avoids multiple recirculation. The queueing models of these two sets of systems are actually identical. We described a MC model for finite-length ODLs and the equations used to calculate PLR and AQR. We also carried out an asymptotic analysis to obtain insight into the performance degradation of ODL buffering for asynchronous switching with high traffic load.

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