Coupled-Core Multi-Core Fibers for Spatial Multiplexing

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***Abstract*— Coupled-core multi-core fibers (MCFs) offer characteristics that are beneficial for long-haul spatially multiplexed transmission. Using full-vector solution of the wave equation, we study the number of propagating modes and several modal characteristics, including intermodal beat lengths, group delay spread, mode-dependent chromatic dispersion, and intramodal and intermodal effective areas. We identify a range of design parameters that simultaneously optimizes these characteristics. Our results demonstrate the limited accuracy of perturbation-based analyses in characterizing MCFs with closely spaced cores.**

***Index Terms*— Multi-core fibers, multi-mode fibers, multi-input multi-output, equalization, mode coupling, modal dispersion, mode-division multiplexing, spatial multiplexing**

I. Introduction

As LONG-HAUL single-mode fiber (SMF) systems approach capacity limits, continued traffic growth has motivated study of spatial multiplexing in new fibers [1]. Multi-mode fibers (MMFs) use a plurality of modes as parallel channels [1]. Multi-input multi-output (MIMO) digital signal processing (DSP) is a concern in MMF systems, since the complexity per data symbol tends to increase, because MIMO dimensionality and group delay (GD) spread both increase with the number of modes. In MMFs supporting two mode groups, very low GD spread can be realized easily [2], but different approaches for controlling GD spread are required for more than two mode groups. One approach is to concatenate MMFs in which lower- and higher-order modes have an opposite ordering of GDs, but this may be practical only for a small number of mode groups [3]. An approach that may scale to a larger number of mode groups combines (i) optimization of the index profile for low uncoupled GD spread with (ii) strong mode coupling [4], [5]. Sophisticated index profiles [5] are required for (i), while additional components or perturbation of the MMF are required for (ii), since typical fibers do not provide sufficient mode coupling [6]. Uncoupled-core multi-core fibers (MCFs), which allow independent parallel transmission in each core enabled by very weak coupling between cores, represent a diametrically opposed approach for spatial multiplexing [7], [8]. While avoiding crosstalk would greatly reduce MIMO DSP complexity, this approach creates a new system challenge of controlling crosstalk along the entire transmission link, which may be difficult in long-haul networks.

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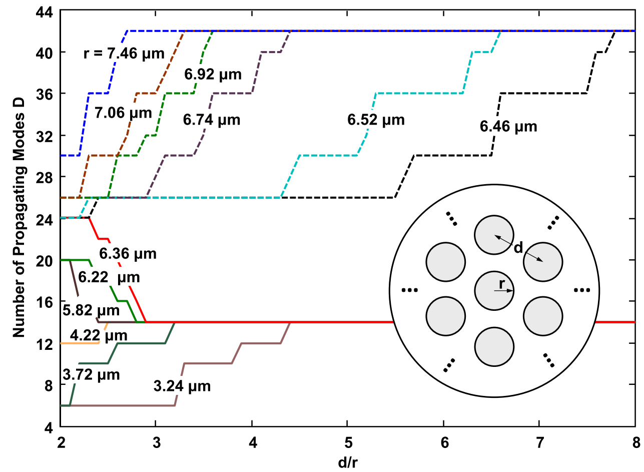

Fig. 1. Number of propagating modes in two polarizations D vs. d/r for step-index seven-core fiber assuming $n_{core} = 1.451$, $n_{clad} = 1.448$ and $\lambda = 1550$ nm. Inset: fiber with cores of radius r on hexagonal lattice with spacing d.

Coupled-core (CC) MCFs present an alternative for spatial multiplexing [9], [10], [11]. They can be considered a form of MMFs, but their lack of circular symmetry enables optimization of modal properties by adjusting the core radius and spacing, without requiring the complex index profiles of advanced MMF designs [5]. Study of CC MCFs and their comparison to MMFs and uncoupled-core MCFs are critically important for spatial multiplexing. Initial studies of CC MCFs [12], [13] employed first-order perturbation theory to characterize "supermodes", which are linear combinations of the individual core modes. However, as we show, perturbation-based analyses can yield incorrect results in certain regimes. In this letter, we use full-vector solution of wave equations to characterize MCFs accurately over a range of coupling regimes. We study the number of propagating modes and several modal characteristics, including intermodal beat lengths, GD spread, mode-dependent chromatic dispersion (CD), and intramodal and intermodal effective areas. We identify a range of design parameters that simultaneously optimizes these modal characteristics.

II. Number of Propagating Modes

The hexagonal lattice is the densest two-dimensional lattice, making it suitable for dense MCF design. The inset of Fig. 1

shows an MCF with identical cores of radius r separated by a spacing d. The ratio d/r affects the strength of interaction between cores and is a key parameter for optimizing MCF characteristics. Throughout this letter, we study a step-index seven-core fiber with core and cladding indices $n_{core} = 1.451$ and $n_{clad} = 1.448$, yielding a numerical aperture $NA = 0.0933$, and assume a wavelength $\lambda = 1550$ nm.

We first study the number of propagating modes as a function of r and d/r. We solve the wave equation numerically using a full-vector finite-difference method [14]. By setting a minimum effective index threshold of n_{clad}, we distinguish propagating modes from evanescent modes or unphysical solutions. Fig. 1 shows the total number of propagating modes D (including spatial and polarization degrees of freedom) as a function of d/r for various values of r. The simplest regime is the uncoupled-core regime, when d/r is sufficiently large (depending on r) that the MCF modes are almost identical to the individual core modes and D is given by the number of cores times the number of modes per core, which depends on the V number of a single core, $V(r) = 2\pi \cdot NA \cdot r/\lambda$. In this regime, D assumes values of 14, 42,.... At the opposite extreme, when $d/r \to 2$, the individual cores touch, and the index profile and modal characteristics resemble those of an ordinary step-index fiber with core radius $3r$. In this extreme case, D is determined approximately by the V number for core radius $3r$, $V(3r) = 6\pi \cdot NA \cdot r/\lambda$. Accordingly, the first few values of D are 6, 12, 20, 24. In the intermediate regime, which is of interest for CC MCFs, D assumes values intermediate between the two extreme regimes. These results demonstrate the limitations of first-order perturbation analyses, which consider D to be fixed as in the uncoupled-core regime, $D = 14, 42, \ldots$. It is essential to know the number of propagating modes to avoid outage or power loss in long-haul systems.

III. Modal Properties for Long-Haul Transmission

In this section, we study modal properties of interest for long-haul transmission in MCFs as a function of the ratio d/r. We fix the core radius at $r = 6.36\,\mu$m, the largest value permitting a single spatial mode per core in the uncoupled-core regime. While larger values of r would yield better modal confinement to reduce propagation loss and larger effective areas to reduce nonlinearity, we wish to avoid multi-mode propagation per core, which is not the regime of interest in this letter. Depending on d/r, the number of propagating modes D varies between 14 and 24, which is evident in Fig. 1.

In the absence of perturbations, the μth mode ($1 \le \mu \le D$) of a CC MCF has an electric field profile $E_\mu(x, y)$ in either of the transverse polarizations. Near angular frequency ω_0, the propagation constant β_μ can be expanded as:

$$\beta_\mu(\omega) \approx \beta_{0,\mu} + \beta_{1,\mu}(\omega - \omega_0) + \beta_{2,\mu}(\omega - \omega_0)^2 \Big/ 2 + \ldots \quad (1)$$

In (1), $\beta_{0,\mu} = \beta_\mu(\omega_0)$ represents modal phase shift per unit length $\beta_{1,\mu} = \left(\partial\beta_\mu/\partial\omega\right)\big|_{\omega=\omega_0}$ represents modal GD per unit length and $\beta_{2,\mu} = \left(\partial^2\beta_\mu/\partial\omega^2\right)\big|_{\omega=\omega_0}$ represents modal CD per unit length. All three parameters have important implications for long-haul systems, as we now explain.

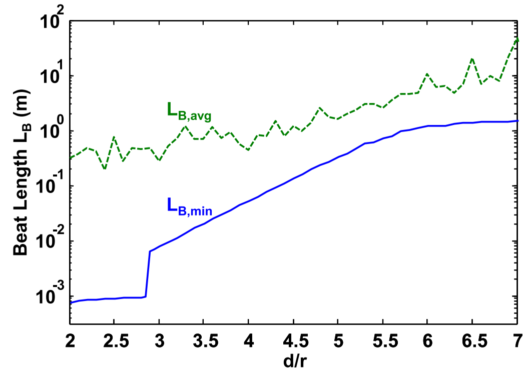

Fig. 2. Minimum (solid line) and average (dashed line) intermodal beat lengths vs. d/r for seven-core fiber, assuming same parameters as Fig. 1 and $r = 6.36\ \mu$m.

The term in (1) involving $\beta_{0,\mu}$ affects the strength of the coupling between MCF modes induced by perturbations of the fiber, including variations in core radii or spacing, bends, or twists [15]. (This coupling between the exact modes of an MCF should not be confused with the coupling between the modes of isolated cores considered in perturbation analyses of MCFs [12], [13].) In the weak-coupling regime, coupling between non-degenerate modes is small, whereas in the strong-coupling regime, coupling between all modes is equally strong [15]. In MMF, strong mode coupling is beneficial, reducing the impact of mode-dependent loss or gain [15], reducing the GD spread and thus the MIMO DSP complexity [4], and possibly reducing the impact of nonlinearity [16]. Strong mode coupling can offer analogous benefits in CC MCFs.

The pairwise coupling between any two modes induced by an index perturbation depends on an overlap integral between the two modal fields and the transverse dependence of the perturbation, and also depends on phase matching between the two propagating modes by the longitudinal dependence of the perturbation [15]. The beat length between modes μ_1 and μ_2 may be defined as $L_{B,(\mu_1,\mu_2)} = 2\pi \Big/ \left|\beta_{0,\mu_1} - \beta_{0,\mu_2}\right|$. Many random perturbations have a longitudinal dependence that is spatially lowpass, such that the pairwise coupling between modes μ_1 and μ_2 scales as $L^4_{B,(\mu_1,\mu_2)}$ to $L^8_{B,(\mu_1,\mu_2)}$ [17].

Fig. 2 characterizes intermodal beat lengths in CC MCFs, showing the minimum $L_{B,min} = \min_{\mu_1,\mu_2,\,\mu_1\neq\mu_2} L_{B,(\mu_1,\mu_2)}$ and the average $L_{B,avg} = \left(D^2 - 2D\right)^{-1} \sum_{\mu_1,\mu_2=1,\,\beta_{0,\mu_1} \neq \beta_{0,\mu_2}}^{D} L_{B,(\mu_1,\mu_2)}$ as a function of the ratio d/r. The minimum beat length is particularly critical in ensuring coupling between all modes. In Fig. 2, when $d/r < 3$, the beat lengths are of the same order as those in step-index MMFs or graded-index MMFs, while at larger d/r, a roughly exponential increase of the beat lengths is observed. These larger beat lengths may represent a significant advantage of CC MCFs over MMFs. Considering the dependence of coupling on spatial overlap between modes, the fields inside a single core have negligible energy

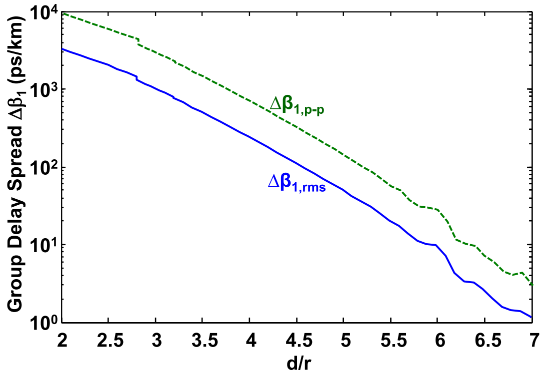

Fig. 3. Peak-to-peak (dashed line) and rms (solid line) GD spreads per unit length vs. d/r for seven-core fiber, assuming same parameters as Fig. 2.

at radii beyond about $3r$, so choosing d/r between about 4 and 6 should ensure strong mode coupling.

Nonuniformity in core radius or spacing can decrease beat lengths in CC MCFs, much like refractive index imperfections in MMFs. For example, at $d/r = 5$, root-mean-square (rms) core radius fluctuations of 0.1%, 0.3% and 1% reduce $L_{B,min}$ on average by factors of about two, four and eight, respectively. Nevertheless, even with 1% rms nonuniformity in core radius, the minimum beat length is an order of magnitude higher than in a graded-index MMF assumed to have an ideal index profile [5].

The term in (1) involving $\beta_{1,\mu}$ causes modal dispersion. The peak-to-peak GD spread of a transmission link must be minimized in order to minimize the MIMO equalizer complexity [4], [15] and enable the equalizer to track fast changes in mode coupling along the link [5]. In the weak-coupling regime, the peak-to-peak GD spread of a link is equal to the peak-to-peak GD spread per unit length $\Delta\beta_{1,p-p} = \max_{\mu}(\beta_{1,\mu}) - \min_{\mu}(\beta_{1,\mu})$ times the total fiber length [4], whereas in the strong-coupling regime, it is proportional to the rms GD spread per unit length $\Delta\beta_{1,rms} = \sqrt{D^{-1}\sum_{\mu=1}^{D}(\beta_{1,\mu} - \beta_{1,avg})^2}$ times the square root of the total fiber length [4], [15]. The mode-averaged GD per unit length is $\beta_{1,avg} = D^{-1}\sum_{\mu=1}^{D}\beta_{1,\mu}$.

Fig. 3 shows the peak-to peak and rms uncoupled GD spreads $\Delta\beta_{1,p-p}$ and $\Delta\beta_{1,rms}$ of CC MCFs as a function of the ratio d/r. Over the range shown, $\Delta\beta_{1,rms}$ decreases roughly exponentially (as $\sim 10^{-(1.5d/r)}$) and $\Delta\beta_{1,p-p}$ is roughly three times larger than $\Delta\beta_{1,rms}$. For d/r exceeding about 3.8, these GD spreads are smaller than those in graded-index MMFs for $D = 12$ [4], while for d/r exceeding about 5.3, they become smaller than those in optimized graded-index MMFs for $D = 12$ [5].

Nonuniformity in core radius or spacing can increase the GD spread in CC MCFs, just as it can decrease the beat lengths. For example, at $d/r = 5$, rms core radius fluctuations of 0.1%, 0.3% and 1% increase $\Delta\beta_{1,rms}$ by factors of about two, three and six, respectively. Nevertheless, even with 1% rms nonuniformity in core radius, the rms GD spread is slightly smaller than that in a graded-index MMF assumed to have an ideal index profile [4].

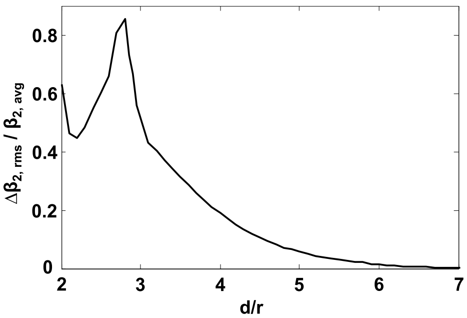

Fig. 4. Rms mode-dependent CD, normalized by mode-averaged CD, vs. d/r for seven-core fiber, assuming same parameters as Fig. 2.

The GD spreads discussed above are those in the absence of mode coupling, and strong mode coupling can reduce them substantially. It may be optimal to choose d/r between about 5 and 6 to reduce the uncoupled GD spread while ensuring strong mode coupling (recall the discussion of beat lengths given above).

The term in (1) involving $\beta_{2,\mu}$ causes CD. Mode-averaged CD, which is substantially fixed, can be compensated using a static (not adaptive) scalar equalizer for each mode [5], as in SMF systems. The mode-averaged CD per unit length $\beta_{2,avg} = D^{-1}\sum_{\mu=1}^{D}\beta_{2,\mu}$ of CC MCFs is similar to that of standard SMFs. Mode-dependent CD should be minimized, since it increases the complexity of the adaptive MIMO equalizer. Fig. 4 shows the rms mode-dependent CD $\Delta\beta_{2,rms} = \sqrt{D^{-1}\sum_{\mu=1}^{D}(\beta_{2,\mu} - \beta_{2,avg})^2}$, normalized by the mode-averaged CD $\beta_{2,avg}$, as a function of d/r. While $\Delta\beta_{2,rms}/\beta_{2,avg}$ is substantial for small d/r, $\Delta\beta_{2,rms}/\beta_{2,avg}$ becomes less than 10% for d/r exceeding about 4.5, and should have a negligible effect on MIMO DSP complexity.

Finally, we study the nonlinear properties of MCFs. These depend on the fiber design through intermodal and intramodal effective areas, which are defined as [18]:

$$A_{eff,(\mu_1,\mu_2)} = \frac{\iint |E_{\mu_1}(x,y)|^2 dx\,dy \cdot \iint |E_{\mu_2}(x,y)|^2 dx\,dy}{\iint |E_{\mu_1}(x,y)|^2 |E_{\mu_2}(x,y)|^2 dx\,dy}. \quad (2)$$

For $\mu_1 \neq \mu_2$, (2) yields the intermodal effective area between modes μ_1 and μ_2, which governs nonlinear interactions between them. For $\mu_1 = \mu_2 = \mu$, (2) reduces to the ordinary intramodal effective area for mode μ, which governs intramodal nonlinear effects. For sufficiently large values of d/r, intermodal effects become negligible and intramodal effects become similar to those of isolated cores.

Fig. 5 shows the minimum intermodal effective area $A_{eff,\min} = \min_{\mu_1,\mu_2,\,\mu_1\neq\mu_2} A_{eff,(\mu_1,\mu_2)}$, the minimum intramodal effective area $A_{eff,\min} = \min_{\mu} A_{eff,(\mu,\mu)}$, and the average intramodal effective area $A_{eff,avg} = D^{-1}\sum_{\mu=1}^{D} A_{eff,(\mu,\mu)}$, as a function of d/r. Minimum intermodal and intramodal effective areas are roughly constant for d/r in the range between about 3 and 5.2, and begin to decline for larger d/r. Over this range of d/r, the average intramodal effective area is 1.4 to 2.7 times higher than the minimum intramodal effective area, which may imply performance disparities between modes.

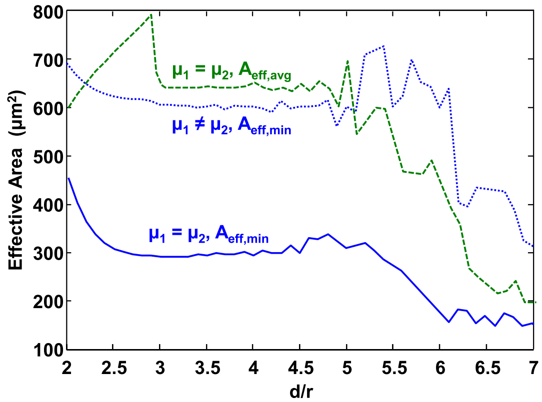

Fig. 5. Minimum intramodal (solid line), minimum intermodal (dotted line) and average intramodal (dashed line) effective areas vs. d/r for seven-core fiber, assuming same parameters as Fig. 2.

Intermodal effective areas between most modes are very large, resulting in negligible nonlinear interactions, and the minimum intermodal effective area is 1.6 to 4.5 times higher than the minimum intramodal effective area. For d/r up to about 6, the intramodal effective areas in CC MCFs are about twice those in graded-index MMFs with $NA = 0.15$ supporting $D = 12$ modes [5]. Overall, d/r should be chosen less than about 6 to benefit from large effective areas. We note that strong mode coupling might further reduce nonlinear effects [16].

IV. Conclusion

CC MCFs offer potential advantages for long-haul spatially multiplexed systems. As compared to MMFs, they offer larger effective areas and may achieve very low uncoupled GD spreads without highly optimized index profiles. As compared to uncoupled-core MCFs, they have smaller cross-sectional areas and avoid requirements for stringent end-to-end control of crosstalk. Using a full-vector mode solver, we have studied CC MCF properties, including intermodal beat lengths, GD spreads, mode-dependent CD and intramodal and intermodal effective areas. We have found that these properties are simultaneously optimized when the ratio of core spacing to core radius lies between about 4.5 and 6, assuming typical values of NA (0.0933) and core radius (6.36 μm). Minimizing variations in core radius and spacing is important to fully realize the benefits of optimized MCF designs. Our work clearly demonstrates the limited accuracy of perturbation-based analyses of "supermodes" in CC MCFs.

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