On Models of Clipping Distortion for Lightwave CATV Systems

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***Abstract*—We prove that the effective transfer function model by Frigo and the traditional series model used to analyze clipping distortion in lightwave CATV systems are identical.**

I. Introduction

Clipping distortion induces a fundamental limit to the capacity of lightwave CATV systems [1], [2]. There are various models [1]–[14] in the literature to analyze clipping distortion. The traditional series model [7], [10], [14] and effective transfer function model [8], [9], [11], [12] have been shown to predict the system performance accurately. In this paper, we prove that these two models are identical, so that they should provide the same results. We will first review both models and then prove that they are identical.

II. Series Model

For an N-channel CATV system, the multiplexed input signal to the laser can be written as

$$i(t) = I_m \sum_{i=1}^{N} \cos(2\pi f_i t + \phi_i), \tag{1}$$

where $I_m = mI_b$ is the modulation current of each subcarrier channel, I_b is the bias current minus the threshold current of the semiconductor laser, m is the modulation index, and f_i and ϕ_i are the frequency and initial phase of each carrier. For a large number of channels, by the central limit theorem, it can be shown that the input signal can be approximated by a Gaussian random process with zero mean and variance $\sigma^2 = NI_m^2/2$. In this paper, we always ignore the bias current of the laser and assume that the clipping nonlinearity operates on the modulation current $i(t)$ directly.

The method to analyze a Gaussian random process passing through a nonlinear function was studied in [15]–[17]. For a Gaussian input random process with input autocorrelation function $R_{\text{in}}(\tau)$ and variance σ^2, after passage through a memoryless nonlinear transfer function, the output autocorrelation function is given by a power series [17, pp. 314–323]

$$R_{\text{out}}(\tau) = \sum_{n=0}^{\infty} \frac{b_n^2}{\sigma^{2n}} [R_{\text{in}}(\tau)]^n, \tag{2}$$

where the coefficients b_n are given by

$$b_n = \frac{1}{\sqrt{2\pi}\sigma n!} \int_{-\infty}^{\infty} g(x) e^{-x^2/2\sigma^2} H_n\left(\frac{x}{\sigma}\right) dx, \tag{3}$$

where $g(\cdot)$ denotes the nonlinear transfer function, and $H_n(x) = (-1)^n e^{x^2/2} \frac{d^n}{dx^n}(e^{-x^2/2})$ is the nth-order Hermite polynomial. In the present case, $R_{\text{in}}(\tau)$ is the autocorrelation function of (1), which is given by

$$R_{\text{in}}(\tau) = \frac{1}{2} I_m^2 \sum_{i=1}^{N} \cos(2\pi f_i \tau). \tag{4}$$

In the series (2), the second term represents the undistorted signal, and all higher-order terms represent composite intermodulation products. The power of the nth-order intermodulation at frequency ν is

$$P_\nu^{(n)} = \frac{1}{2} b_n^2 \left(\frac{I_m}{2\sigma}\right)^{2n} D_\nu^{(n)}, \tag{5}$$

where $D_\nu^{(n)}$ is the number of intermodulation products of $[\sum_{n=1}^{N}(e^{-j2\pi f_n \tau} + e^{j2\pi f_n \tau})]^n$ falling at frequency ν. We can show that [12]

$$D_\nu^{(n)} = \sum_{p_1,\cdots,p_n} \frac{n!}{\prod |p_i|!}, \quad \sum_{i=1}^{N} p_i f_i = \nu, \quad \sum_{i=1}^{N} |p_i| = n. \tag{6}$$

The series model has been used to analyze clipping distortion in lightwave CATV systems [7], [15] and in discrete multitone systems [10]. The technique is very general and can be applied to any nonlinear multicarrier system.

III. Effective Transfer Function Model

The effective transfer function model was first proposed by Frigo [8], [9], [11] and later proved by Mazo [12] rigorously. The model transforms the transfer function of a nonlinear function to a modified effective transfer function.

This modified transfer function incorporates the multicarrier input, first, by assuming that the N subcarriers can be represented as a noise process with Gaussian probability density function of variance σ^2, and second, by calculating the expected output for any input signal in the presence of

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the noise. For a nonlinear transfer function $g(\cdot)$, the effective transfer function is [12]

$$f(a) = E_x(g(a+x)), \tag{7}$$

where x is the Gaussian random process. Even if $g(\cdot)$ is not a smooth function, $f(a)$ will be a smooth function. Ignoring the bias current, the output $f(i(t))$ can be expanded as a Taylor series

$$f(i(t)) = \sum_{k=0}^{\infty} \chi^{(n)}(i(t))^n, \tag{8}$$

where $\chi^{(n)} = f^{(n)}(0)/n!$. In [8], [9], [11], the effective transfer function $f(a)$ is first evaluated and is then expanded to find $\chi^{(n)}$. Each nth-order intermodulation term has an amplitude of $\chi^{(n)}(I_m/2)^n$, so that the nth-order intermodulation power at frequency ν is

$$P_\nu^{(n)} = \frac{1}{2}\left[\frac{\chi^{(n)} I_m^n}{2^n}\right]^2 D_\nu^{(n)} \tag{9}$$

where $D_\nu^{(n)}$ is defined in (6).

IV. Equivalence of Models

We would like to prove that the series model and effective transfer function model are identical. To do so, we will find the relation between $\chi^{(n)}$ and b_n, and show that they are equivalent.

The effective transfer function can be evaluated as

$$\begin{aligned} f(a) &= E_x(g(a+x)) \\ &= \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty} g(x)\exp\left(-\frac{(x-a)^2}{2\sigma^2}\right)dx. \end{aligned} \tag{10}$$

Using the generating function of Hermite polynomials [17, pp. 66–69], [18, 8.957]

$$\exp(-t^2/2 + tx) = \sum_{n=0}^{\infty} \frac{t^n}{n!} H_n(x) \tag{11}$$

and exchanging the order of summation and integration, we have

$$f(a) = \sum_{n=0}^{\infty} \frac{a^n}{\sqrt{2\pi}\sigma^{n+1} n!}\int_{-\infty}^{\infty} g(x)e^{-x^2/2\sigma^2} H_n\left(\frac{x}{\sigma}\right)dx. \tag{12}$$

Comparing (8) and (11) term by term, the coefficients $\chi^{(n)}$ are

$$\chi^{(n)} = \frac{1}{\sqrt{2\pi}\sigma^{n+1} n!}\int_{-\infty}^{\infty} g(x)e^{-x^2/2\sigma^2} H_n\left(\frac{x}{\sigma}\right)dx. \tag{13}$$

or

$$\chi^{(n)} = b_n/\sigma^n. \tag{14}$$

Comparing the intermodulation powers given by expressions (5) and (9), using the relation $\chi^{(n)} = b_n/\sigma^n$, it is obvious that the effective transfer function model and the series expansion model are identical.

V. Conclusion

We have proved that in the analysis of clipping distortion in lightwave CATV systems, the effective transfer model and the series model are identical. Because we make no assumption regarding the form of the nonlinearity, both models can be applied to multicarrier systems with any kind of nonlinear distortion.

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