Capacity of coherent free-space optical links using atmospheric compensation techniques

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Abstract: We analyze the ergodic capacity and ε-outage capacity of coherent optical links through the turbulent atmosphere. We consider the effects of log-normal amplitude fluctuations and Gaussian phase fluctuations, in addition to local oscillator shot noise, for both passive receivers and those employing active modal compensation of wavefront phase distortion. We study the effect of various parameters, including the ratio of receiver aperture diameter to wavefront coherence diameter, the strength of the scintillation index, and the number of modes compensated.

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1. Introduction

In this study we study the maximal rate at which the information may be transferred through free-space optical communication links using coherent detection. Evaluating the performance of a heterodyne or homodyne receiver in the presence of atmospheric turbulence is generally difficult because of the complex ways that turbulence, acting as a time-varying, multiplicative noise, affects the coherence of the received signal that is to be mixed with the local oscillator.

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The downconverted heterodyne or homodyne power is maximized when the spatial field of the received signal matches that of the local oscillator. Any mismatch between the amplitudes and phases of the two fields will result in a reduction of the downconverted power, i.e., fading. In the case of coherent modulation, phase fluctuations can severely degrade performance unless measures are taken to compensate for them at the receiver. Here, we assume that after homodyne or heterodyne downconversion is used to obtain an electrical signal, the receiver is able to track any phase fluctuations caused by turbulence (as well as those caused by laser phase noise), performing ideal coherent (synchronous) demodulation. Under this assumption, analyzing the receiver performance requires knowledge of only the envelope statistics of the downconverted electrical signal.

The classical theory of communications was developed originally in the context of linear channels with additive noise [1]. The capacity C of a communication channel is the maximal rate at which the information may be transferred through the channel without error. For an additive white Gaussian noise (AWGN) channel, the complex baseband representation is $y[t]=x[t]+n[t]$, where $x[t]$ and $y[t]$ are the complex input and output at time t, and $n[t]$ is the additive Gaussian noise, which is independent over time. In the classical capacity formula for the AWGN channel with average power constraint P and noise power spectral density $N_0/2$, given by $C=B\ \log_2(1+\gamma_0)$, the spectral bandwidth B, which has units of Hz, multiplies the maximal spectral efficiency $\log_2(1+\gamma_0)$, which has units of bits/s/Hz. Here, $\gamma_0=P/N_0B$ is the signal-to-noise ratio (SNR) per unit bandwidth. When the SNR γ_0 is low, the capacity increases linearly with the received power P, but when the SNR γ_0 is high, the capacity increases logarithmically with P but depends more strongly on the spectral bandwidth B.

In free-space optical communication through the turbulent atmosphere, we must consider fading channels, which are a class of channels with multiplicative noise. The complex baseband representation of an atmospheric channel $y[t]=\alpha[t]x[t]+n[t]$ includes the multiplicative effect of the fading process $\alpha[t]$ at time t. We let α^2 denote the atmospheric channel power gain and $(P/N_0B)\alpha^2=\gamma_0\alpha^2$ denote the instantaneous received SNR per symbol. Conditional on a realization of the atmospheric channel described by α, this is an AWGN channel with instantaneous received SNR $\gamma=\gamma_0\alpha^2$ and the maximum rate of reliable communication supported by this channel is $\log_2(1+\gamma_0\,\alpha^2)$ bits/s/Hz. This quantity is a function of the random channel power gain α^2, and is therefore random. The statistical properties of the atmospheric random channel fade α^2, with probability density function (PDF) $p_{\alpha 2}(\alpha^2)$, provide a statistical characterization of the SNR $\gamma=\gamma_0\alpha^2$ and, consequently, of the maximal spectral efficiency achievable for the free-space optical link. Although information theory has been applied to free-space optical communication links using direct detection [2-4], the ultimate classical information capacity when coherent (homodyne or heterodyne) detection is used needs to be properly considered.

In [5], we study in a unified framework the effects of both wavefront distortion and amplitude scintillation on the performance of synchronous (coherent) receivers utilizing wavefront compensation. The effects ascribed to turbulence are random and subsequently must be described in a statistical sense. Early works quantified turbulence-induced fading through its mean and variance [6,7], although these are not adequate to fully characterize system performance. Later analyses have attempted to overcome these limitations and fully characterize the statistics of heterodyne optical systems by assuming a highly simplified model of atmospheric effects [8,9]. An alternate approach, aimed at overcoming the limitations of previous work, is based on numerical simulation of heterodyne optical systems [10]. Unfortunately, none of these prior works have resulted in an accurate statistical description of the performance of phase-compensated homodyne or heterodyne systems. In [5], we define a mathematical model for the received coherent signal after propagation through the atmosphere. By noting that the downconverted signal current can be characterized as the sum of many contributions from different coherent regions within the aperture, we show that the PDF of this current can be well-approximated by a modified Rice distribution. In our model, the parameters describing the PDF depend on the turbulence conditions and the degree of modal compensation applied in the receiver. We provide analytical expressions and use them to study the effect of various parameters on performance, including turbulence level, signal strength, receive aperture size, and the extent of wavefront compensation.

Although atmospheric fades are random, for the usual case of wind-driven turbulence, they are approximately constant on time intervals no shorter than one millisecond. Because data rates can exceed a billion symbols per second, a block of several million symbols can experience substantially similar fading conditions. There are several different information-theoretic notions of capacity [11]. Without any delay constraints, we can code over many channel realizations and achieve reliable communication at rates up to the ergodic capacity, defined as the average maximum mutual information per unit time, where an ensemble average is taken over the random gains. The ergodic capacity is the expectation with respect to the gains of the instantaneous capacity. The ergodic assumption requires that the codeword extent over at least several atmospheric coherence times, which allows coding across both deep and shallow fade channel realizations. When delay constraints prevent averaging over deep and shallow channel realizations, and the codeword is restricted to just one coherence time, strictly speaking, the channel capacity is zero, because there is a chance that the fading might be so severe that the instantaneous capacity is below any desired rate. In this case, a more appropriate measure of capacity is the probability that the channel can support a desired rate. In Section 2, we consider the capacity of coherent single-input single –output links for a given outage probability. In Section 3, we consider the ergodic capacity of such links. We present conclusions in Section 4.

2. ε-outage capacity

As data rates increase, atmospheric communication channels become better described as slow-fading channels. In terms of information theory this is equivalent to communication over channels where there is a nonzero probability that any given transmission rate cannot be supported by the channel. Consequently, in many practical situations, where delay constraints prevent using an extended codeword and averaging over deep fade channel realizations is not possible, an appropriate measure of capacity is the probability that the channel can support a given rate R, i.e., $p_{out}(R)=\mathbb{P}\{\log_2(1+\gamma)<R\}$. Here, the operator $\mathbb{P}\{\mathrm{A}\}$ indicates the probability of an event A. Let γ_R denote the SNR that is required to support a rate R. As the channel capacity is monotically increasing with received power for a given channel state, the probability of outage can be expressed in terms of SNR as $p_{out}(R)=\mathbb{P}\{\gamma<\gamma_R\}$. This result can be expressed in terms of the complementary cumulative distribution function (CCDF) of the SNR γ as $p_{out}(R)=1-F_c(\gamma_R)$. From here, we can solve $p_{out}(R)=\varepsilon$ to obtain the SNR γ producing a ε-outage probability $F_c(\gamma_R)=1-\varepsilon$, i.e., $\gamma_R=F_c^{-1}(1-\varepsilon)$. Then, by definition, the ε-outage capacity becomes

$$C_\varepsilon = B\log_2\left(1+\gamma_R\right) = B\log_2\left[1+F_c^{-1}\left(1-\varepsilon\right)\right]. \tag{1}$$

It is clear that the atmospheric outage capacity depends on the statistical distribution of SNR γ through its CCDF $F_c(\gamma_R)$. We have already modeled the impact of atmospheric turbulence-induced phase and amplitude fluctuations on free-space optical links using synchronous detection and found that the SNR γ is described by a noncentral chi-square distribution with two degrees of freedom [5]:

$$p_\gamma\left(\gamma\right) = \frac{1+r}{\overline{\gamma}}\exp\left(-r\right)\exp\left[-\frac{\left(1+r\right)\gamma}{\overline{\gamma}}\right]I_0\left[2\sqrt{\frac{\left(1+r\right)r\gamma}{\overline{\gamma}}}\right], \tag{2}$$

where the average SNR $\overline{\gamma}$ is giving by $\overline{\gamma}=\gamma_0\overline{\alpha^2}$ and the parameters $\overline{\alpha^2}$ and $1/r$ describe turbulence effects through the relations

$$\overline{\alpha^2} = \sigma_r^2 + \sigma_i^2 + \overline{\alpha}_r^2$$
$$\frac{1}{r} = \frac{\sigma_r^2 + \sigma_i^2 + \overline{\alpha}_r^2}{\left[\overline{\alpha}_r^4 + 2\overline{\alpha}_r^2\left(\sigma_i^2 - \sigma_r^2\right) - \left(\sigma_i^2 - \sigma_r^2\right)^2\right]^{1/2}} - 1. \quad (3)$$

In this model, the signal is characterized as the sum of a constant (coherent) term and a random (incoherent) residual halo. The contrast parameter *1/r* is a measure of the strength of the residual halo to the coherent component. The mean $\overline{\alpha}_r$ and variances σ_r^2, σ_r^2 in Eq. (3) are obtained with the help of [5]

$$\overline{\alpha}_r = \exp\left(-\frac{1}{2}\sigma_\chi^2\right)\exp\left(-\frac{1}{2}\sigma_\phi^2\right)$$
$$\overline{\alpha}_i = 0$$
$$\sigma_r^2 = \frac{1}{2N}\left[1 + \exp\left(-2\sigma_\phi^2\right) - 2\exp\left(-\sigma_\chi^2\right)\exp\left(-\sigma_\phi^2\right)\right] \quad (4)$$
$$\sigma_i^2 = \frac{1}{2N}\left[1 - \exp\left(-2\sigma_\phi^2\right)\right].$$

Here, log-normal amplitude fluctuations and Gaussian phase fluctuations are characterized by their respective statistical variances, σ_χ^2 and σ_ϕ^2,

$$\sigma_\chi^2 = \log_e\left(1 + \sigma_\beta^2\right)$$
$$\sigma_\phi^2 = C_J\left(\frac{D}{r_0}\right)^{5/3} \quad (5)$$

The intensity variance σ_β^2 is often referred to as the scintillation index [12]. The coefficient C_J depends on the number *J* of Zernike terms corrected by a receiver employing active modal compensation [13]. Phase-compensated receivers offer the potential for overcoming atmospheric limitations by adaptive tracking of the beam wave-front and consequent correction of atmospherically-induced aberrations. The modal compensation method is a correction of several modes of an expansion of the total phase distortion in a set of basics functions. Here, we have considered a model for a modal compensation system, a hypothetical device whose response functions are components of some expansion basis. Different sets of functions can be used for the expansion although most often they are Zernike polynomials, a set of orthonomal basis modes defined on a unit circle and that are related to the classical Seidel aberrations [14]. The modes are a product of angular functions and radial polynomials when polar coordinates are considered. We will assume that the modal compensation system has infinite spatial resolution in the correction of phase distortions. In Eq. (5), the receiver aperture diameter *D* is normalized by the wavefront coherence diameter r_0, which describes the spatial correlation of phase fluctuations in the receiver plane [6]. The model leading to the PDF in Eq. (2) is based on the observation that the downconverted signal current can be characterized as the sum of many contributions from *N* different coherent regions within the aperture [5]

$$N = \left\{1.09\left(\frac{r_0}{D}\right)^2 \Gamma\left[\frac{6}{5}, 1.08\left(\frac{D}{r_0}\right)^{5/3}\right]\right\}^{-1}, \quad (6)$$

where $\Gamma(a,x)$ is the lower incomplete gamma function. For the limiting case in which the receiver aperture is much greater than the coherence diameter r_0, i.e., $D \gg r_0$, to a good

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approximation, Eq. (6) leads to an aperture consisting of $(D/r_0)^2$ independent cells, each of diameter r_0. In the opposite extreme, for an aperture much smaller than the coherence diameter, $D \ll r_0$, the number of cells described by Eq. (6) approaches unity. Values of $N<1$ are not possible.

The PDF Eq. (2) modeling the impact of atmospheric turbulence on coherent links can be integrated to obtain the corresponding cumulative distribution function. After some algebra, we obtain the CCDF

$$\begin{aligned} F_c(\gamma_R) &= 1 - \int_0^{\gamma_R} d\gamma \; p_\gamma(\gamma) \\ &= Q\left(\sqrt{2r}\,, \sqrt{\frac{2(1+r)}{\bar{\gamma}}\gamma_R}\right) \end{aligned} \tag{7}$$

where $Q(a,b)$ is the first-order Marcum Q function. In order to estimate the ε-outage capacity in Eq. (1), we need to determine the inverse of this CCDF. However, there is no known elementary inverse of the Marcum Q Marcum's function. Hence, we represent the Marcum Q function by a strict upper Chernoff bound [15], $Q(a,b) \leq exp[-(b-a)^2/2]$, allowing us to obtain an upper bound on the CCDF

$$F_c(\gamma_R) \leq \exp\left[-\frac{1}{2}\left(\sqrt{\frac{2(1+r)}{\bar{\gamma}}\gamma_R} - \sqrt{2r}\right)^2\right]. \tag{8}$$

Equation (7) can be easily inverted to obtain the argument γ_R. By denoting $F_c(\gamma_R) \equiv p$,

$$F_c^{-1}(p) = \gamma_R \leq \frac{\bar{\gamma}}{2(1+r)}\left(\sqrt{-2\log_e p} + \sqrt{2r}\right)^2. \tag{9}$$

We can thus obtain a tight upper bound for the ε-outage capacity

$$C_\varepsilon \leq B\log_2\left[1 + \frac{\bar{\gamma}}{2(1+r)}\left(\sqrt{-2\log_e(1-\varepsilon)} + \sqrt{2r}\right)^2\right]. \tag{10}$$

The applicability of this result is shown by noting that, in the regime of weak turbulence when the signal coherent term is very strong ($r \to \infty$), the density function (2) becomes highly peaked around the mean value γ_0, and there is no fading to be considered. In this case, as should be expected, Eq. (10) tends towards the Shannon limit $\log_2(1+\gamma_0)$. On the other hand, we note that in the regime of strong turbulence, the coherent part of the signal is very weak, $r \to 0$, and the fading PDF (2) becomes a negative-exponential distribution, i.e. $p_\gamma(\gamma) = 1/\bar{\gamma}\exp(-\gamma/\bar{\gamma})$ and $F_c(\gamma_R) = \exp(-\gamma/\bar{\gamma})$. The corresponding ε-outage capacity $C_\varepsilon = B\log_2\left[1 - \bar{\gamma}\log_e(1-\varepsilon)\right]$ coincides with Eq. (10) in the limit of strong turbulence $r \to 0$.

We check the tightness of the upper bound Eq. (10) by considering the corresponding lower Chernoff bound to the Marcum Q function [15], $Q(a,b) \geq \exp[-(b+a)^2/2]$. The corresponding lower bound to the ε-outage capacity is similar to Eq. (10) except for the symbol of the term $\sqrt{2r}$

$$C_\varepsilon \geq B\log_2\left[1 + \frac{\bar{\gamma}}{2(1+r)}\left(\sqrt{-2\log_e(1-\varepsilon)} - \sqrt{2r}\right)^2\right] \tag{11}$$

Both upper and lower bounds to the ε-outage capacity are extremely close to each other in most situations considered in this study, which justifies the utility of the bound described by Eq. (10).

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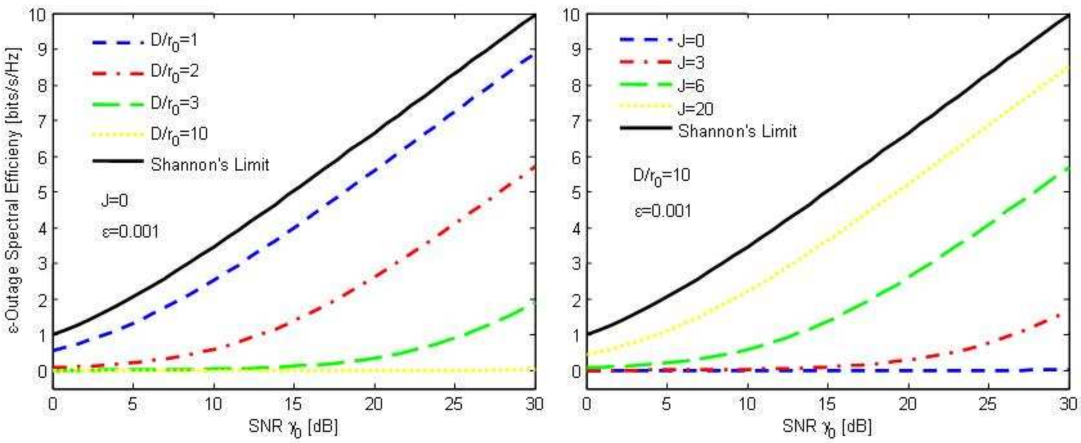

Fig. 1. ε-outage spectral efficiency vs. turbulence-free SNR per symbol γ_0 for coherent detection and additive white Gaussian noise (AWGN). Performance is shown for different values of: (a) the normalized receiver aperture diameter D/r_0, and (b) the number of modes J removed by adaptive optics. The outage probability is fixed at ε=0.001. Amplitude fluctuations are neglected by assuming σ_β^2=0. Turbulence is characterized by the phase coherence length r_0. In (a), D/r_0 ranges from 0.1 (weak turbulence) to 10 (strong turbulence). In (b), the compensating phases are expansions up to tilt (J=3), astigmatism (J=6), and 5th-order aberrations (J=20). The no-correction case (J=0) is also considered. The AWGN Shannon limit is indicated by black lines.

Figures 1-3 illustrate the effect of atmospheric turbulence on the information capacity of channels in the slow-fading scenario when coherent detection and modal-compensated heterodyne or homodyne receivers are considered. We study the ε-outage capacity C_ε Eq. (10) for different outage probabilities ε as a function of the average turbulence free SNR γ_0, the receiver aperture diameter D, the number of spatial modes J removed by the compensation system, and the strength of atmospheric turbulence. Turbulence is quantified by two parameters: the phase coherence length r_0 and the scintillation index σ_β^2. We consider two nonzero values of the scintillation index. The value σ_β^2 = 0.3 corresponds to relatively low scintillation levels, while σ_β^2 = 1 corresponds to strong scintillation, but still below the saturation regime. When the turbulence reaches the saturation regime, wavefront distortion becomes so severe that it would be unrealistic to consider phase compensation.

Figure 1 presents the ε-outage spectral efficiency C_ε/B vs. turbulence-free SNR γ_0. Figure 1(a) shows the capacity for different values of the normalized aperture diameter D/r_0, while Fig. 1(b) shows the performance for different values of J, the number of modes compensated. In all cases, we use a small outage probability ε=0.001. We assume no scintillation, σ_β^2 = 0, so the effect of turbulence is simply to reduce the coherence length r_0. For a fixed aperture diameter D, as r_0 is reduced, the normalized aperture diameter D/r_0 increases, and turbulence reduces the heterodyne or homodyne downconversion efficiency. Just using a normalized aperture diameter D/r_0=10, turbulence reduces the capacity of the atmospheric channel to very small values for all values of the SNR γ_0 considered. When phase correction is used, as in Fig. 1(b), in most situations compensation of just a few modes yields a substantial performance improvement. Compensation of J = 20 modes yields significant improvement for even the largest normalized apertures considered. For example, for a normalized aperture D/r_0=10, the 0.001-outage spectral efficiency can be as large as 8 bits/s/Hz for the higher values of SNR γ_0 considered.

Figure 2 considers the effect of aperture diameter on the ε-outage spectral efficiency. It presents the spectral efficiency as a function of the normalized aperture D/r_0 for a constant phase coherence length r_0. For the smallest aperture diameter considered, the turbulence-free

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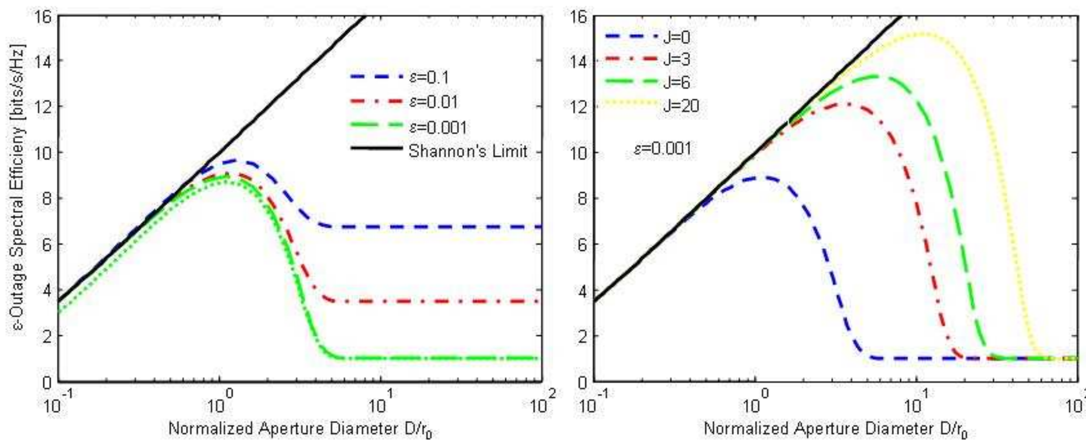

Fig. 2. ε-outage spectral efficiency vs. normalized receiver aperture diameter D/r_0 for coherent detection and AWGN. In (a), no phase compensation is employed, and performance is shown for different values of the outage probability ε. In (b), the outage probability is fixed at ε=0.001, and performance is shown for different values of *J*, the number of modes corrected by adaptive optics. In all cases, the turbulence-free SNR per symbol γ_0 is proportional to the square of the aperture diameter *D*. For the smallest aperture considered, we assume $\gamma_0 = 10$ dB. In (a), ε ranges from 0.1 (large outage probability) to 0.001 (small outage probability). In (b), the compensating phases are expansions up to tilt (*J*=3), astigmatism (*J*=6), and 5th-order aberrations (*J*=20). The AWGN Shannon limit is indicated by black lines. In (a), for ε=0.001, the dotted line considers the ε-outage spectral efficiency when the scintillation index is not neglected but fixed at $\sigma_\beta^2 = 1$

SNR has a value $\gamma_0 = 10$ dB. For any other aperture diameter, the value of γ_0 is proportional to D^2. Figure 2(a) presents capacity for different values of the outage probability ε. The dependence on scintillation index σ_β^2 is very weak, as it can be seen in Fig. 2(a) for ε=0.001, where the outage capacity for σ_β^2=0 (solid line) and σ_β^2=1 (dashed line) are compared. In Fig. 2(a), the existence of an optimal aperture diameter in coherent free-space links is apparent. This optimal aperture diameter maximizes the ε-outage capacity. When the aperture is larger than the optimal value, phase distortion cannot be overcome by the increase on collected power and we observe a decrease in capacity. When larger normalized apertures D/r_0 are considered in Fig. 2(a), the capacity tends toward an asymptotic value that is independent of normalized aperture diameter D/r_0. In this regime, defined by the negative-exponential distribution, the ε-outage capacity is given by $C_\varepsilon = B\log_2\left[1-\overline{\gamma}\log_e\left(1-\varepsilon\right)\right]$. In Fig. 2(b), we consider a small outage probability ε=0.001 and show the ε-outage spectral efficiency for different values of *J*, the number of modes compensated. As we increase *J*, the optimized value of D/r_0 increases, and the optimized capacity improves appreciably. Even for such small outage probability, with compensation of *J*=20 modes, and optimized D/r_0, an outage spectral efficiency of 14 bits/s/Hz is obtained.

The result expressed in Eq. (10) indicates that, to achieve the same rate as the AWGN channel, the atmospheric channel needs an extra power equal to

$$P_{\mathrm{dB}} = -10\log_{10}\left[\frac{\overline{\alpha^2}}{2\left(1+r\right)}\left(\sqrt{-2\log_e\left(1-\varepsilon\right)}+\sqrt{2r}\right)^2\right] \tag{12}$$

expressed in decibels (dB). This fade margin is the same regardless of the turbulence-free SNR γ_0. For a reasonably low outage probability ε, we can make the approximation that $\log_e(1-\varepsilon)\approx -\varepsilon$, and the fade margin Eq. (12) reduces to

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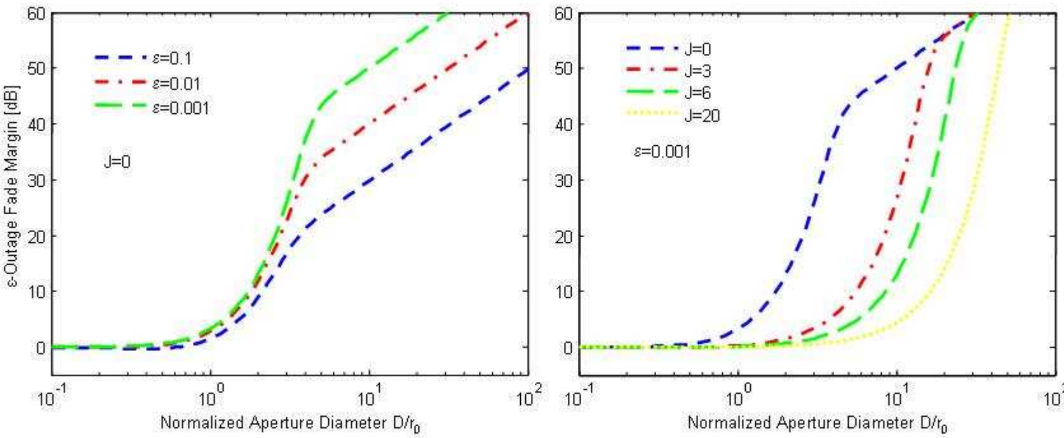

Fig. 3. ε-outage fade margin vs. normalized receiver aperture diameter D/r_0 for coherent detection and AWGN. In (a), no phase compensation is employed, and performance is shown for different values of the outage probability ε. In (b), the outage probability is fixed at ε=0.001 and performance is shown for different values of *J*, the number of modes corrected by adaptive optics. In all cases, the turbulence-free SNR per symbol γ_0 is proportional to the square of the aperture diameter *D*. For the smallest aperture considered, we assume γ_0 = 10 dB. In (a), ε ranges from 0.1 (large outage probability) to 0.001 (small outage probability). In (b), the compensating phases are expansions up to tilt (*J*=3), astigmatism (*J*=6), and 5th-order aberrations (*J*=20).

$$P_{\text{dB}} = -10\log_{10}\left[\frac{\overline{\alpha^2}}{1+r}\left(\sqrt{\varepsilon}+\sqrt{r}\right)^2\right]. \tag{13}$$

In weak turbulence, $r\to\infty$ and $P_{\text{dB}} \approx -10\log_{10}\left(\overline{\alpha^2}\right)$ tends towards zero. On the other hand, under strong turbulence conditions, $r\to 0$ and $P_{\text{dB}} \approx -10\log_{10}\left(\varepsilon\,\overline{\alpha^2}\right)$. In this regime, for a given outage probability *ε*, the fade margin penalty increases steadily with the intensity of the random fade α^2. Consequently, an increase of the receiver aperture diameter translates into an increase of the fade penalty. Figure 3 shows the ε-outage fade margin Eq. (12) vs. the normalized receiver aperture diameter D/r_0. Figure 3(a) shows the fade margin when no phase compensation is employed, and performance is shown for different values of the outage probability *ε*, while in Fig. 3(b) the outage probability is fixed at *ε*=0.001 and fade penalties are shown for different values of *J*, the number of modes corrected by adaptive optics. For a fixed *ε*-outage probability, as the normalized aperture D/r_0 increases, the turbulence fade margin penalty increases. Even using a convenient normalized aperture D/r_0=10, turbulence introduces a fade margin penalty of over 50 dB at *ε*=0.001. When phase correction is used, as in Fig. 3(b), compensation of a few modes yields to a substantial fade penalty decrease. For example, for the normalized aperture D/r_0=10, compensation of *J*=20 modes eliminates almost completely the 50-dB fade margin penalty at *ε*=0.001.

3. Ergodic capacity

In the slow fading scenario, the atmospheric channel remains constant over the transmission duration of the codeword. Without any delay constraints, if the codeword spans several coherence periods, then time diversity is achieved and the outage probability improves. When the codeword spans many coherence periods, we are in the fast fading regime and, proper coding and interleaving, allows us to express the capacity as an average over many independent fades of the atmospheric channel:

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$$\mathrm{E}\left[C\right]=B\int_0^\infty d\gamma\,\log_2\left(1+\gamma\right)\,p_\gamma\left(\gamma\right). \tag{14}$$

Here, the PDF $p_\gamma(\gamma)$ is given by Eq. (2). This ergodic capacity $E[C]$, defined as an ensemble average, is a meaningful way to study the information theory aspects of fast fading atmospheric channels. The ergodic capacity is the expectation with respect to the gains of the instantaneous capacity and, considering that the gain from the times when the channel fades are shallow cannot compensate for the loss when channel fades are deep because the spectral efficiency term $\log_2(1+\gamma)$ is a convex function, Jensen's inequality provides an upper bound for the ergodic capacity, $\mathrm{E}\left[C\right]\le B\log_2\left(1+\overline{\gamma}\right)$. Ergodicity makes certain that the time-average SNR $\overline{\gamma}=\gamma_0\overline{\alpha^2}$ converges to the same limit for all realizations of the atmospheric fading process.

Although the integral (14) cannot be put in a closed form for coherent atmospheric channels, we are able to estimate the ergodic capacity by expanding the spectral efficiency $\log_2(1+\gamma)$ in a Taylor series about the expected value of the SNR, $\overline{\gamma}$

$$\log_2\left(1+\gamma\right)=\log_2\left(1+\overline{\gamma}\right)+\log_2 e\sum_{m=1}^{M}\frac{\left(-1\right)^{m-1}}{m\left(1+\overline{\gamma}\right)^m}\left(\gamma-\overline{\gamma}\right)^m+R_{M+1}, \tag{15}$$

where e is the base of the natural logarithm. The Lagrange form of the remainder R_{M+1}, for a number ξ between $\overline{\gamma}$ and γ,

$$R_{M+1}=\log_2 e\,\frac{\left(-1\right)^M}{\left(M+1\right)\xi^{M+1}}\left(\gamma-\overline{\gamma}\right)^{M+1} \tag{16}$$

is negative when M is odd and, consequently, the truncated Mth order approximation (15) is an upper bound of the capacity for any value of γ. Note that the Jensen upper bound coincides with the first term of (15). Applying the expectation integral (14) to the Taylor series (15) yields to the M-order approximation to the ergodic capacity:

$$\mathrm{E}\left[C\right]=B\log_2\left(1+\overline{\gamma}\right)+B\log_2 e\sum_{m=2}^{M}\frac{\left(-1\right)^{m-1}}{m\left(1+\overline{\gamma}\right)^m}\overline{\left(\gamma-\overline{\gamma}\right)^m}. \tag{17}$$

By using the binomial formula giving the expansion of powers of sums, the mth central moment $\overline{\left(\gamma-\overline{\gamma}\right)^m}$ of the SNR γ is converted to moments about the origin $\overline{\gamma^m}$. These can be expressed in closed form. After some algebra, and using the PDF given by Eq. (2), we obtain the moments about the origin of the atmospheric SNR γ

$$\overline{\gamma^m}=\int_0^\infty d\gamma\,\gamma^m\,p_\gamma\left(\gamma\right)=\frac{\Gamma\left(1+m\right)}{\left(1+r\right)^m}L_m\left(-r\right)\left(\overline{\gamma}\right)^m \tag{18}$$

in terms of the simple Laguerre polynomials L_m for the parameter r describing the ratio of the coherent component to the residual halo in the atmospheric coherent signal collected by the receiving aperture [5]. In Eq. (18), Γ is the complete gamma function.

Figures 4 and 5 show the effect of atmospheric turbulence on the information capacity of channels in the fast-fading scenario when coherent detection and modal-compensated heterodyne or homodyne receivers are considered. We study the ergodic capacity $E[C]$ in Eq. (17) as a function of the average turbulence free SNR γ_0, the receiver aperture diameter D, the number of spatial modes J removed by the compensation system, and the strength of atmospheric turbulence. These results consider the ergodic capacity Eq. (17) to a fourth-order

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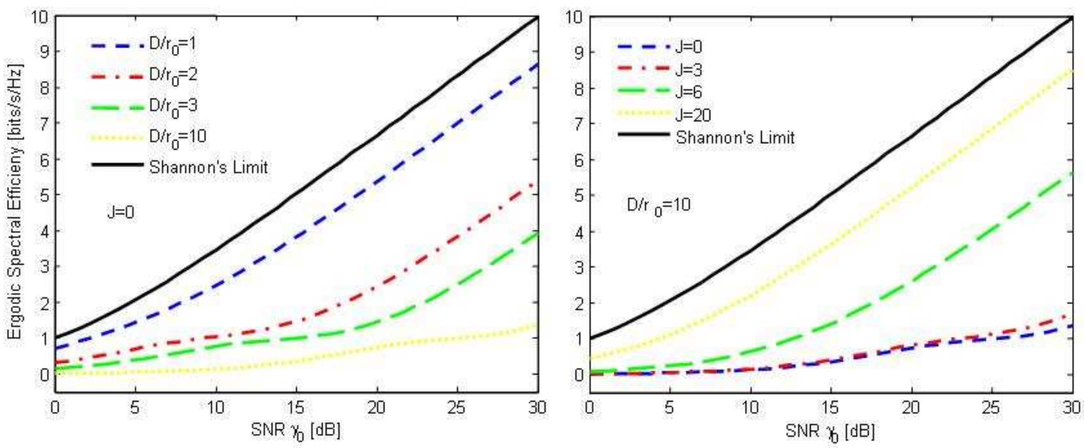

Fig. 4. Ergodic spectral efficiency vs. turbulence-free SNR per symbol γ_0 for coherent detection and additive white Gaussian noise (AWGN). Performance is shown for different values of: (a) the normalized receiver aperture diameter D/r_0, and (b) the number of modes J removed by adaptive optics. Amplitude fluctuations are neglected by assuming $\sigma_\beta^2=0$. Turbulence is characterized by the phase coherence length r_0. In (a), D/r_0 ranges from 0.1 (weak turbulence) to 10 (strong turbulence). In (b), the compensating phases are expansions up to tilt (J=3), astigmatism (J=6), and 5th-order aberrations (J=20). The no-correction case (J=0) is also considered. The AWGN Shannon limit is indicated by black lines.

approximation, which yields high accuracy. For this analysis, the central moments of order m=2, 3, and 4, related to variance, skewness, and kurtosis, respectively, of the fading SNR γ, need to be considered.

Figure 4 shows the ergodic spectral efficiency $E[C]/B$ vs. turbulence-free SNR γ_0 for different values of the normalized aperture diameter D/r_0. As in Figure 1, we assume no scintillation, $\sigma_\beta^2=0$, so the effect of turbulence is simply to reduce the coherence length r_0. Figure 4(a), where no phase compensation is employed J=0, illustrates how even relatively small normalized aperture diameters D/r_0 are not able to achieve the same rate as the AWGN channel. For example, for a SNR γ_0=30dB and normalized aperture D/r_0=10, the ergodic spectral efficiency is slightly higher than 1 bits/s/Hz. This value should be contrasted with the 10-bits/s/Hz AWGN spectral efficiency. When phase correction is used, as in Fig. 4(b), this turbulence penalty reduces considerably. For a normalized aperture D/r_0=10, a mere 20-mode compensation brings the ergodic spectral efficiency to less than 1 bit/s/Hz of the AWGN Shannon limit for most values of the SNR γ_0 considered.

Figure 5 analyzes the effect of aperture diameter on the ergodic spectral efficiency. For a constant phase coherence length r_0 and constant scintillation index σ_β^2, it shows $E[C]/B$ as a function of the normalized aperture diameter D/r_0. As in Fig. 2, for the smallest aperture diameter considered, the turbulence-free SNR has a value of γ_0 =10 dB, while for any other aperture diameter, the value of γ_0 scales proportional to D^2. We again observe two different regimes. For relatively small apertures, only scintillation is of significance, but when the aperture is larger, phase distortion becomes dominant. In Fig. 5(a), no phase compensation is used and the performance is presented for different values of the scintillation index σ_β^2. Here, the optimal aperture is close to D/r_0=1, reaching a maximum spectral efficiency value of almost 9 bits/s/Hz. The dependence on amplitude scintillation is weak. For larger apertures, moving into the negative-exponential regime, efficiency goes down to an asymptotic value of 7 bits/s/Hz. In Fig. 5(b), we consider strong scintillation σ_β^2=1, and show the spectral efficiency for different values of the number of modes compensated J. As expected, the optimized value of D/r_0 and the corresponding ergodic spectral efficiency increase considerably. Even for such a strong scintillation, compensation of J=20 modes translates into

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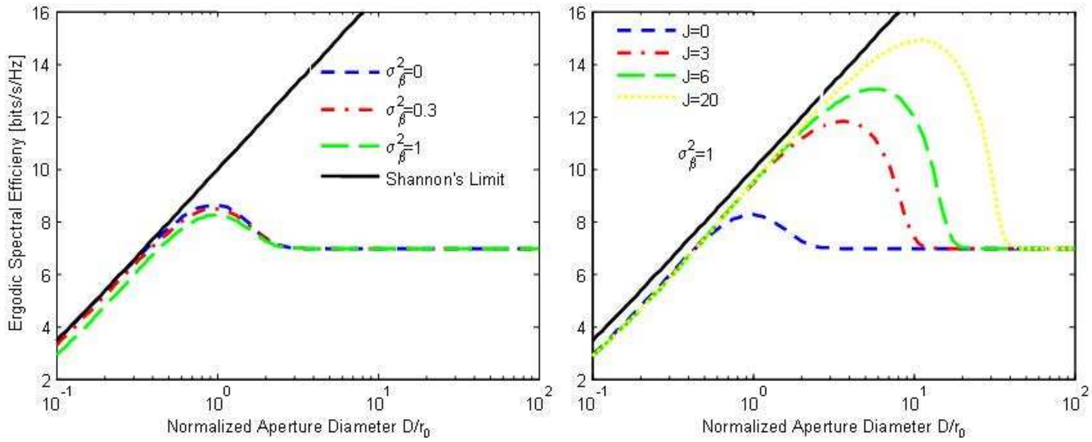

Fig. 5. Ergodic spectral efficiency vs. normalized receiver aperture diameter D/r_0 for coherent detection and AWGN. In (a), no phase compensation is employed, and performance is shown for different values of the scintillation index σ_β^2. In (b), the scintillation index is fixed at $\sigma_\beta^2 = 1$, and performance is shown for different values of J, the number of modes corrected by adaptive optics. In all cases, the turbulence-free SNR per symbol γ_0 is proportional to the square of the aperture diameter D. For the smallest aperture considered, we assume $\gamma_0 = 10$ dB. In (a), σ_β^2 ranges from 0.3 (weaker turbulence) to 1 (stronger turbulence). In (b), the compensating phases are expansions up to tilt (J=3), astigmatism (J=6), and 5th-order aberrations (J=20). The AWGN Shannon limit is indicated by black lines.

an optimal value D/r_0=10 and an ergodic spectral efficiency close to 15 bits/s/Hz. The 7-bits/s/Hz asymptotic value is now achieved for much larger values of the normalized aperture D/r_0.

4. Conclusions

We have developed analytical expressions for the ergodic and outage capacities for free-space optical communication links using coherent detection and active modal compensation of wavefront phase distortion to overcome turbulence-induced fading. We have studied the effect of various parameters, including turbulence level, signal strength, receive aperture size, and the extent of wavefront compensation. We have separately quantified the effects of amplitude fluctuations and phase distortion, and have identified the impact of the number of modes compensated on the maximal rate at which the information may be transferred. In most situations considered, amplitude fluctuations effects become negligible, and phase distortion become the dominant effect, so phase compensation becomes effective in increasing link capacity. We have examined information-theoretic bounds on the outage capacity, and have obtained simple and tight bounds on the ergodic capacity. For typical turbulence conditions, large gains in achievable rate are realizable by correcting a fairly small number of modes and using optimum receiving aperture diameters.

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