Capacity of coherent free-space optical links using diversity-combining techniques

Aniceto Belmonte[1] and Joseph M. Kahn[2]

[1]Technical University of Catalonia, Department of Signal Theory and Communications, 08034 Barcelona, Spain
[2] Stanford University, Department of Electrical Engineering, Stanford, CA 94305, USA
belmonte@tsc.upc.edu

Abstract: We study the performance of diversity combining techniques applied to synchronous laser communication through the turbulent atmosphere. We assume that a single information-bearing signal is transmitted over two or more statistically independent fading channels, and that the multiple replicas are combined at the receiver to improve detection efficiency. We consider the effects of log-normal amplitude fluctuations and Gaussian phase fluctuations, in addition to local oscillator shot noise. We study the effect of various parameters, including the ratio of receiver aperture diameter to wavefront coherence diameter, the scintillation index, and the number of independent diversity branches combined at the receiver. We consider both maximal-ratio combining (MRC) and selective combining (SC) diversity schemes. We derive expressions for the outage Shannon capacity, thus placing upper bounds on the spectral efficiency achievable using these techniques.

OCIS codes: (010.1330) Atmospheric turbulence; (060.4510) Optical communications; Coherent receivers; Diversity combining; Shannon capacity.

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1. Introduction

In this work, we analyze the spectral efficiency of optical communication over the clear turbulent atmosphere. Defined as the average transmitted data rate per unit bandwidth for a specific average transmit power and probability of outage or error, achievable spectral efficiency is an important performance measure for free-space optical communication systems. As the use of multilevel phase modulation schemes increase spectral efficiency by sending multiple bits per symbol, optical coherent reception provides an alternative to direct detection schemes for free-space optical communication applications. Certainly, both spectral efficiency and power efficiency need to be considered in the development of transmitter and receiver systems that extent the performance of free-space optical communication systems and provide suitable long-haul and space links. As we intent to focus on spectral efficiency, in this paper we do not address the receiver sensitivities in terms of photon efficiency.

A coherent receiver is more complicated than a direct-detection receiver [1]. Accurate wavefront matching between the incoming signal and the local oscillator is needed to ensure effective coherent reception. Imperfect spatial mode matching can lead to destructive interference and, consequently, to degraded system performance. Any mismatch between the amplitudes and phases of the two fields will result in a reduction of the downconverted power, i.e., fading. The presence of atmospheric turbulence further complicates the problem. In general, when atmospheric turbulence is taken into consideration, the efficiency of a single-aperture coherent receiver is limited by the phase coherence length. For systems employing small receiver apertures, the required wavefront alignment accuracy can be achieved because, if each receiver is smaller than the scale on which the signal wavefront varies, the local-oscillator phase can be accurately matched to the signal to achieve effective coherent reception. The wavefront distortions due to the atmosphere are then negligible. However, for systems using large collecting apertures, such as those designed to maintain a high signal-to-noise ratio (SNR) over a long range, the difficulty in maintaining good spatial mode matching across the input aperture can present a serious problem in achieving effective coherent reception. The performance of large-aperture coherent receivers generally does not improve with increasing collecting size.

The coherent optical receiver down-converts the whole optical signal linearly to an electrical signal by means of heterodyne or homodyne detection. In the case of coherent modulation, phase fluctuations can severely degrade performance unless measures are taken to compensate for them at the receiver. Here, we assume that after homodyne or heterodyne downconversion is used to obtain an electrical signal, the receiver is able to track any temporal phase fluctuations caused by turbulence (as well as those caused by laser phase noise), performing ideal coherent (synchronous) demodulation. Under this assumption, analyzing the receiver performance requires knowledge of only the envelope statistics of the downconverted electrical signal.

Atmospheric fading leads to serious degradation in the receiver sensitivity measured in terms of received photons per symbol, resulting in either a higher error rate or a higher required transmit power for a given multilevel modulation technique. The simplest fading compensation technique may comprise an increased link budget margin or code interleaving [2]. Still, these techniques are designed to deal with the worst channel conditions and, consequently, under minor fading conditions there is a poor utilization of the channel capacity. Adapting the receiver to the channel fading can lead to better utilization of the channel capacity. Specifically, by using a deformable mirror to multiply the field received over the receiving aperture by the conjugate of its phase prior to diffraction-limited reception, we obtain a phase-compensated coherent communication receiver. This fade compensation technique uses high-bandwidth adaptive optics to correct for the atmospherically induced wavefront distortion before wavefront matching between the incoming signal and the local oscillator [3,4]. In principle, this adaptive coherent receiver may also be implemented by

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using channel measurements and an adaptive local oscillator spatially matched to the field entering the diffraction-limited optical system. Phase-compensated receivers offer the potential for overcoming atmospheric limitations by adaptive tracking of the beam wave-front and consequent correction of atmospherically-induced aberrations. For typical turbulence conditions, large gains in achievable rate of reliable communication supported by the atmospheric channel are realizable by correcting a fairly small number of modes and using optimum receiving aperture diameters [4].

As an alternative to a single monolithic-aperture coherent receiver with a full-size collecting area, a large effective aperture can be achieved by combining the output signal from an array of smaller receivers. The advantage of a coherent array in terms of the coupling efficiency is that the number of turbulence speckles over each subaperture in the array is much smaller than it would be over a single large aperture. Because each receiver can now be smaller than the scale on which the signal wavefront varies, the local oscillator phase can be matched to the signal to achieve effective coherent reception. Output signals from these receivers can then be combined electronically to improve the detection statistics. In general, the performance of a combined system should improve with an increasing number of receivers and, consequently, given a fixed collecting area, the combined system can offer superior performance. It is important to note that diversity combining consists of receiving redundantly the same information signal over two or more fading channels and to exploit the low probability of concurrence of deep fades in all the diversity channels. These multiple replicas can be obtained by extracting the signals via different optical paths in space by using multiple receivers, as mentioned above, but also in wavelength by using multiple-wavelength channels separated by at least the coherence bandwidth of the channel or in time, by using multiple time slots separated by at least the coherence time of the atmospheric channel.

In atmospheric optical communications, channel variations are typically much slower than the signaling period. As such, we need to model the channel as a non-ergodic block-fading channel, for which a given sequence of symbols in a code undergoes only a finite number of fading realizations. Turbulence-induced random fluctuations are a slow time-varying process, with a coherence time on the order of milliseconds, and it is therefore appropriate to analyze the outage probability of the channel. To some extent, this has been done in the case of direct-detection receivers [5–12]. In these works, the outage probability and capacity of the atmospheric channel is analyzed for ideal (i.e. a Poisson process) and non-ideal photodetection, and diverse bandwidth constraints. In general, these works also make the further assumption of on-off keying transmission or pulse-position modulation. In this paper we study the outage capacity of the clear turbulent atmospheric channel using diversity under the assumptions of coherent (synchronous) and shot-noise limited photodetection at the receiver. The shot-noise limited receiver sensitivity can be achieved with a sufficient local oscillator power. We model the channel as a quasi-static block fading channel in accordance with which communication takes place over a finite number of blocks and each block of transmitted symbols experiences an identically distributed fading realization.

We consider two types of receive diversity combining. First we assume the receiver has knowledge of the instantaneous channel state, making perfect maximal ratio combining (MRC) diversity possible. In this diversity scheme, the receiver co-phases the intermediate signals, adjusts their amplitudes separately, and sums them to obtain a composite signal with improved SNR. The rate at which phase and amplitude must be adjusted will be dictated by the rate at which the atmospheric turbulence fluctuates, generally no higher than 1 kHz. The MRC receiver is the optimal combining technique in that it yields a carrier with the highest mean SNR and lowest SNR fading. The optimum electronic gain for each receiver should be proportional to the received signal field amplitude. Note that when the electronic gains and phase delays are back-propagated into the LO, the optimum gain and phase adjustments would result in an amplitude and phase match of this synthetic LO field to the distorted signal field. When high-bandwidth adaptive optics are considered, we are only able to match the signal phase, and not the signal amplitude, to correct for the atmospherically induced wavefront distortion.

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Although it yields the highest possible heterodyne efficiency, the implementation of a MRC receiver requires the development of signal processing that adjust both the delay and the gain elements of the IF signal outputs. Then we consider the case when selective combining (SC) diversity schemes is used. A selection receiver is quite different from the coherent MRC summing receiver as the single strongest intermediate signal is switched to the output, while all the other array element signals are discarded. Although this simple diversity technique clearly does little to improve the mean SNR value of the collected signal, it does provide a reduction in the signal fading. In Section 2, we define a statistical model to describe the signal collected by the receiver combiner after propagation through the atmosphere. In Section 3, we consider the outage capacity of such channels under diversity-combining techniques. We present conclusions in Section 4.

2. Outage probability of coherent atmospheric links using diversity combining

The classical theory of communications was developed originally in the context of linear channels with additive noise [13]. The capacity C of a communication channel is the maximal rate at which the information may be transferred through the channel without error. For an additive white Gaussian noise (AWGN) channel, in the classical capacity formula with average power constraint P and noise power spectral density $N_0/2$, given by $C = B\log_2(1+\gamma_0)$, the spectral bandwidth B, which has units of Hz, multiplies the maximal spectral efficiency $\log_2(1+\gamma_0)$, which has units of bits/s/Hz. Here, $\gamma_0 = P/N_0B$ is the SNR per unit bandwidth B. The SNR γ_0 for a quantum or shot-noise limited signal can be interpreted as the detected number of photons (photocounts) per symbol when $1/B$ is the symbol period [14]. Coherently detected signals are modeled as narrowband RF signals with additive white Gaussian noise (AWGN). In free-space optical communication through the turbulent atmosphere, we must consider fading channels, which are a class of channels with multiplicative noise. In the fading AWGN channel with average power constraint P and noise power spectral density $N_0/2$, we let α^2 denote the atmospheric channel power fading and $(P/N_0B)\alpha^2 = \gamma_0\alpha^2$ denote the instantaneous received SNR per symbol. For a shot-noise-limited coherent optical receiver, the SNR of the envelope detector can be taken as the number of signal photons detected on the receiver aperture γ_0 multiplied by a heterodyne mixing efficiency α^2: In addition to the effective delivery of the signal to the detector, the performance of the optical link also depends on the receiver sensitivity measured in terms of received photons. For systems with perfect spatial mode matching the heterodyne mixing efficiency is equal to 1. When the spatial modes are not properly matched, the contribution to the current signal from different parts of the receiver aperture can interfere destructively and result in the reduced instantaneous heterodyne mixing and consequent fading.

Conditional on a realization of the atmospheric channel described by α, this is an AWGN channel with instantaneous received SNR $\gamma = \gamma_0\alpha^2$. This quantity is a function of the random channel power gain α^2, and is therefore random. The statistical properties of the atmospheric random channel fade α^2, with probability density function (PDF) $p_{\alpha 2}(\alpha^2)$, provide a statistical characterization of the SNR $\gamma = \gamma_0\alpha^2$ and, consequently, of the maximal spectral efficiency achievable for the free-space optical link. We have already modeled the impact of atmospheric turbulence-induced phase and amplitude fluctuations on free-space optical links using synchronous detection and found that the SNR γ for a single monolithic-aperture coherent receiver is described by a noncentral chi-square probability distribution function (PDF) with two degrees of freedom [3]:

$$p_\gamma(\gamma) = \frac{1+r}{\bar{\gamma}}\exp(-r)\exp\left[-\frac{(1+r)\gamma}{\bar{\gamma}}\right] I_0\left[2\sqrt{\frac{(1+r)r\gamma}{\bar{\gamma}}}\right], \qquad \gamma \geq 0, \qquad (1)$$

where the average SNR (or average detected photocounts) $\bar{\gamma}$ and the parameter $1/r$ consider turbulence effects. The model leading to the PDF in Eq. (1) is based on the observation that the downconverted signal current can be characterized as the sum of many contributions from

N different coherent regions within the aperture [3]. In this model, the signal is characterized as the sum of a constant (coherent) term and a random (incoherent) residual halo. The contrast parameter $1/r$ is a measure of the strength of the residual halo relative to the coherent component. The parameter r ranges between 0 and ∞. It can be shown that when the constant term is very weak ($r \to 0$), turbulence fading makes the SNR to become negative-exponential-distributed, just as in a speckle pattern. Likewise, when the dominant term is very strong ($r \to \infty$), the density function becomes highly peaked around the mean value $\bar{\gamma}$, and there is no fading to be considered.

In order to assess the impact of turbulence on the heterodyne mixing and fading, the field amplitude without the effect of turbulence in the pupil plane must be modified by a multiplicative factor $\exp\left[\chi(\boldsymbol{r}) - j\phi(\boldsymbol{r})\right]$ where $\chi(\boldsymbol{r})$ and $f(\boldsymbol{r})$ represent the log-amplitude fluctuations (scintillation) and phase variations (aberrations), respectively, introduced by atmospheric turbulence. Consequently, both $\bar{\gamma}$ and $1/r$ are described in terms of log-normal amplitude fluctuations and Gaussian phase fluctuations as characterized by their respective statistical variances, σ_χ^2 and σ_f^2,

$$\sigma_\chi^2 = \log_e\left(1 + \sigma_\beta^2\right)$$
$$\sigma_\phi^2 = 1.0299\left(\frac{D}{r_0}\right)^{5/3} \tag{2}$$

The intensity variance σ_β^2 is often referred to as the scintillation index [15]. The coefficient 1.0299 in the phase variance σ_f^2 assumes that no terms are corrected by a receiver employing active modal compensation [16]. In Eq. (2), the receiver aperture diameter D is normalized by the wavefront coherence diameter r_0, which describes the spatial correlation of phase fluctuations in the receiver plane [1].

We consider both MRC and SC diversity combining of the received signal. MRC schemes assume perfect knowledge of the branch amplitudes and phases, require independent processing of each branch, and need that the individual signals from each branch be weighted by their signal to noise power ratios then summed coherently. A receiver with MRC will coherently combine the diversity branches by weighting them by the complex conjugate of their respective fading gains and adding them. The instantaneous SNR γ_{MRC} for a summing coherent MRC combiner is the power ratio of the phase-coherent addition of the signal amplitudes from each element of the combiner to the incoherent addition of the noise. If an optimum voltage gain proportional to the amplitude of the signal itself is assumed for each receiver in the combiner, and if equal noise powers are assumed, the resultant composite SNR for an L-element MRC combiner is [17]

$$\gamma_{MRC} = \sum_{l=1}^{L} \gamma_l \ . \tag{3}$$

Therefore, the instantaneous MRC combiner SNR γ_{MRC} is the sum of the component array element SNR's. For independent branch signals and equal average branch SNR $\bar{\gamma}$,

$$\bar{\gamma}_l = \bar{\gamma} \qquad \text{for all } \ l \in \{1, 2, ..., L\} \ , \tag{4}$$

the PDF of the received SNR γ_{MRC} at the output of a perfect L-branch MRC coherent combiner in the atmosphere would be described a sum of L independent and identically distributed non-central chi-squared random variables with two degrees of freedom. This random variable has a noncentral chi-square distribution with $2L$ degrees of freedom:

$$p_\gamma\left(\gamma_{MRC}\right) = \left(\frac{1+r}{\bar{\gamma}}\right)^{\frac{L+1}{2}} \left(\frac{1}{Lr}\right)^{\frac{L-1}{2}} \exp\left(-Lr\right) \exp\left[-\frac{(1+r)\gamma_{MRC}}{\bar{\gamma}}\right] I_{L-1}\left[2\sqrt{\frac{L(1+r)r\gamma_{MRC}}{\bar{\gamma}}}\right]. \tag{5}$$

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Here, the mean SNR $\overline{\gamma}_{MRC}$ is simply L times the single-element mean SNR $\overline{\gamma}$ or $L\overline{\gamma}$. Equivalently, this result can be expressed in terms of the cumulative distribution function (CDF) of the SNR γ_{MRC}, which is defined as the probability that the output SNR γ_{MRC} falls below a certain specified threshold γ_t, or outage probability. After some algebra, we obtain the CDF

$$\begin{aligned} F_{MRC}(\gamma_t) &= \int_0^{\gamma_t} d\gamma_{MRC}\; p_\gamma(\gamma_{MRC}) \\ &= 1-\left(\frac{1}{2Lr}\right)^{\frac{L-1}{2}} Q_L\left(\sqrt{2Lr}\,,\sqrt{\frac{2(1+r)}{\overline{\gamma}}\gamma_t}\right), \end{aligned} \tag{6}$$

where $Q_L(a,b)$ is the generalized L-order Marcum Q function.

Now, since some of the diversity branches may be too weak to play a part, instead of combining signals from all diversity branches as in MRC, SC only processes one of the diversity branches. Specifically, the combiner chooses the branch with the highest number of signal photons received and the coherent sum of the individual branch signals is not required. Thus, although this diversity technique is simpler than MRC, it also yields suboptimal performance since too much information may be lost with L–1 diversity branches released. Assuming independent branch signals and equal average branch SNR $\overline{\gamma}$, the outage probability of the received signal photons γ at the output of an L-branch SC coherent combiner is found to be given by the Lth power of the CDF corresponding to a single-branch receiver [18]: The probability that the SNR of the l-th branch is less than or equal to any specified value γ_t is given by its CDF, so that the probability that the SNR of a perfect L-branch SC coherent combiner is simultaneously less than γ_t must be the product of the corresponding CDFs. Using Eq. (6) with $L = 1$, it results the following expression for the SC CDF:

$$F_{SC}(\gamma_t) = \left[1-Q\left(\sqrt{2r}\,,\sqrt{\frac{2\;(1+r)}{\overline{\gamma}}\;\gamma_t}\right)\right]^L, \tag{7}$$

where Q now refers to the first-order Marcum Q function.

3. Outage capacity numerical results and comparisons

As data rates increase, atmospheric communication channels become better described as slow-fading channels. In terms of information theory this is equivalent to communication over channels where there is a nonzero probability that any given transmission rate cannot be supported by the channel. Consequently, in many practical situations, where delay constraints prevent using an extended codeword and averaging over deep fade channel realizations is not possible, an appropriate measure of capacity is the probability that the channel can support a given rate R, i.e., $p_{out}(R) = \mathrm{P}\{\log_2(1+\gamma)<R\}$. Here, the operator P{A} indicates the probability of an event A. If γ_R denotes the SNR that is required to support a rate R, the probability of outage can be expressed in terms of SNR as $p_{out}(R) = \mathrm{P}\{\gamma<\gamma_R\}$. This result can be expressed in terms of the cumulative distribution function CDF of the SNR γ as $p_{out}(R) = F(\gamma_R)$. From here, we can solve $p_{out}(R) = \varepsilon$ to obtain the SNR γ producing a ε-outage probability $F(\gamma_R) = \varepsilon$, i.e., $\gamma_R = F^{-1}(\varepsilon)$. Then, by definition, the ε-outage spectral efficiency becomes

$$\frac{C_\varepsilon}{B} = \log_2(1+\gamma_R) = \log_2\left[1+F^{-1}(\varepsilon)\right]. \tag{8}$$

It is clear that the atmospheric outage spectral efficiency depends on the statistical distribution of SNR γ through its CDF $F(\gamma_R)$. Note that Eq. (8) requires the inversion of the CDFs given by

Eqs. (6) and (7) modeled the impact of atmospheric turbulence-induced phase and amplitude fluctuations on free-space optical links using synchronous detection and MRC and SC diversity combining of the received signal. However, there is no known elementary inverse of the Marcum Q functions of any order L. For $L>1$, even the use of alternative representations of the generalized Marcum Q-functions does not alleviate this difficulty. Instead, we approach the problem numerically by using Newton's method to converge to the solution of $F^{-1}(\varepsilon)$ for both MRC and SC diversity combining.

In Figs. 1-3, we start by comparing the spectral efficiency of an AWGN channel with the spectral efficiency of a fading atmospheric channel with various diversity combining techniques. We consider a L-aperture heterodyne receiver system where the receivers are separated by more than one coherence length. Each receiver has a pupil area $1/L$ times the pupil area of the single receiver system so that the received signal power in both the single and multiple receiver systems is the same. Equivalently, as it was noted before, diversity can also be obtained with just one of these reduced apertures by using L wavelength channels separated by at least the coherence bandwidth of the channel or in time, by using L time slots separated by the coherence time of the atmospheric channel.

Figure 1 considers the effect of aperture diameter on the ε-outage spectral efficiency. It presents the spectral efficiency as a function of the normalized aperture D/r_0 for a constant phase coherence length r_0. In (a), MRC combining is employed. In (b), a SC combiner is considered. In all cases, the outage probability is small and fixed at $\varepsilon = 0.001$, and the channel capacity per unit bandwidth is shown for different values of the number L of combiner branches. The case $L = 1$ corresponds to no receive diversity. The area πD^2 describes the combined, multi-aperture system equivalent aperture. When no receive diversity is considered, D equals the receiver aperture diameter. If a L-aperture system is analyzed, each one of the aperture diameters equals $D/\sqrt{L}$. For the smallest aperture considered, we assume γ_0 equal to 10 photons-per-symbol. For any other aperture diameter, the value of γ_0 is proportional to D^2. Comparing Figs. 1(a) and 1(b) we see that, as expected, the SC scheme provides less diversity gain and a lower rate of improvement than the MRC scheme. The dependence on scintillation index σ_β^2 is strong, as it can be seen in the plots, where the outage capacity for $\sigma_\beta^2 = 0$ (solid line) and $\sigma_\beta^2 = 1$ (dashed line) are compared. For relatively small apertures, amplitude scintillation is dominant. A receiver with MRC will coherently combine the diversity branches by weighting them by the complex conjugate of their respective fading gains and adding them. Consequently, it will provide some degree of protection against scintillation. On the other hand, SC combining does not allow for this kind of scintillation compensation and performance is less improved by the degree of diversity considered in the analysis.

Note the diminishing capacity returns that are obtained as the number of branches increases. Although as we increase L the outage spectral efficiency improves appreciably, the greatest improvement is still obtained in going from single- to two-branch combining. In any case, even for such small outage probability $\varepsilon = 0.001$, a multiple receiver system with $L = 8$ apertures reach an outage spectral efficiency of over 10 bits/s/Hz, very close to the optimal spectral rate of a single branch AWGN channel as defined by Shannon.

When a single ($L = 1$) or dual ($L = 2$) receiver system is considered, the existence of an optimal aperture diameter in coherent free-space links is apparent. This optimal aperture diameter maximizes the ε-outage capacity. When the aperture is larger than the optimal value, phase distortion cannot be overcome by the increase in collected power, and we observe a decrease in capacity. For $L>2$ receiver systems, or under strong scintillation conditions, no optimal value can be identified.

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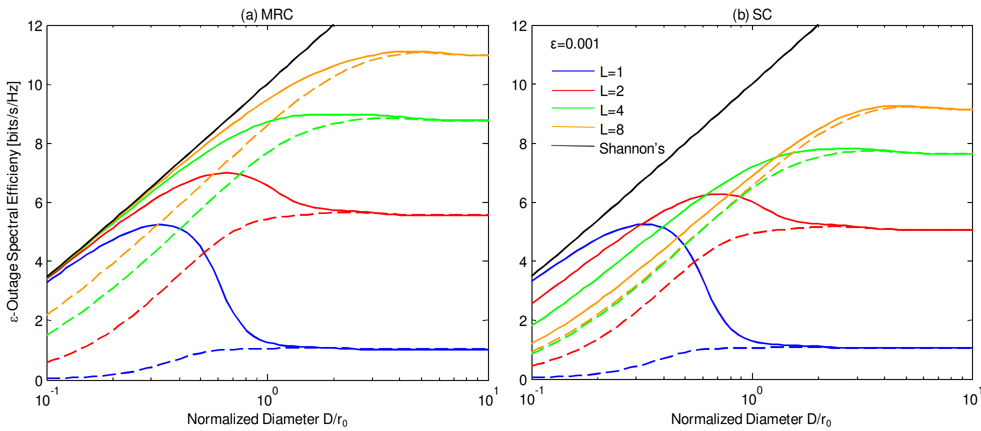

Fig. 1. ε-outage spectral efficiency vs. normalized receiver aperture diameter D/r_0 for coherent detection and AWGN. In (a), MRC combining is employed. In (b), a SC combiner is considered. In all cases, the outage probability is fixed at ε=0.001, and the channel capacity per unit bandwidth is shown for different values of the number L of combiner branches. The case L=1 corresponds to no receive diversity (blue lines). The area πD^2 describes the combined, multi-aperture system equivalent aperture. When no receive diversity is considered, D equals the receiver aperture diameter. The turbulence-free SNR per symbol γ_0 is proportional to the square of the aperture diameter D. For the smallest aperture considered, we assume γ_0 equal to 10 photons per symbol. Solid lines neglect amplitude fluctuations by assuming σ_β^2=0. In this case, turbulence is characterized by the phase coherence length r_0. The dotted lines consider the ε-outage spectral efficiency when the scintillation index is not neglected but fixed at $\sigma_\beta^2 = 1$. The AWGN Shannon limit is indicated by black lines.

In all cases, and both MRC and SC diversity combining, when larger normalized apertures D/r_0 are considered, the capacity tends toward an asymptotic value that is independent of normalized aperture diameter D/r_0. In this regime, dominated by wavefront distortions, and where amplitude fluctuations and the level of scintillation σ_β^2 are of little influence, the signal coherence term is weak and $r\rightarrow 0$. Now, the fading PDF Eq. (5), when L independent signals are combined in a MRC combiner, becomes a gamma distribution, i.e.,

$$F_c\left(\varepsilon\right)=\frac{\Gamma\left(L,\frac{\varepsilon}{\overline{\gamma}}\right)}{\Gamma\left(L\right)}. \qquad (9)$$

Here $\Gamma(a,x)$ is the lower incomplete gamma function and $\Gamma(a) = \Gamma(a,0)$ is the ordinary gamma function. It is easy to shown that, for reasonable small outage probabilities $\varepsilon \rightarrow 0$, the outage capacity per unit bandwidth Eq. (8) results, after some algebra,

$$\frac{C_\varepsilon}{B}=\log_2\left\{1+\overline{\gamma}\ \left[\ \varepsilon\,\Gamma\left(1+L\right)\ \right]^{\frac{1}{L}}\right\}. \qquad (10)$$

Equation (10) estimates the asymptotic values observed in Fig. 1(a). Also, in this regime with $r\rightarrow 0$, the PDF Eq. (1) for a single receiver system becomes a negative-exponential distribution and, consequently, the fading CDF in a L-branch SC combiner is given by

$$F_c\left(\gamma_R\right)=\exp\left(-L\ \frac{\gamma}{\overline{\gamma}}\right). \qquad (11)$$

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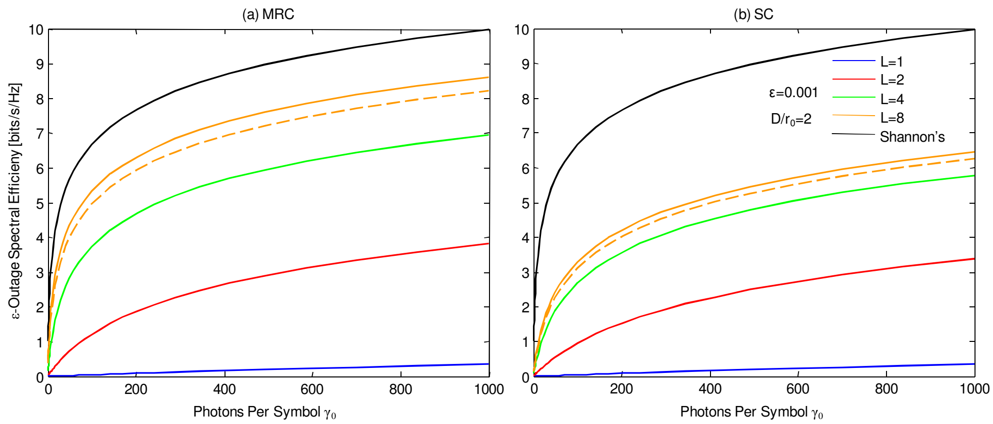

Fig. 2. ε-outage spectral efficiency vs. turbulence-free photons per symbol γ_0 for coherent detection and additive white Gaussian noise (AWGN). In (a), MRC combining is employed. In (b), a SC combiner is considered. In all cases, the outage probability is fixed at ε=0.001, and the channel capacity per unit bandwidth is shown for different values of the number L of combiner branches. The case L=1 corresponds to no receive diversity (blue lines). Amplitude fluctuations are neglected by assuming σ_β^2=0. Turbulence is characterized by a moderate phase coherence length r_0 such as D/r_0=2. The area πD^2 describes the combined, multi-aperture system equivalent aperture. The AWGN Shannon limit is indicated by black lines. The dotted lines consider the ε-outage spectral efficiency when the scintillation index is not neglected but fixed at $\sigma_\beta^2 = 1$.

Once again, when outage probabilities are small $\varepsilon \to 0$, the outage spectral efficiency becomes

$$\frac{C_\varepsilon}{B} = \log_2\left(1+\frac{\overline{\gamma}}{L}\ \varepsilon\right), \tag{12}$$

which calculates the asymptotic values observed in Fig. 1(b).

Interestingly, the results expressed in Eqs. (10) and (12) indicate that, to achieve the same rate as the MRC combiner, the SC combiner needs an extra power (in dB) equal to

$$\Delta P_{\mathrm{dB}} = L\ \left(\varepsilon^{\,1-\mathrm{L}} L!\right)^{\frac{1}{L}}\ , \tag{13}$$

where we have used the relation, valid for all natural numbers a, $\Gamma(1 + a) = a!$. This fade margin increases with the number of branches in the combiner and the outage probability ε, but is the same regardless of the mean SNR $\overline{\gamma}$. Equation (13) allows us to compare fade margins for the MRC and SC combiners. For example, for a 2-branch combiner and a small outage probability ε = 0.001, the SC strategy needs almost 100 photons per symbol more than the MRC to reach the same spectral rate. Figure 2 explicitly presents these results by plotting the ε-outage spectral efficiency as a function of turbulence-free photons per symbol γ_0. The normalized aperture $D/r_0 = 2$ remains constant for all the cases considered in the figure which corresponds, in most situations, to the exponential regime $r \to 0$. From Eq. (13) and the study of Fig. 2, it becomes clear the superior power efficiency of the MRC combiner. As a further example, if the combining system is assumed to collect γ_0 200 photons per symbol in the absence of turbulence, then the maximum spectral rate that can be achieved using a 4-branch SC combining is C_ε/B = 3.5 bits/s/Hz, while when utilizing a 4-branch MRC combining is a 34% higher at C_ε/B = 4.7 bits/s/Hz. Still, it is a spectral efficiency nearly a 40% smaller than it could be expected without turbulence, with an estimated Shannon's rate of almost 8 bits/s/Hz, and extra gains in achievable rates are within reach by increasing the combiner complexity L.

In Fig. 3, the tradeoff between the outage probability ε and the maximum achievable rate C_ε/B is analyzed for MRC and SC combining. The ε versus C_ε/B tradeoff is governed by Eq.

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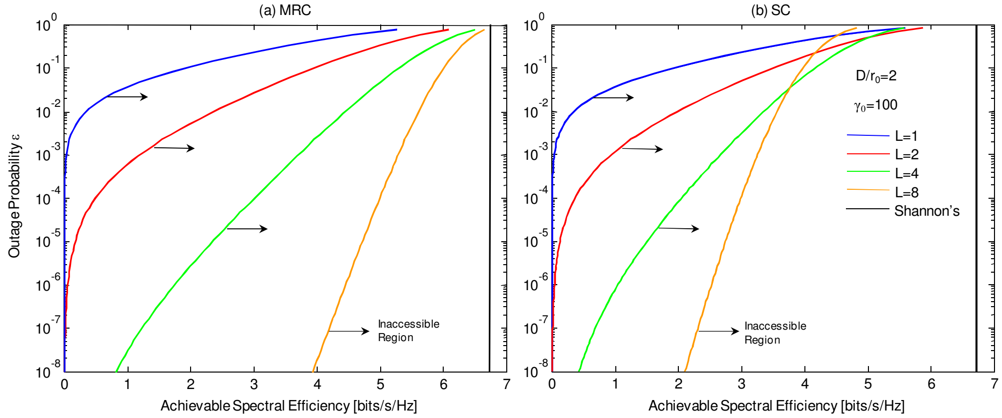

Fig. 3. Probability of outage versus channel capacity per unit bandwidth for coherent detection and additive white Gaussian noise (AWGN). We consider both MRC (in a) and SC (in b) combining of the received signal and the trade-off between the outage probability and the maximum achievable rate is analyzed for different values of the number of branches L in the combiner. The case L=1 corresponds to no receive diversity (blue lines). In all cases, we assume the number of photons per symbol γ_0 equals to 100. Amplitude fluctuations are neglected by assuming σ_β^2=0. Turbulence is characterized by a moderate phase coherence length r_0 such that D/r_0=2. The area πD^2 describes the combined, multi-aperture system equivalent aperture. The AWGN Shannon limit corresponding to γ_0 = 100 photons-per-symbol is indicated by vertical black lines.

(8) and parameterized by the turbulence-free photons per symbol γ_0, the normalized aperture diameter D/r_0, and the number of branches L at the combiner. These information-theoretic limits quantify the optimum tradeoff between the pair ε and C_ε/B, and show those set of pairs for which is impossible to find a reliable error-correcting codes which can realize any arbitrarily small probability of error. It is clear from Fig. 3 that, for a given probability of outage, there is a significant gain in the achievable rate when utilizing the MRC combiner over the SC combiner. For example, if the system is designed to meet $\varepsilon = 10^{-4}$, then the maximum code rate that can be reliably transmitted over this channel using a 4-branch SC combining is C_ε/B = 2.1 bits/s/Hz, while when utilizing a 4-branch MRC combining, C_ε/B = 3.1 bits/s/Hz, which is an increase of close to 50% in the achievable rate. Once again, sizeable additional gains in achievable rate are on hand by increasing the combiner branch number L, and the optimal tradeoff between ε and C_ε/B shown in Fig. 3 can be used to select code rates in feasible coherent FSO channels.

4. Conclusion

This paper considers the analysis of coherent free-space optical diversity-combining channels corrupted by atmospheric turbulence from an information theory perspective. We analyze how these channels, by providing redundant replicas of the transmitted message to the receiver, each corrupted independently by the atmosphere, produce reliable communication because of the low probability of concurrence of deep fades in all the diversity channels. New statistical models are presented where the ratio of receiver aperture diameter to wavefront coherence diameter, the strength of the scintillation index, and the number of independent diversity branches combined at the receiver are considered. These models are used to derive fundamental limits on outage probability and achievable rates for synchronous coherent communication links.

We have separately quantified the effects of amplitude fluctuations and phase distortion, and have identified the impact of the combiner number of branches on the maximal rate at which the information may be transferred. In most situations considered, phase distortion becomes the dominant effect on the coherent performance and amplitude fluctuations are of less importance. We have examined information-theoretic limits on the outage capacity, and

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have obtained simple analytical results. For typical turbulence conditions, sizeable gains in achievable rate are realizable by allowing for a rather small number of apertures. Also, the MRC combiner was shown to offer performance significantly better than the SC combiner.

It has been demonstrated that MRC and SC diversity combining of the received signal leads to significant increases in the channel capacity subject to outage. Furthermore, for a given probability of outage, there is a significant gain in the achievable rate when utilizing the MRC combiner over the SC combiner. This study allows for the optimization of channel capacity over a variety of turbulence conditions and for the selection of maximum code rates at a given probability of outage.

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