Channel Capacity of WDM Systems Using Constant-Intensity Modulation Formats

Keang-Po Ho and Joseph M. Kahn

Abstract— **By eliminating self- and cross-phase modulation, constant-intensity modulation techniques can improve the spectral efficiency of WDM systems. In the linear regime, the spectral efficiency with constant-intensity modulation is found to be about 1.10 bit/s/Hz more than half the Shannon limit. In WDM systems limited by cross-phase modulation, constant-intensity modulation allows the launching of higher optical power and yields increased spectral efficiency.**

I. Introduction

Recently, Mitra and Stark [1] calculated the maximum spectral efficiency of dense wavelength-division-multiplexed (WDM) systems that are limited by both optical amplifier noise and fiber nonlinearities. Mitra and Stark argued that the capacity of WDM systems is limited most fundamentally by cross-phase modulation (XPM), in which the intensity of each signal perturbs the fiber refractive index, thereby modulating the phase of all the other signals. In addition, as the signals propagate, fiber dispersion converts XPM-induced phase modulation to intensity noise. With constant-intensity modulation, such as phase or frequency modulation [2]-[4] (or, to a certain degree, polarization modulation [5]), both self-phase modulation (SPM) and XPM cause only non-time-variant phase shifts, eliminating both phase and intensity distortion. Under the assumptions in [1], increasing launched power leads to a monotonic increase in spectral efficiency, leading to a higher spectral efficiency than that limited by XPM. However, laser intensity fluctuations or imperfect phase/frequency modulation cause intensity noise [3] and fiber dispersion converts phase modulation to amplitude variation [4]. In this paper, we calculate the maximum spectral efficiency that can be achieved using constant-intensity modulation techniques, assuming the use of low-noise lasers and nearly ideal phase/frequency modulators, and assuming careful control of fiber dispersion.

The work of K.-P. Ho was supported in part by an Earmarked Grant from the Research Grant Council of Hong Kong Special Administrative Region, under project number CUHK 4225/00E. The work of J. M. Kahn was supported in part by the DARPA STAB Program under contract DAAH01-00-C-0089.

K.-P. Ho is with StrataLight Communications, Campbell, CA 95008, on leave from Department of Information Engineering, The Chinese University of Hong Kong, Shatin, NT, Hong Kong (e-mail: kpho@stratalight.com).

J. M. Kahn is with StataLight Communications, Campbell, CA 95008 and Department of Electrical Engineering and Computer Sciences, University of California, Berkeley, CA 94720 (e-mail: jmk@stratalight.com).

II. Spectral Efficiency of Constant-Intensity Signals

Calculation of the maximum spectral efficiency can be reduced, using the sampling theorem, to the problem of maximizing the mutual information of a discrete-time continuous-amplitude channel [6]:

$$C = \max_{P_X(x)} E\left\{\log_2 \frac{P_{Y|X}(y|x)}{P_Y(y)}\right\}. \tag{1}$$

In (1), $P_X(x)$ and $P_Y(y)$ are the probability density functions (p.d.f.'s) of the channel input X and output Y, $P_{Y|X}(y|x)$ is the conditional p.d.f. and $E\{.\}$ denotes expectation. With additive Gaussian noise, the maximum spectral efficiency is given by the well-known Shannon limit of [6]:

$$C = \log_2(1 + \mathrm{SNR}), \tag{2}$$

where SNR is the signal-to-noise ratio of the channel.

With the constraint of constant-intensity modulation, X is a complex-valued representation of electric field, and assumes values along a circle of radius A, as shown in Fig. 1a. The optical amplifier noise is additive Gaussian noise. In the linear regime, $Y = X + N$, as shown in Fig. 1b, where N is a two-dimensional Gaussian variable with variance σ_n^2, i.e.,

$$P_{Y|X}(y|x) = P_N(y-x) = \frac{1}{2\pi\sigma_n^2}\exp\left(-\frac{(y_1-x_1)^2+(y_2-x_2)^2}{2\sigma_n^2}\right)$$

and

$$E\{\log_2 P_{Y|X}(y|x)\} = -\log_2 2\pi e\sigma_n^2. \tag{3}$$

The conditional entropy (3) is independent of the input p.d.f. $P_X(x)$. The mutual information (1) is maximized when X is uniformly distributed along the circle, in which case the output p.d.f. $P_Y(y)$ is

$$P_Y(y_1, y_2) = \frac{1}{2\pi}\int_{-\pi}^{+\pi} P_N(y_1 - A\cos\theta, y_2 - A\sin\theta)\,d\theta \tag{4}$$

or

$$P_Y(y_1, y_2) = f(r) = \frac{1}{2\pi\sigma_n^2}\exp\left(-\frac{A^2+r^2}{2\sigma_n^2}\right) I_0\left(\frac{Ar}{\sigma_n^2}\right), \quad r = \left(y_1^2 + y_2^2\right)^{1/2} \tag{5}$$

where $I_0(.)$ is the zero-order modified Bessel function of the first kind. The output entropy is

$$E\{\log_2 P_Y(y)\} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} P_Y(y_1, y_2)\log_2 P_Y(y_1, y_2)\,dy_1 dy_2 \tag{6}$$

or

Presented at Optical Fiber Communications Conference, Anaheim, CA, March 17-22, 2002

$$E\{\log_2 P_Y(y)\} = 2\pi \int_0^{+\infty} r f(r) \log_2 f(r)\, dr. \tag{7}$$

The spectral efficiency under the constant-intensity constraint is

$$C = -2\pi \int_0^{+\infty} r f(r) \log_2 f(r)\, dr - \log_2 2\pi e \sigma_n^2. \tag{8}$$

The spectral efficiency given by (8) can be evaluated numerically for all values of optical SNR. At high SNR, an approximation to (8) can be derived using the asymptotic expression $I_0(u) \sim e^u / \sqrt{2\pi u}$. After some algebra, (5) is simplified to

$$f(r) \sim \frac{1}{(2\pi)^{3/2} A \sigma_n} \exp\left(-\frac{(r-A)^2}{2\sigma_n^2}\right). \tag{9}$$

Aside from a constant multiplicative factor, the asymptotic expression (9) is a Gaussian p.d.f. with mean A and variance σ_n^2. Substituting (9) into (7), we obtain

$$E\{\log_2 P_Y(y)\} \sim -\tfrac{1}{2} \log_2 \left[(2\pi)^3 e A^2 \sigma_n^2\right].$$

Combined with (1) and (3), we obtain the asymptotic spectral efficiency under a constant-intensity constraint:

$$\begin{aligned} C &\sim \log_2 \frac{A}{\sigma_n} + \frac{1}{2} \log_2 \frac{2\pi}{e}, \\ &= \tfrac{1}{2} \log_2 \mathrm{SNR} + 1.10 \end{aligned} \tag{10}$$

where SNR = $A^2/2\sigma_n^2$. The asymptotic spectral efficiency of (10) is 1.10 bit/s/Hz higher than half the Shannon limit (2). In the absence of a constant-intensity constraint, using direct detection, the asymptotic spectral efficiency [7] is

$$\tfrac{1}{2} \log_2 \mathrm{SNR} - 1.00. \tag{11}$$

The asymptotic spectral efficiency of (10) is 2.10 bit/s/Hz larger than that of (11).

III. Numerical Results and Discussion

Fig. 2 plots the spectral efficiency as a function of optical SNR, including the Shannon limit (2), numerical evaluation of (8), and the asymptotic expressions (10) and (11). Fig. 2 indicates that (10) agrees well with (8) over a wide range of SNR values.

The spectral efficiencies shown in Fig. 2 are valid in the regime in which fiber nonlinearities do not degrade system performance. For modulation techniques with constant intensity, neither SPM nor XPM cause signal distortion in the fiber. Under the assumptions made in [1], the spectral efficiencies shown in Fig. 2 are applicable. If there are guard bands between adjacent channels, the spectral efficiency is scaled by the channel utilization ratio.

The spectral efficiency of constant-intensity modulation is limited by four-wave-mixing (FWM). Assuming equally spaced channels, the FWM component generated by channels p, q, and r falls in channel $n = p + q - r$. For an N-channel WDM system with $-(N-1)/2 \le p, q, r, n \le (N-1)/2$, the middle channel ($n$ = 0) has the largest total FWM power [3][8] per fiber span, given by

$$\sigma_{n,\mathrm{FWM}}^2 = \sum_{\substack{p,q,p\neq 0,q\neq 0 \\ |p+q|\le (N-1)/2}} \frac{\gamma^2 P_p P_q P_{p+q} (D_{pq}/3)^2}{\alpha^2 + \Delta k_{pq}^2}, \tag{12}$$

where γ is the nonlinear coefficient, P_q, P_q, and P_{p+q} are optical powers at channels p, q, and $p+q$, respectively, $D_{pq}/3$ = 1 if $p = q$ and $D_{pq}/3$ = 2 if $p \neq q$, and $\Delta k_{pq} = 2\pi \lambda^2 D \Delta f^2 qp / c$, where D is the dispersion coefficient, λ is the optical wavelength, c is the speed of light, and Δf the channel spacing. In (12), each fiber span is assumed to be much longer than the effective nonlinear distance and all FWM components are combined incoherently by ignoring the phase dependence between FWM components [8].

We model FWM as additive Gaussian noise occupying the same bandwidth as the signal. Fig. 3 shows the spectral efficiency as a function of input power density, considering the Shannon limit (2), the expression (8) for constant-intensity modulation, and the results of Mitra and Stark [1]. In the absence of FWM, spectral efficiency increases monotonically with power density for the Shannon limit (2) and for constant-intensity modulation (8), but the spectral efficiency computed following [1] reaches a maximum value limited by XPM. When FWM is considered, spectral efficiencies for the Shannon limit (2) and for constant-intensity modulation reach maximum values limited by FWM, while the spectral efficiency computed following [1] remains unchanged. In the presence of FWM, the maximum spectral efficiency of constant-intensity modulation is about 2.8 bit/s/Hz, compared with 2.3 bit/s/Hz computed following [1].

The system of Fig. 3 has 101 channels and n_s = 10 fiber spans; uses optical fiber having attenuation coefficient α = 0.2 dB/km, nonlinear coefficient of $\gamma = 1.24$ /W/km, and dispersion coefficient D = 17 ps/km/nm; operates around the wavelength of λ = 1.55 μm with channel bandwidth B = 40 GHz, and channel separation Δf = 1.5B; uses optical amplifiers with noise figure of 4 dB and gain of 30 dB. Using an overall effective length of $L_{\mathrm{eff}} = n_s/\alpha$, the nonlinear intensity scale of [1] is I_0 = 11.2 mW. In Fig. 3, we assume that the FWM components from individual fiber spans combine incoherently, giving a total FWM noise power of $n_s \sigma_{n,\mathrm{FWM}}^2$.

Fig. 3 shows that constant-intensity techniques can provide higher spectral efficiencies than those of [1] in the regime in which XPM dominates over FWM. Because FWM decreases more rapidly than XPM as channel spacing is increased, the improvement obtained using constant-intensity modulation is more significant for systems having large channel spacing. To the limit of very small channel spacing, the maximum spectral efficiency following [1] may be much higher than that for constant-intensity modulation. However, the impact of FWM becomes comparable to XPM, contrary to the assumptions of [1]. Both FWM and XPM decrease with an increase of fiber dispersion. FWM dominates over XPM for zero-dispersion optical fiber. In practical systems using non-zero-dispersion fiber, dispersion tends to convert phase/frequency modulation to intensity modulation [2][4]. Fiber dispersion must be compensated carefully to maintain constant intensity.

As in [1], we implicitly assume that coherent detection is used. In principle, incoherent (or direct) detection can be used for some constant-intensity techniques. However, incoherent detection usually yields spectral efficiency inferior to that of coherent detection.

In the linear regime, the spectral efficiencies of Fig. 2 can be doubled by launching independent signals in two orthogonal polarizations and using polarization-resolved detection. In the presence of polarization-mode dispersion, two signals can propagate independently if launched into the two time-varying principal states of polarization (PSP). The transmitter must be able to adaptively lock to the PSP. Launching adjacent channels with orthogonal polarizations is also a well-known technique to reduce fiber nonlinearities.

Polarization modulation [5] also has constant intensity, and can reduce SPM and XPM. Due to differences between the nonlinear coefficients of signals with parallel and orthogonal polarizations, polarization modulation is subject to some residual time-varying XPM.

Fiber nonlinearities are all deterministic effects and can, in principle, be compensated. Delayed time-reversal using phase conjugation can undo all fiber nonlinearities in principle [9]. Because fiber loss must be compensated precisely using distributed gain [10], complete compensation of nonlinearities remains far from practical.

IV. Conclusions

Constant-intensity modulation techniques eliminate the effects of both SPM and XPM, potentially increasing the limits to spectral efficiency in WDM systems. In the linear regime at high SNR, the spectral efficiency of constant-intensity modulation is 1.1 bit/s/Hz larger than half the Shannon limit. In practice, the improvement yielded by constant-intensity modulation techniques over unconstrained modulation techniques depends on the relative strengths of XPM and FWM. A representative 10-span WDM system shows an improvement from 2.3 to 2.8 bit/s/Hz.

References

[1] P. P. Mitra and J. B. Stark, "Nonlinear limits to the information capacity of optical fibre communications," *Nature,* vol. 411, pp. 1027-1030, June 28, 2001.

[2] J. M. Kahn and K.-P. Ho, "A bottleneck for optical fibres," *Nature,* vol. 411, pp. 1007-1010, June 28, 2001.

[3] A. R. Chraplyvy, "Limitation on lightwave communications imposed by optical-fiber nonlinearities," *J. Lightwave Technol.,* vol. 8, pp. 1548-1557, 1990.

[4] S. Norimatsu and K. Iwashita, "The influence of cross-phase modulation on optical FDM PSK homodyne transmission systems," *J. Lightwave Technol.,* vol. 11, pp. 795-804, 1993.

[5] S. Betti, F. Curti, G. de Marchis, and E. Iannone, "Multilevel coherent optical system based on Stokes parameters modulation," *J. Lightwave Technol.,*vol. 8, pp. 1127-1136, 1990.

[6] C. E. Shannon, "A mathematical theory of communications," *Bell Syst. Tech. J.* vol. 27, pp. 379-423, pp. 623-656, 1948.

[7] A. Mecozzi and M. Shtaif, "On the capacity of intensity modulated systems using optical amplifiers," *IEEE Photon. Technol. Lett.,* vol. 13, pp. 1029-1031, 2001.

[8] M. Eiselt, "Limits on WDM systems due to four-wave mixing: A statistical approach," *J. Lightwave Technol.,* vol. 17, pp. 2261-2267, 1999.

[9] D. M. Pepper and A. Yariv, "Compensation for phase distortions in nonlinear media by phase conjugation," *Opt. Lett.,* vol. 5, pp. 59-60, 1980.

[10] I. Brener, *et al.,* "Cancellation of all Kerr nonlinearities in long fiber spans using a $LiNbO_3$ phase conjugator and Raman amplification," in *Opt. Fiber Comm. Conf.,* vol. 4, pp. 266-268, 2000.

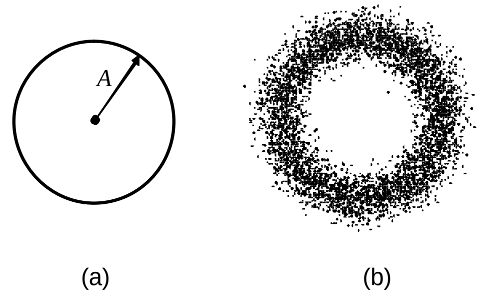

Fig. 1. Constant-intensity modulation with additive Gaussian noise: (a) input *X* and (b) output *Y*.

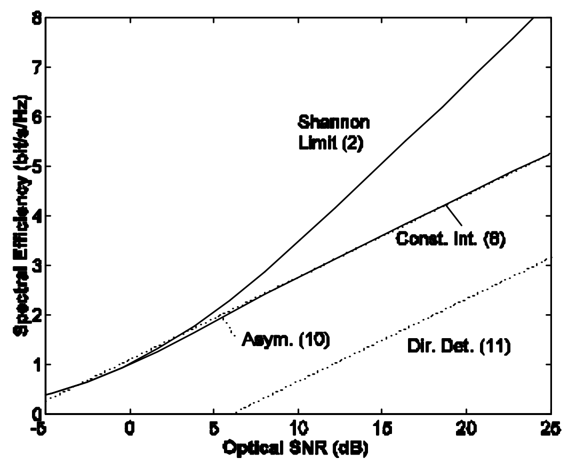

Fig. 2. Spectral efficiency vs. optical SNR in linear regime.

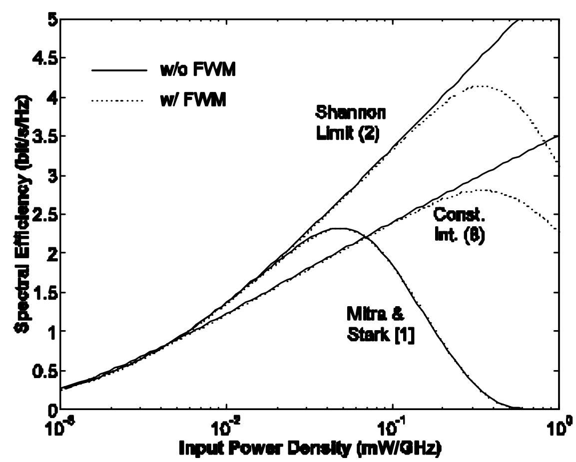

Fig. 3. Spectral efficiency vs. input power density for a 10-span WDM system.