Modal Multiplexing and Atmospheric Turbulence Mitigation in Free-Space Optical Communications

Joseph M. Kahn[1] and Aniceto Belmonte[2]

1. E. L. Ginzton Laboratory, Department of Electrical Engineering
Stanford University, USA

2. Department of Signal Theory and Communications
Technical University of Catalonia, Spain

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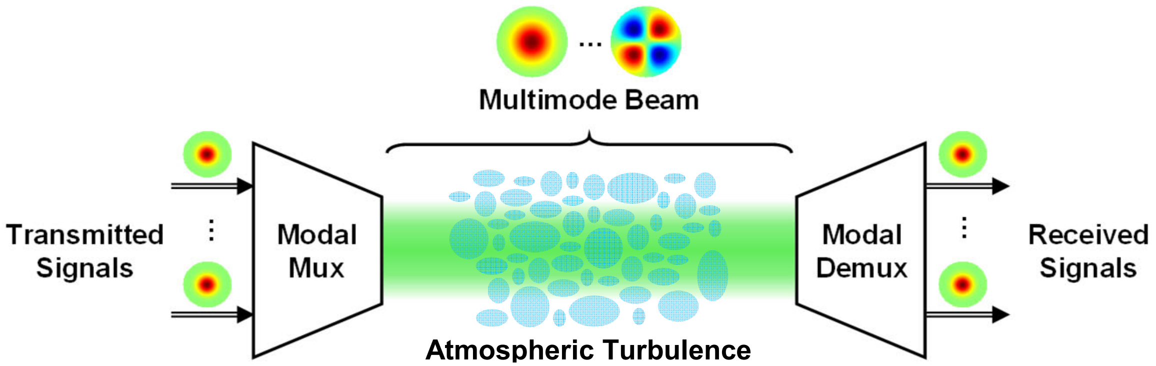
Modal Multiplexing in Near-Field Free-Space Optical Links



Near-field links

- Beam size in receiver plane is smaller than receiver aperture, preserving orthogonality between modes.
- Can multiplex signals in orthogonal spatial (and polarization) modes to increase link capacity and/or reliability.

Questions we address

- How can we realize the modal mux/demux and the necessary MIMO signal processing?
- In the absence of atmospheric turbulence, what are the best spatial modes to use?
  What capacity and degrees of freedom do they achieve?
- In the presence of atmospheric turbulence, what are the best spatial modes to use?
  What capacity and degrees of freedom do they achieve?

Outline

Modal multiplexing technologies

- Modal multiplexers and demultiplexers
- Multi-input multi-output signal processing

Modal multiplexing in the absence of turbulence[1]

- Space-bandwidth constraints
- Physical comparison of mode sets
- Information-theoretic comparison of mode sets

Modal multiplexing in the presence of turbulence[2]

- Atmospheric turbulence and its impact
- Optimal mode set for turbulence channels
- Information-theoretic comparison of mode sets

1. N. Zhao, X. Li, G. Li and J. M. Kahn, “Capacity Limits of Spatially Multiplexed Free-Space Communication”, *Nature Photonics* **9** (2015).
2. A. Belmonte and J. M. Kahn, “Optimal Modes for Spatially Multiplexed Free-Space Communication in Atmospheric Turbulence”, *Optics Express* **29** (2021).

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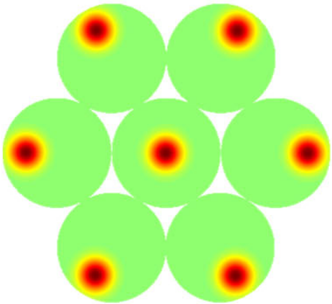
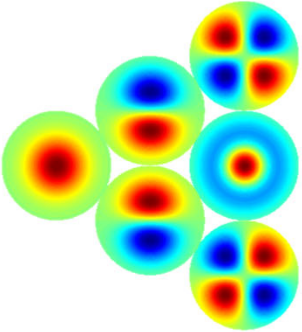
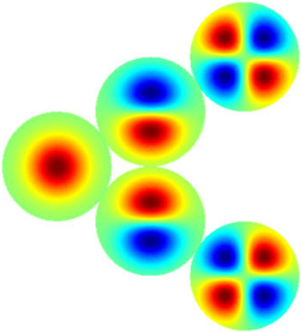
Mode Sets for Modal Multiplexing



Parallel
Gaussian
Beams



Laguerre-
Gauss
Modes



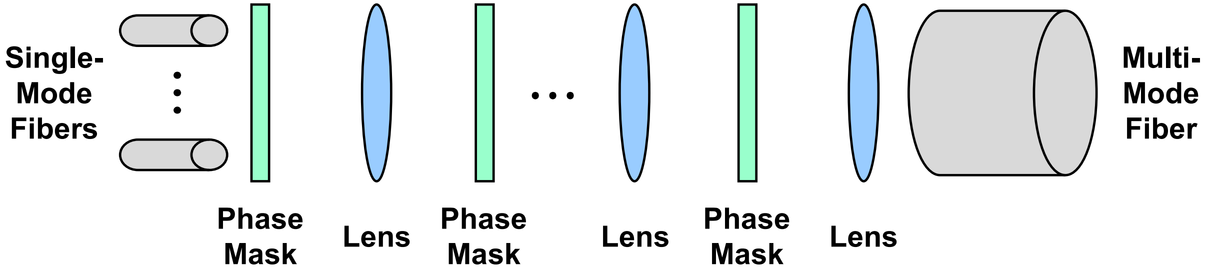
Orbital Angular
Momentum
Modes

Modal Multiplexers and Demultiplexers

Device	Loss	Crosstalk
Waveplates + beamsplitters	Mux + demux scales as N^2	Low inter- and intra-group
Multi-plane light converter	Fundamentally lossless	Low inter- and intra-group
Photonic lantern	Fundamentally lossless	High inter- and intra-group
Mode-selective photonic lantern	Fundamentally lossless	Low inter-group, but strong intra-group
Fiber array	Fundamentally lossless	Low for parallel Gaussian beams

- *Low crosstalk*: one-to-one mapping between inputs/outputs and modes (requires proper alignment and orientation).
- All these devices are *reciprocal*: a demultiplexer is a multiplexer operated in reverse.

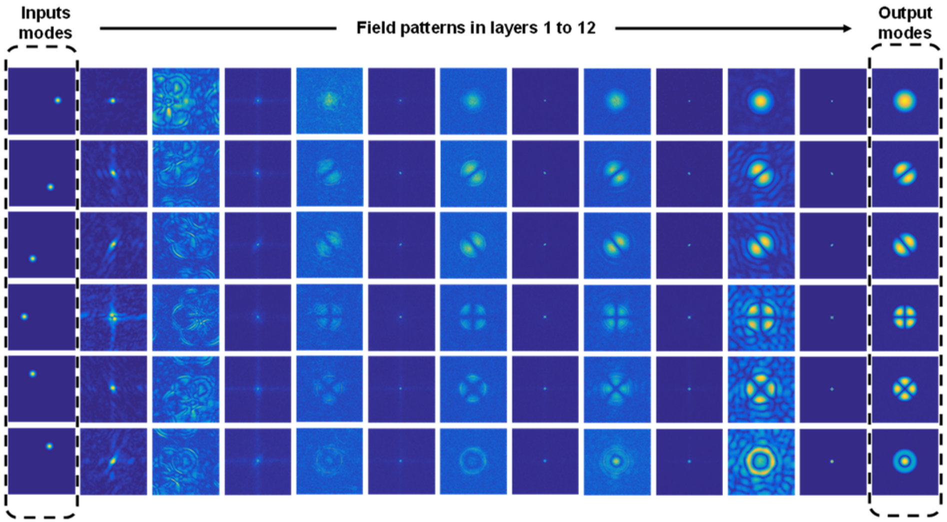
Multi-Plane Light Converter (1/2)

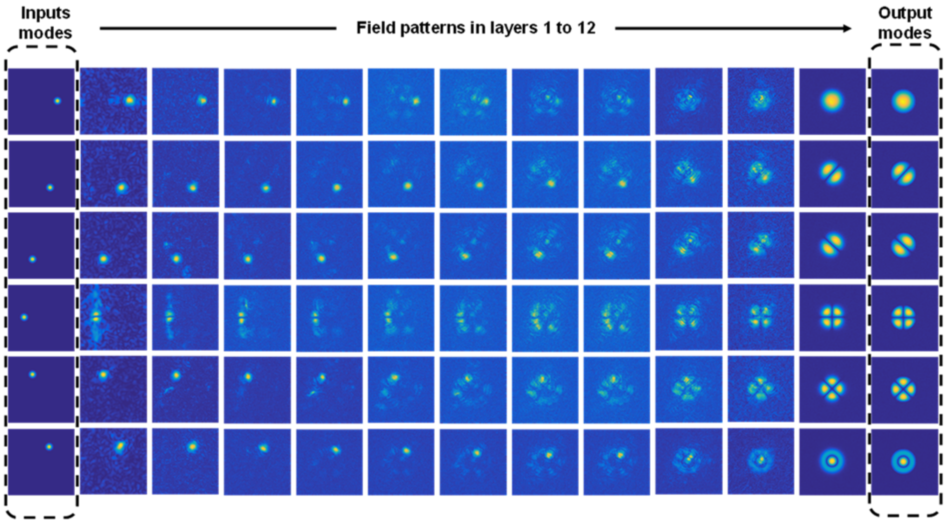


- Based on field pattern transformation by successive phase masks.
- Can be designed for L-G, H-G, OAM or any mode set.
  Can yield low crosstalk.
- Can be realized in folded geometry, replacing lenses by curved mirrors.

G. Labroille *et al*., *Optics Express* **22** (2014).

Multi-Plane Light Converter (2/2)





Diffractive design

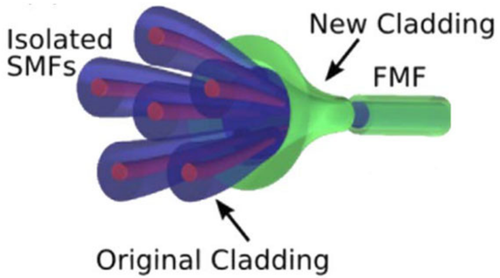
- Field patterns alternate between real space and Fourier space.

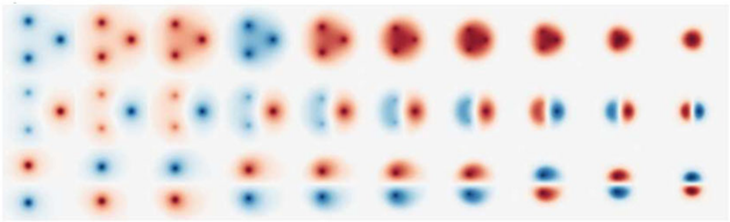
Adiabatic design

- Field patterns evolve continuously.

K. Choutagunta, unpublished (2017).

Photonic Lantern

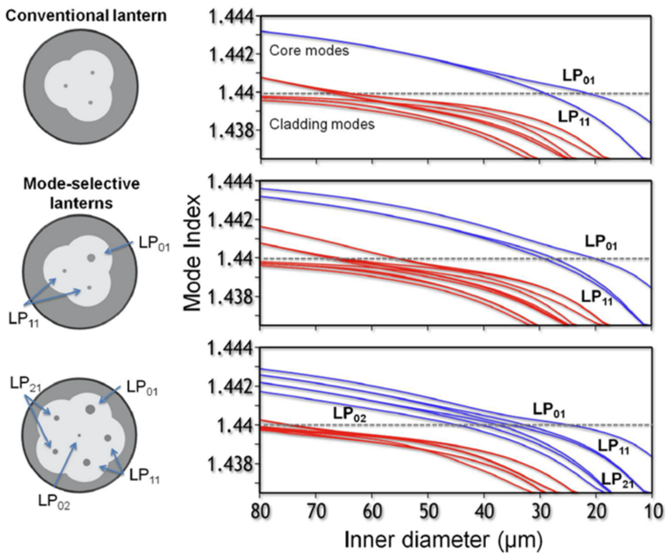




- Based on adiabatic mode conversion.
- Can be designed for L-G or H-G modes.
  Yields inter- and intra-group crosstalk, necessitating MIMO equalization.

S. G. Leon-Saval *et al.*, *Optics Letters* **30** (2005).
N. K. Fontaine *et al.*, *Optics Express* **20** (2012).

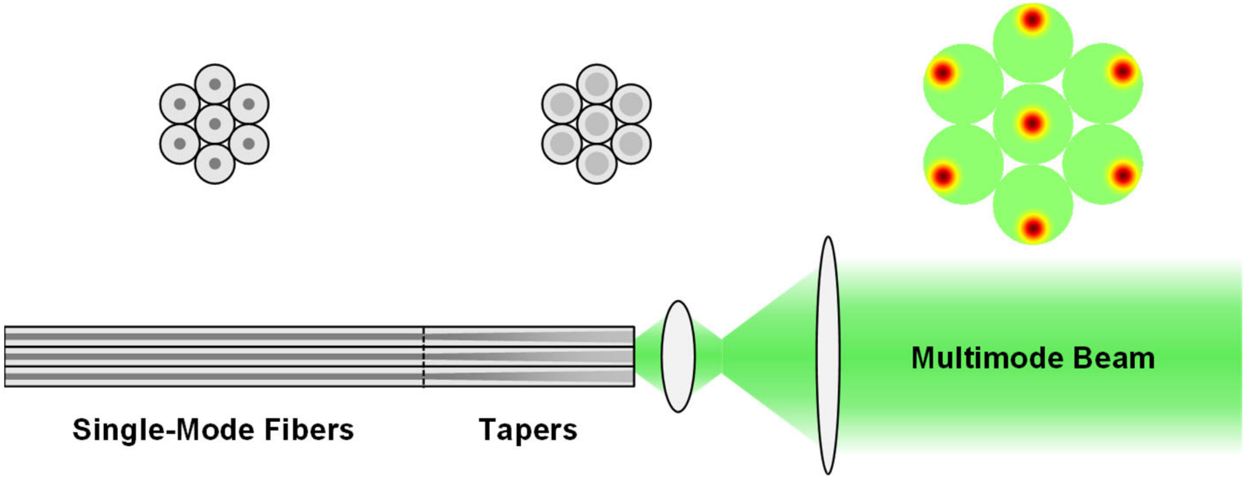
Mode-Selective Photonic Lantern



- Based on adiabatic mode conversion guided by propagation constant matching.
- Can be designed for L-G or H-G modes. Yields low inter-group crosstalk.
  Yields intra-group crosstalk, necessitating intra-group MIMO equalization.

S. G. Leon-Saval *et al.*, *Optics Express* **22** (2014).

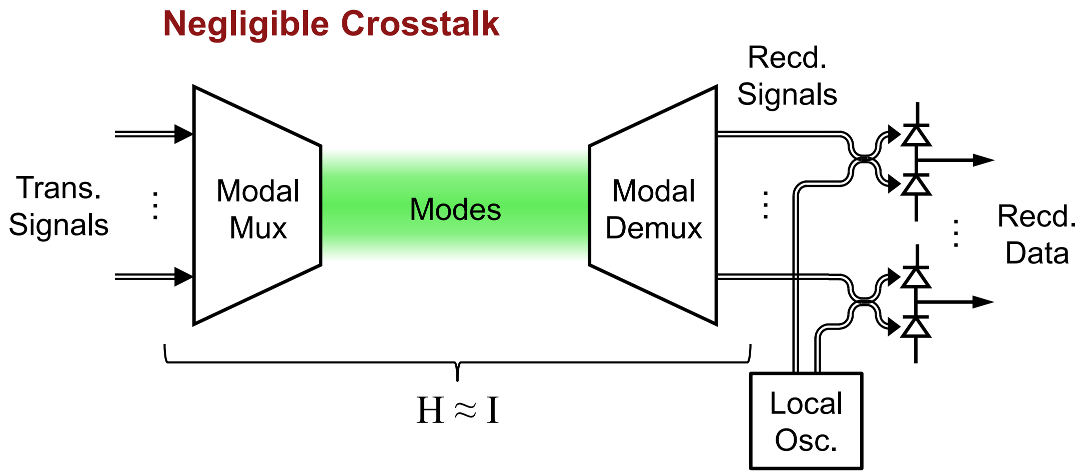
Fiber Array



- Based on imaging and mode-size conversion.
- Can yield low crosstalk for parallel Gaussian beams.
- Can demultiplex other mode sets, but resulting crosstalk necessitates MIMO equalization.
  A similar demultiplexing strategy is considered in analysis below.

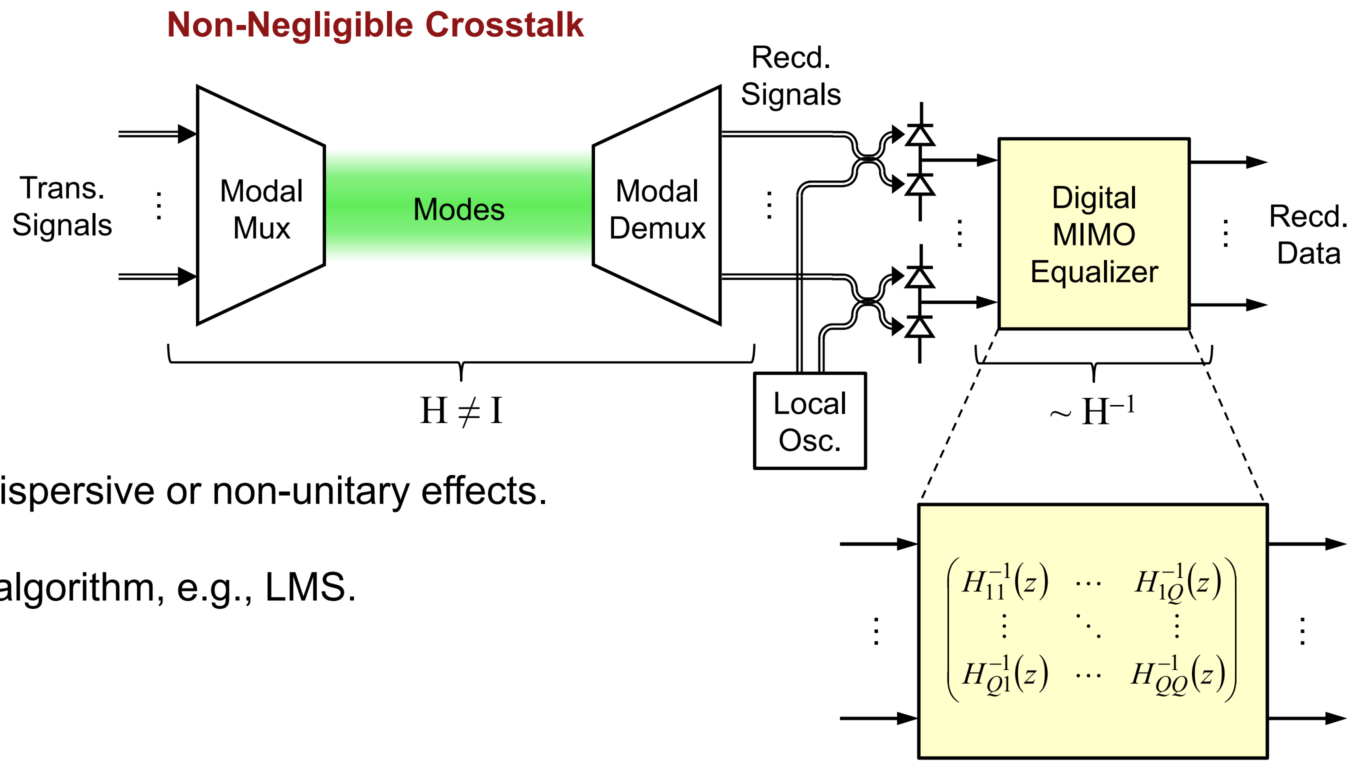
Coherent Detection Links

- Superior spectral efficiency and receiver sensitivity.



Coherent Detection Links

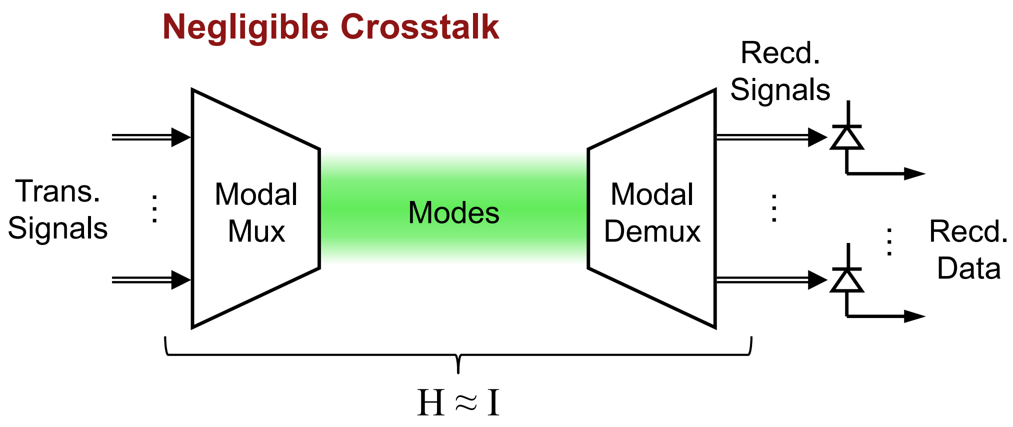
- Superior spectral efficiency and receiver sensitivity.



- Easy to compensate dispersive or non-unitary effects.
- Adapt using standard algorithm, e.g., LMS.

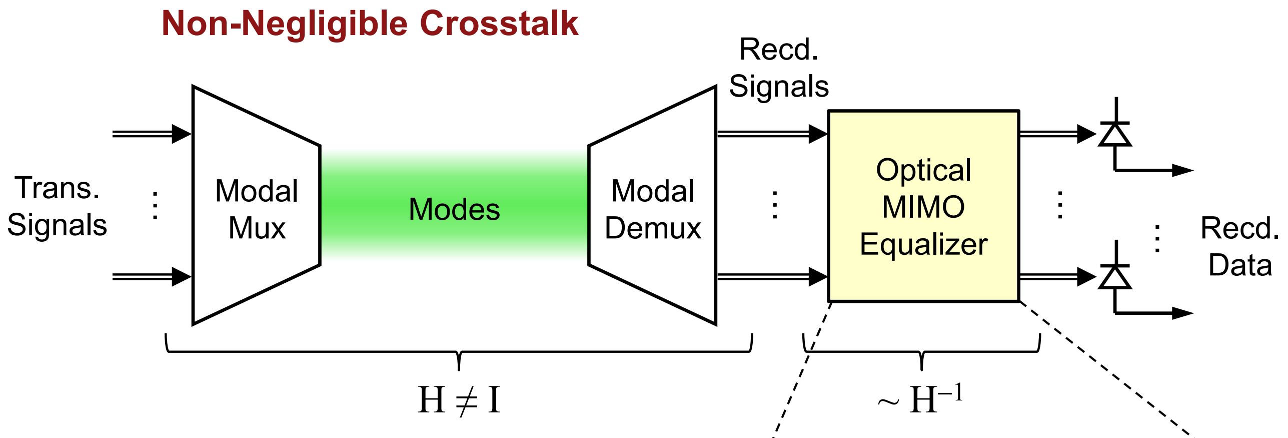
Direct Detection Links

- Inferior spectral efficiency and receiver sensitivity.

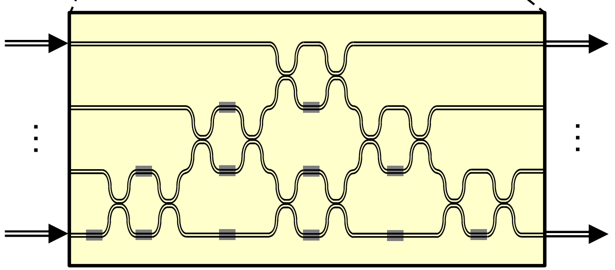


Direct Detection Links

- Inferior spectral efficiency and receiver sensitivity.



- More difficult to compensate dispersive or non-unitary effects.
- Adapt using non-standard algorithm.



(equalizer for unitary **H** shown)

D. A. B. Miller, *Photonics Research* **1** (2013).
K. Choutagunta *et al.*, *J. Lightw. Technol.* **38** (2020).

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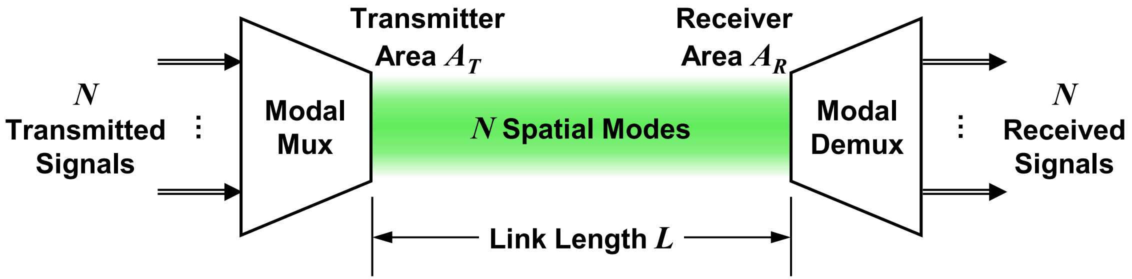
Space-Bandwidth Constraints



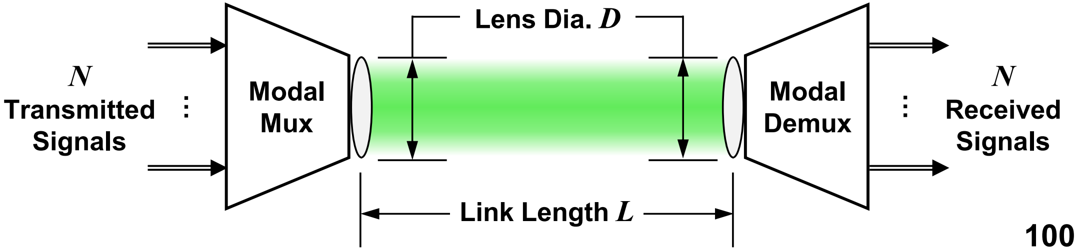
- Any free-space link has constraints on transverse spatial dimensions and spatial frequency bandwidth.
- Wish to multiplex N data streams in N orthogonal spatial modes (can use 2 polarizations to multiplex $2N$).
- Maximum number of strongly coupled spatial modes is approximated by product of transmitter and receiver Fresnel numbers:

$$N \leq N_F = \frac{A_T A_R}{\lambda^2 L^2}$$

- If $N_F \leq 1$, spatial mode multiplexing is not possible.

J. H. Shapiro *et al.*, *J. Optical Networking* **4** (2005).
D. A. B. Miller, *J. Lightwave Technology* **35** (2017).

Symmetric Two-Lens Link

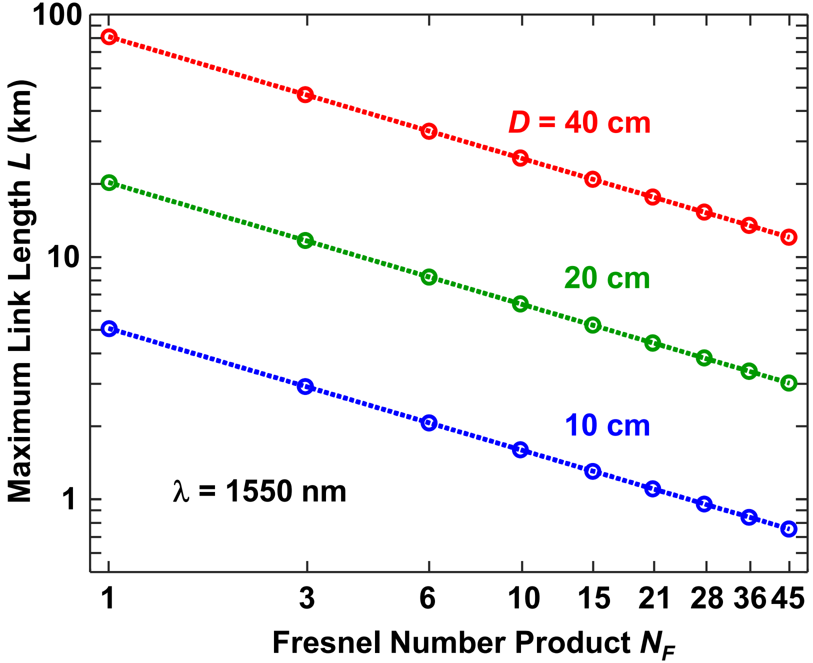


- Maximum number of strongly coupled spatial modes:

$$N \le N_F = \frac{\pi^2 D^4}{16\lambda^2 L^2}$$

- Maximum link length supporting N_F spatial modes:

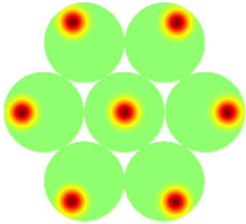
$$L \le \frac{\pi D^2}{4\lambda\sqrt{N_F}}$$



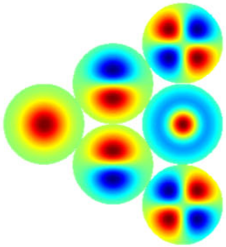
Heuristic Comparison of Mode Sets

- The following mode sets have similar space-bandwidth products.
  For the L-G or OAM modes, this corresponds to the number of mode groups M.

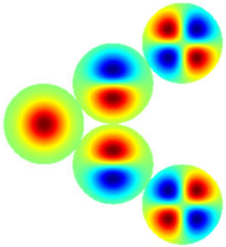
M = 3 Mode Groups



Gaussian Beams
$N = 7$



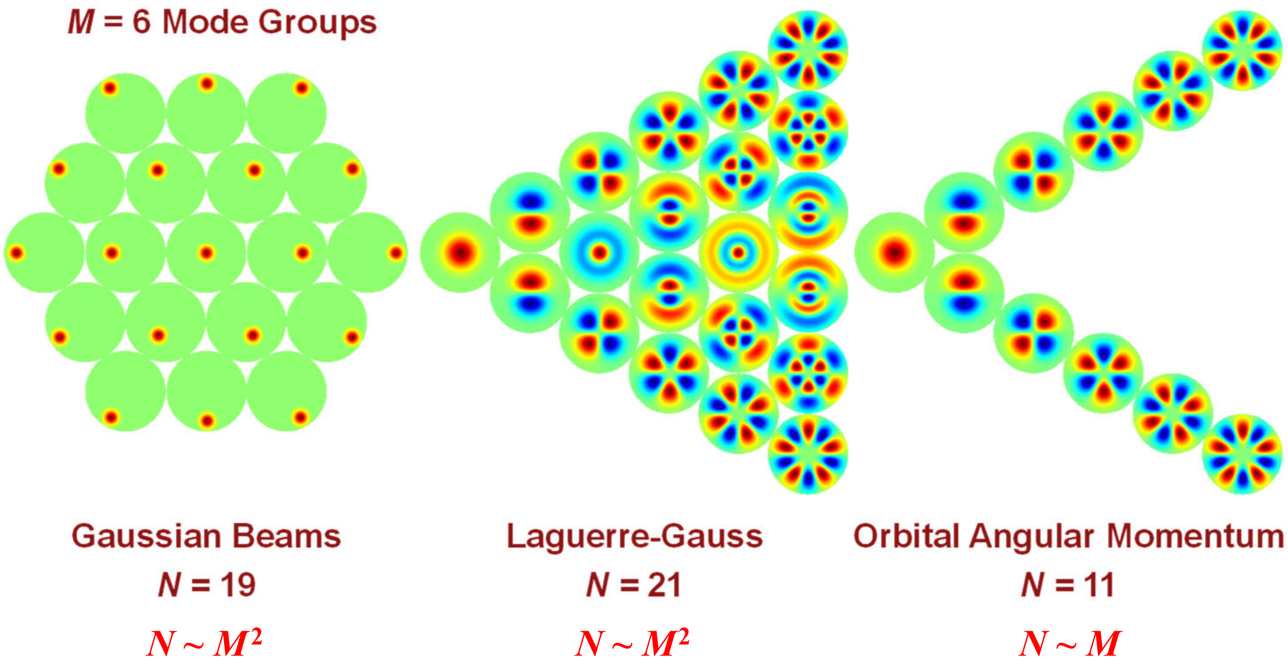
Laguerre-Gauss
$N = 6$



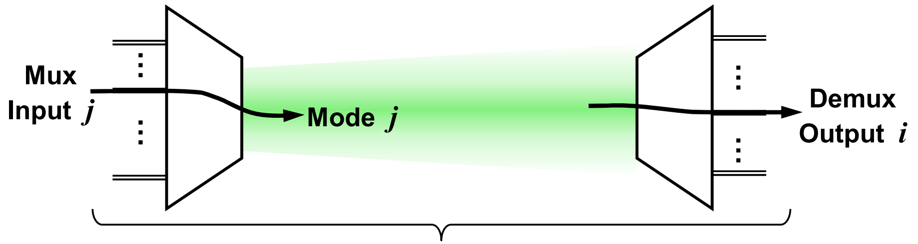
Orbital Angular Momentum
$N = 5$

Heuristic Comparison of Mode Sets

- The following mode sets have similar space-bandwidth products.
  For the L-G or OAM modes, this corresponds to the number of mode groups M.



Rigorous Comparison of Mode Sets (1/3)



Transmission Matrix

- $\mathbf{H}_{ij}$ is transmission coefficient between mode j and output i, including diffraction and crosstalk.
- $\mathbf{H}$ includes modes far beyond nominal cutoff determined by $N_F = A_T A_R / \lambda^2 L^2$.

L-G, OAM and Gaussian Modes

- Consider pixel-based demultiplexer similar to that for Gaussian beams (pessimistic for L-G modes).
- Diffraction loss + crosstalk → $\mathbf{H}$ is non-diagonal and non-unitary.

OAM Modes Only

- Also consider ideal OAM demultiplexer (optimistic for OAM modes).
- Diffraction loss → $\mathbf{H}$ is diagonal and non-unitary.

Rigorous Comparison of Mode Sets (2/3)

- Perform a singular value decomposition of the transmission matrix:

$$\mathbf{H} = \mathbf{V}\,\mathbf{D}\,\mathbf{U}^H$$

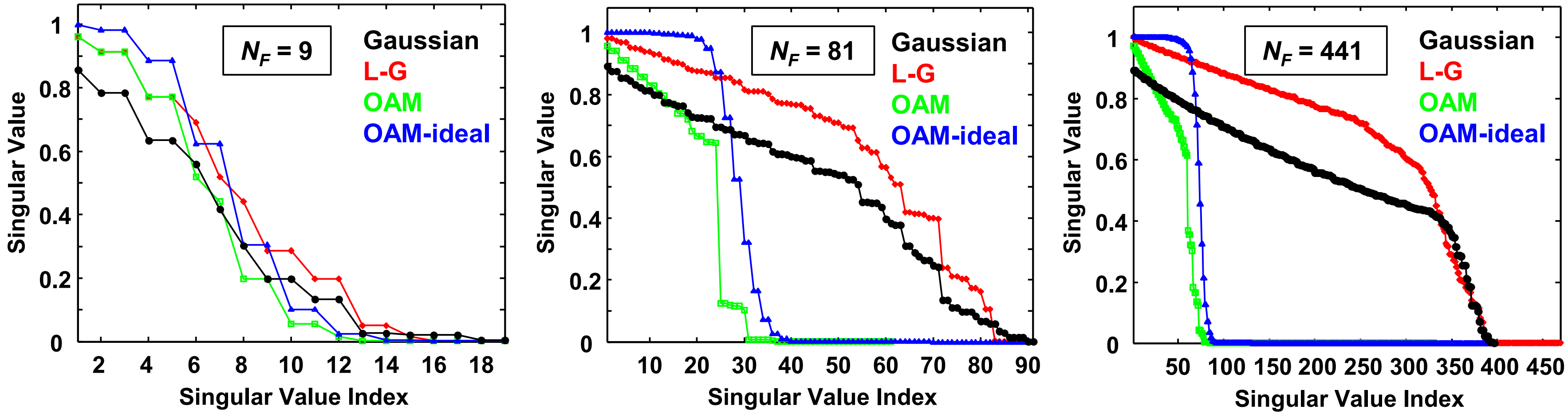
- $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices. Their columns are transmit and receive bases that diagonalize $\mathbf{H}$ into uncoupled spatial subchannels.

- $\mathbf{D}$ is a diagonal matrix of the singular values:

$$\mathbf{D} = \mathrm{diag}\left(\sqrt{\lambda_1},\ldots,\sqrt{\lambda_N}\right)$$

- $\{\lambda_1,\ldots,\lambda_N\}$ are eigenvalues of $\mathbf{H}\mathbf{H}^H$, representing power gains of spatial subchannels.

Rigorous Comparison of Mode Sets (3/3)



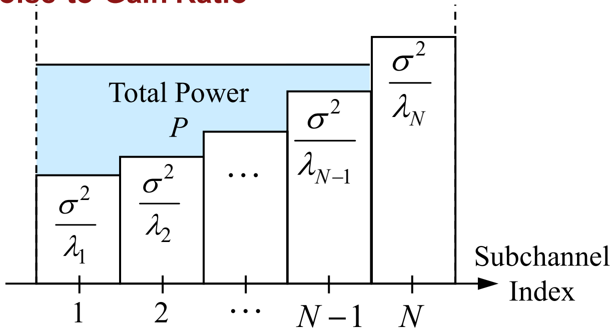
- For $N_F = 9$: all three mode sets yield a similar number of spatial subchannels.
- For $N_F > 9$: L-G modes and Gaussian beams yield $N_{LG} \approx N_{Gaussian} \approx N_F$.
  OAM modes yield $N_{OAM} << N_F$.
- The comparison was intentionally pessimistic for L-G modes.
  A more ideal demux would yield more equal singular values for L-G modes.

Achieving Channel Capacity (1/2)

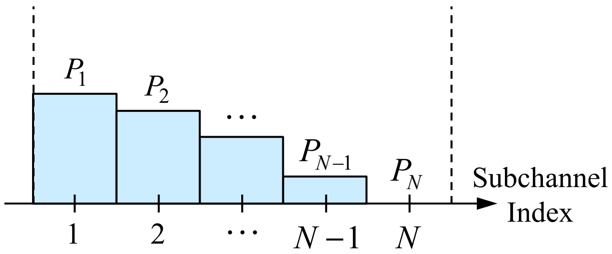
Assume:

- Coherent detection with equal noise power σ^2 per receiver.
- Constraint on total transmit power $P = \sum_{n=1}^{N} P_n$.
- Total SNR is $SNR = \frac{1}{\sigma^2} \sum_{n=1}^{N} P_n$.
- Receiver knows spatial decoding matrix $\mathbf{V}$.
- Transmitter knows spatial precoding matrix $\mathbf{U}$ and $\{\lambda_1,\ldots,\lambda_N\}$, can optimize power allocation $\{P_1,\ldots,P_N\}$.

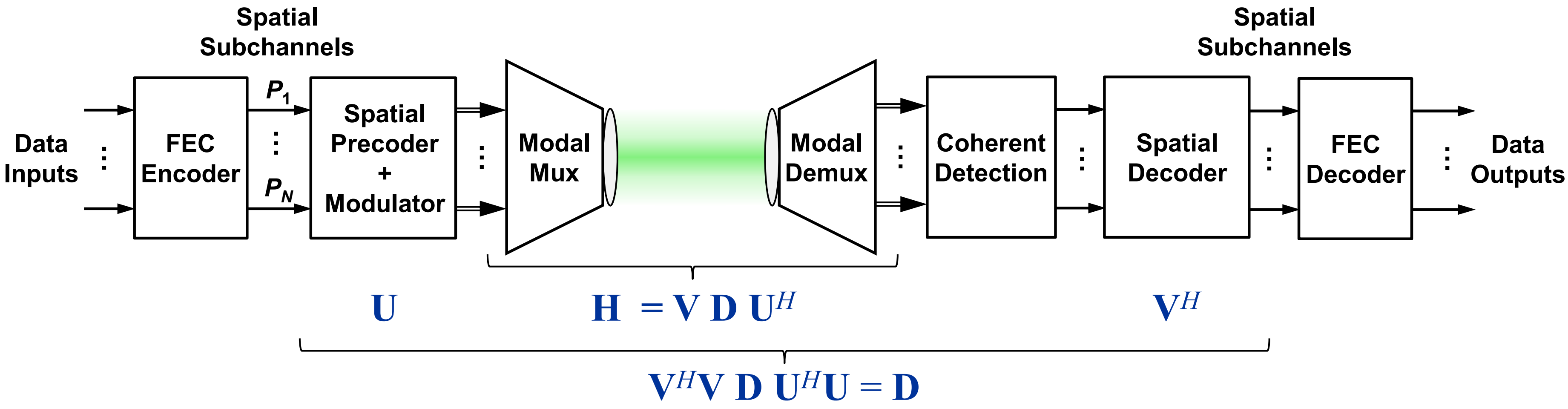
Noise-to-Gain Ratio



Optimal Power Allocation



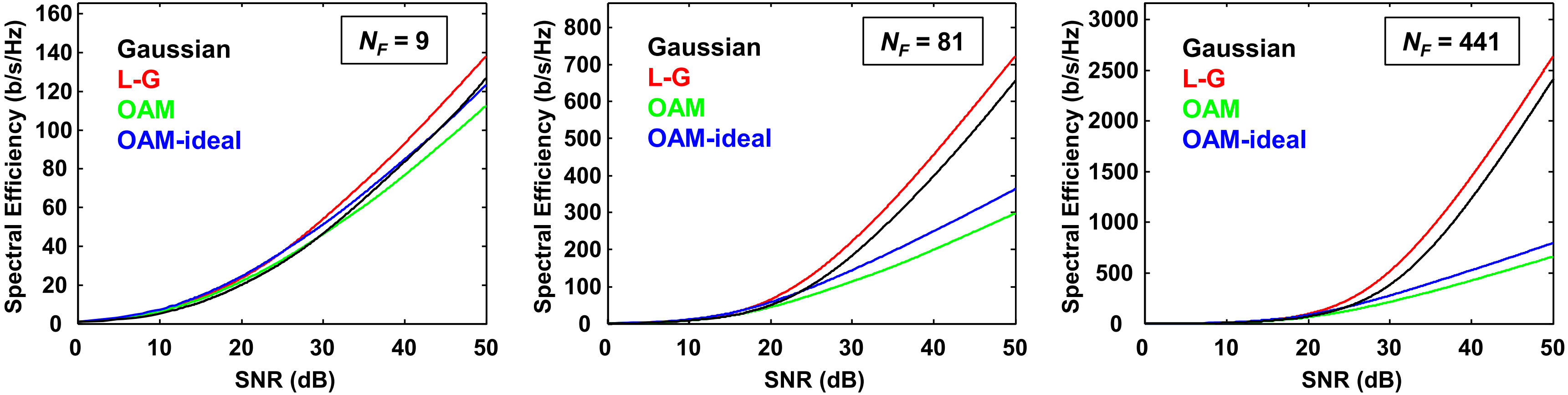
Achieving Channel Capacity (2/2)



- Require a mode-selective mux to enable optimal power allocation and spatial precoding.
- Obtain a set of N uncoupled parallel channels with power gains given by the λ_n.
- Capacity per unit bandwidth:

$$SE = \sum_{n=1}^{N} \log_2\left(1 + \frac{\lambda_n P_n}{\sigma^2}\right)$$

Channel Capacity

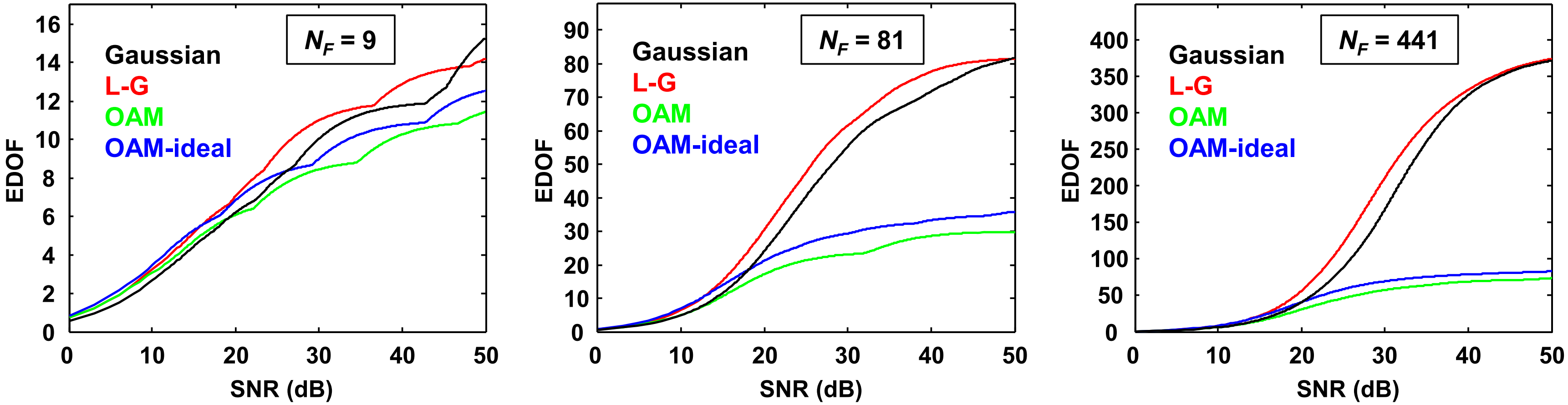


- Low SNR: power-limited, capacity depends weakly on N.
- High SNR: mode-limited, capacity depends strongly on N.

Effective Degrees of Freedom

- Number of spatial subchannels effectively conveying information:

$$EDOF = \frac{d}{d\delta} SE\left(2^{\delta} P\right)\Big|_{\delta=0}$$



- Low SNR: power-limited, EDOF depends weakly on N.
- High SNR: mode-limited, EDOF approaches N.

D.-S. Shiu *et al.*, *Trans. on Commun*. **48** (2000).

Discussion

- Laguerre-Gauss or parallel Gaussian beams offer higher capacity than the OAM subset of Laguerre-Gauss modes, except at small N_F.
- Hermite-Gauss modes (which have no OAM) offer the same capacity as Laguerre-Gauss modes. The two sets are related by a unitary transformation.
- Polarization multiplexing can double link capacity. In the low-NA/weak-guidance regime, LP modes are sufficient. Vector modes (or complex polarization) offer no benefit.
- In choosing a mode set for modal multiplexing, one should consider:
  - Completeness of the set.
  - Ease of implementation.
  - Performance in atmospheric turbulence.

  Whether or not the set includes OAM has no effect on capacity.

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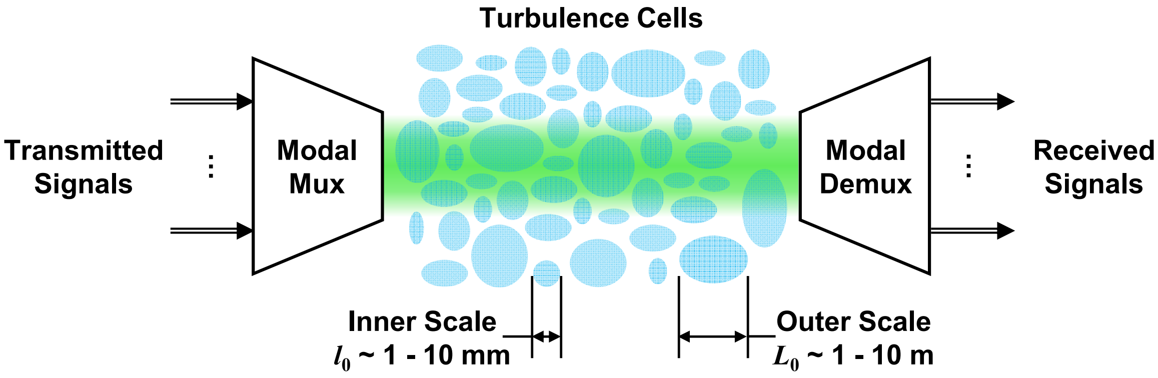
Atmospheric Turbulence and Its Impact



Major effects

- Beam wander.
- Beam spreading.
- Phase distortion: *most important effect in near-field links*.
- Amplitude scintillation: caused by interference over long paths.
- Wind- and thermal-induced motion of turbulence cells cause these effects to be *time-varying*.

Modeling Propagation in Atmospheric Turbulence

- Optical field in one polarization (suppressing z dependence):

$$U(\boldsymbol{\rho}) = U_0(\boldsymbol{\rho}) \quad \exp\left(\Phi(\boldsymbol{\rho})\right)$$

profile in absence of turbulence, e.g., Gaussian

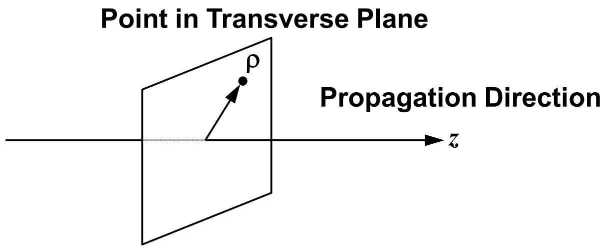
distortion by atmosphere

- Atmospheric distortion exponent:

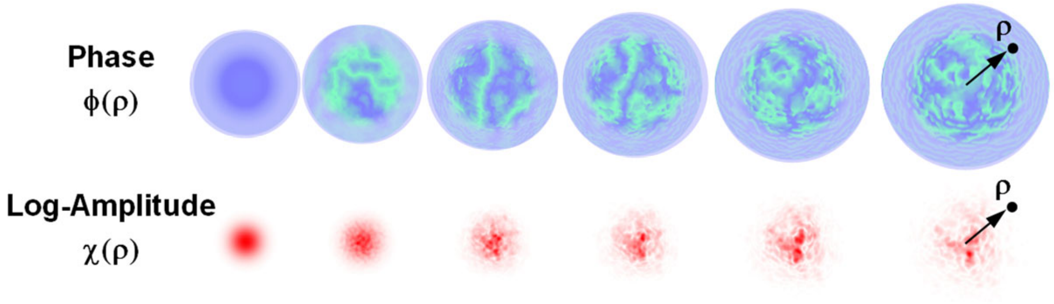
$$\Phi(\boldsymbol{\rho}) = \chi(\boldsymbol{\rho}) + j\phi(\boldsymbol{\rho})$$

log-amplitude fluctuations (scintillation)

phase fluctuations (aberrations)



- Evolution of $\phi(\rho)$ and $\chi(\rho)$ along z:



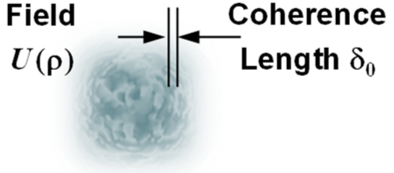
Optimal Modes in Atmospheric Turbulence (1/2)

Assumed scenario

- Receiver can track instantaneous state of atmospheric distortion.
- Transmitter cannot obtain accurate, up-to-date information on atmospheric distortion, because of:
  - Imperfect reciprocity.
  - Transmission delay.
- Receiver and transmitter do know the statistics of atmospheric turbulence, characterized by a *field coherence length*

$$\delta_0 = \frac{r_0}{6.88^{3/5}}$$

where r_0 is the Fried parameter.



Optimal Modes in Atmospheric Turbulence (2/2)

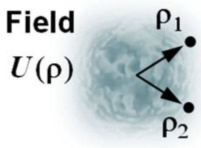
- Among all mode sets, the optimal modes suffer the least degradation by turbulence.
- Among all sets of N orthogonal modes, the *optimal modes*

$$\left\{\psi_n(\boldsymbol{\rho}),\ n=1,\ldots,N\right\}$$

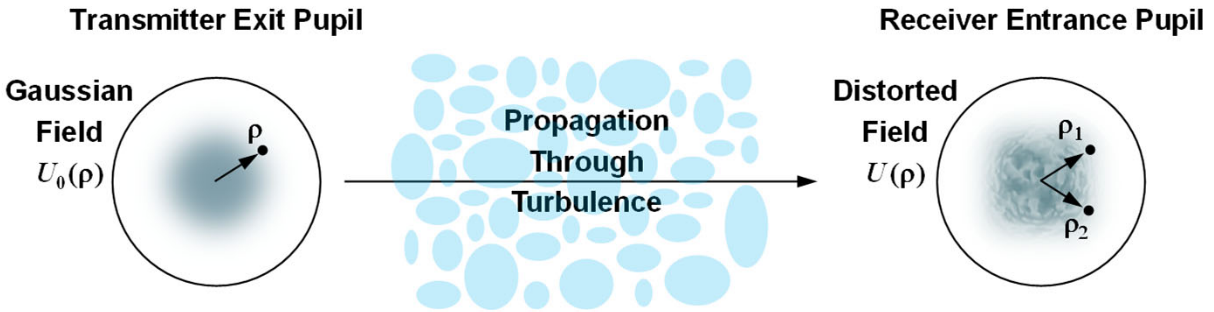
  can approximate a received field $U(\rho)$ with minimum mean-square error.

- Optimal modes are best basis set for transmission, reception and MIMO signal processing when number of modes is constrained.
- Optimal modes are determined by performing a Karhunen-Loève expansion of received field $U(\rho)$.
- Optimal modes are eigenfunctions of the *mutual intensity function* in the receiver plane

$$\Gamma(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)=E\left(U(\boldsymbol{\rho}_1)U^*(\boldsymbol{\rho}_2)\right)$$



Derivation of Optimal Modes (1/3)



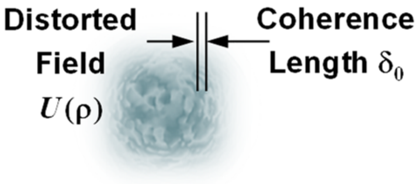
- Assume Gaussian field profile with *reference beam waist* ω_0:

$$U_0(\boldsymbol{\rho}) = \exp\left(-\frac{|\boldsymbol{\rho}|^2}{2\omega_0^2}\right)$$

ω_0 describes soft apertures at transmitter and receiver pupil planes.

- Assuming Kolmogorov turbulence, mutual intensity in receiver plane is

$$\Gamma(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = \exp\left(-\frac{|\boldsymbol{\rho}_1|^2 + |\boldsymbol{\rho}_2|^2}{2\omega_0^2}\right)\exp\left(-\frac{|\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2|^{5/3}}{2\delta_0^{5/3}}\right)$$



- Recall *field coherence length* $\delta_0 = r_0 / 6.88^{3/5}$, where r_0 is Fried parameter.

Derivation of Optimal Modes (2/3)

- To facilitate analysis, approximate received field distortion by $(\omega_0/\delta_0)^2$ independent cells, each of radius δ_0. The distortions are highly correlated within each cell. Similar methods are used to model imaging in random media.*
- The cells are centered at $\mathbf{v} \sim N\left(0,\, \omega_0^2\, \mathbf{I}_{2\times 2}\right)$ and are described by elemental functions of form $\exp\left(-\frac{|\boldsymbol{\rho}-\mathbf{v}|^2}{2\delta_0^2}\right)$.
- This leads to a mutual intensity of the form

$$\Gamma\left(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2\right)=\frac{1}{2\pi\omega_0^2}\int \exp\left(-\frac{|\boldsymbol{\rho}_1-\mathbf{v}|^2}{2\delta_0^2}\right)\exp\left(-\frac{|\boldsymbol{\rho}_2-\mathbf{v}|^2}{2\delta_0^2}\right)\times\exp\left(-\frac{|\mathbf{v}|^2}{2\omega_0^2}\right)d\mathbf{v}$$

Performing the integration yields

$$\Gamma\left(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2\right)=\exp\left(-\frac{|\boldsymbol{\rho}_1|^2+|\boldsymbol{\rho}_2|^2}{2\sigma_g^2}\right)\exp\left(-\frac{|\boldsymbol{\rho}_1-\boldsymbol{\rho}_2|^2}{2\sigma_s^2}\right)$$

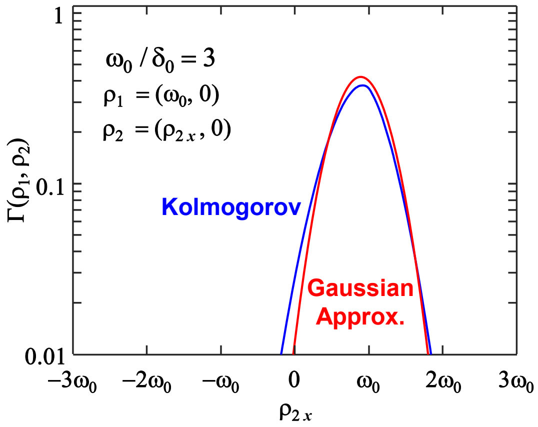
- We have defined

$$\sigma_g^2=\omega_0^2+\delta_0^2 \qquad \sigma_s^2=\frac{\delta_0^2}{\omega_0^2}\left(\omega_0^2+\delta_0^2\right)$$

*J. W. Goodman, *Statistical Optics*, 2nd Ed., Wiley (2015).

Derivation of Optimal Modes (3/3)

- We obtain a good approximation of the Kolmogorov mutual intensity.



- Now expand mutual intensity in series of orthogonal functions:

$$\Gamma(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2) = \sum_n \lambda_n \psi_n^*(\boldsymbol{\rho}_1)\psi_n(\boldsymbol{\rho}_2)$$

- Find eigenmodes $\psi_n(\boldsymbol{\rho})$ and corresponding eigenvalues λ_n by solving Fredholm integral equation:

$$\int_{\substack{\text{receiver}\\ \text{plane}}} \Gamma(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\psi_n(\boldsymbol{\rho}_2)\,d\boldsymbol{\rho}_2 = \lambda_n \psi_n(\boldsymbol{\rho}_1)$$

- By substitution in Fredholm integral, can verify that optimal modes in receiver plane are *complete set of Laguerre-Gauss modes* with appropriately chosen beam waist ω.

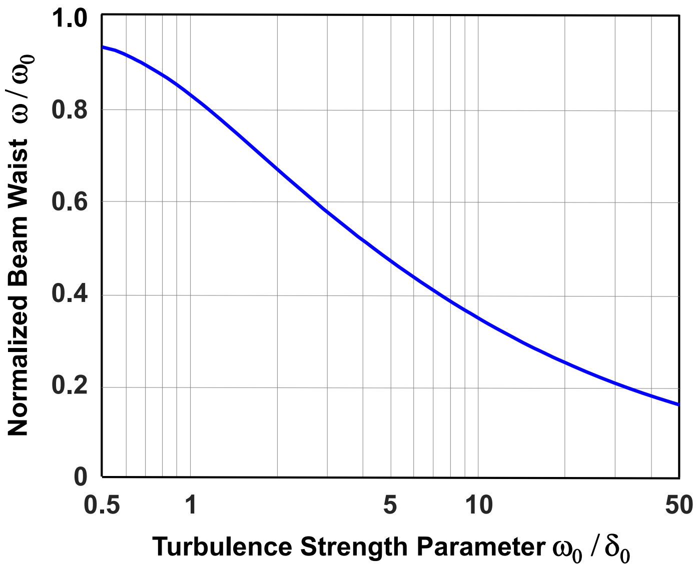
The derivation yields two key parameters of optimal modes that depend on ω_0 and δ_0:

- Optimized beam waist ω.
- Turbulence eigenvalue λ_n.

Optimized Beam Waist

- Optimized waist ω cannot exceed reference waist ω_0. In strong turbulence, ω should be not much larger than δ_0.

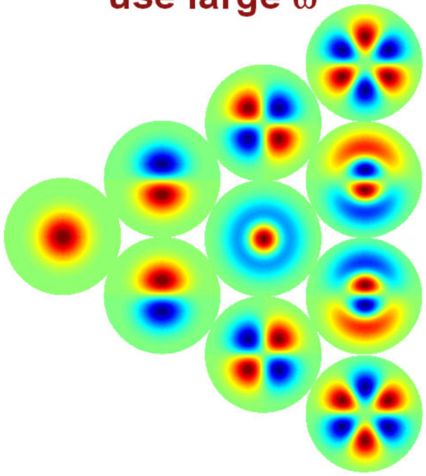
$$\frac{1}{\omega^2} = \frac{1}{\omega_0^2} + \frac{1}{2\delta_0^2}\frac{\sqrt{1+2(\omega_0/\delta_0)^2}}{1+(\omega_0/\delta_0)^2}$$



Weak Turbulence Strong Turbulence

Weak Turbulence

use large ω

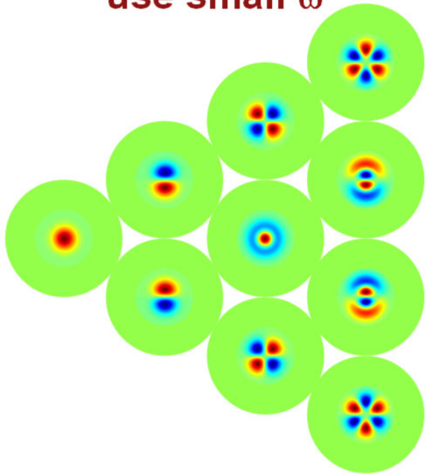


$$\frac{\omega_0}{\delta_0} \to 0$$

$$\omega \approx \omega_0$$

Strong Turbulence

use small ω



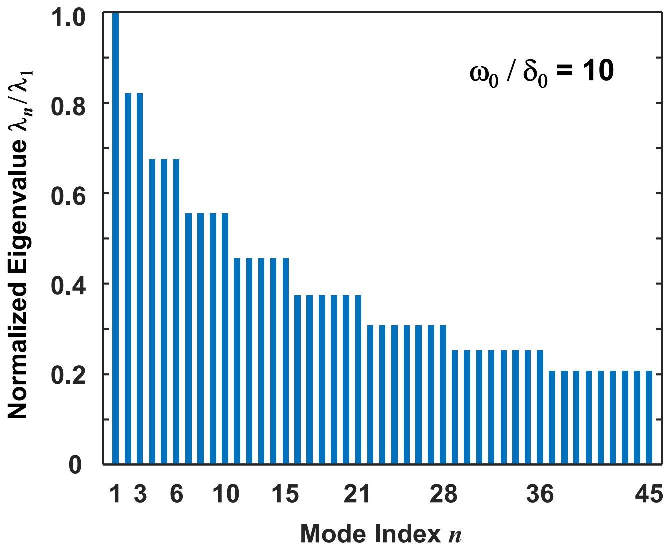
$$\frac{\omega_0}{\delta_0} \to \infty$$

$$\omega \approx \left(\sqrt{2}\omega_0\delta_0\right)^{1/2}$$

Turbulence Eigenvalue

- Modes with large λ_n are, on average, less degraded by turbulence than modes with small λ_n.
- Turbulence eigenvalue λ_n depends on (p, l) via $2p+|l|$, like mode group index $m = 2p+|l|+1$.

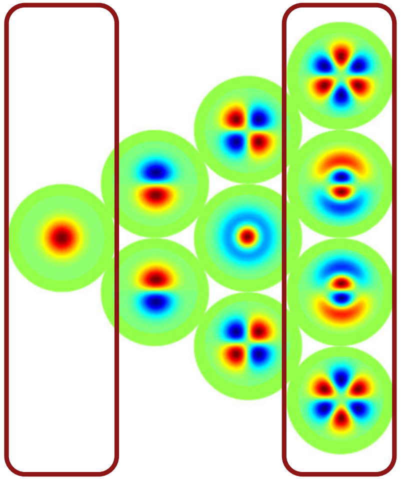
$$\lambda_n = \frac{2}{\xi+\sqrt{2\xi-1}}\left(\frac{\xi-\sqrt{2\xi-1}}{\xi+\sqrt{2\xi-1}}\right)^{2p+|l|} \qquad \xi = 1+\left(\omega_0/\delta_0\right)^2$$



Strong Modes **Weak Modes**

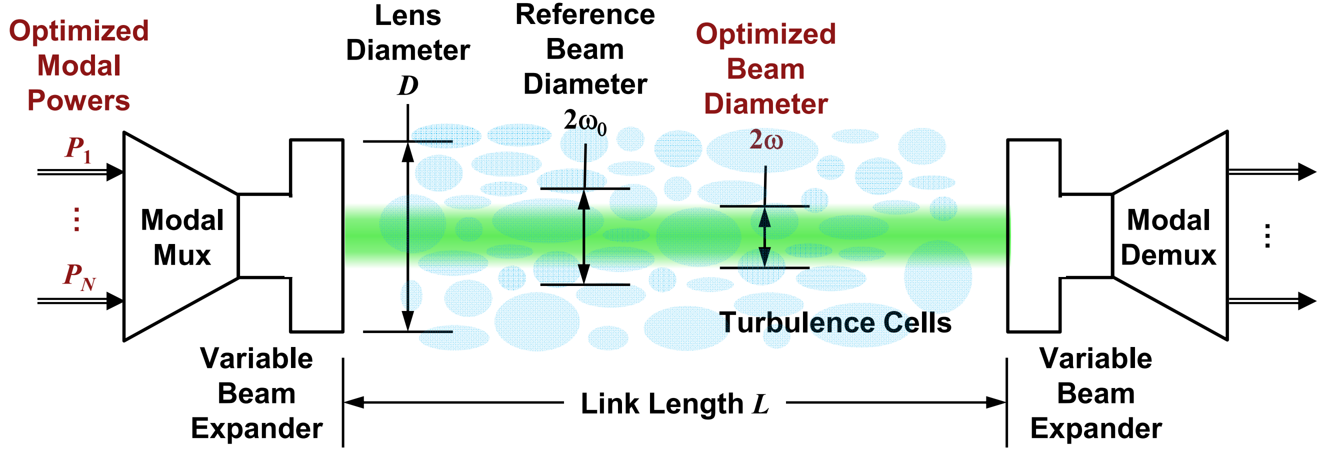
Strongest Mode allocate most power

Weaker Modes allocate less power



$1 \quad 2p+|l|+1 \longrightarrow 4$

Optimized Link Design (1/2)



- Choose number of mode groups M. Determines number of modes:

$$N_{LG} = \tfrac{1}{2} M (M+1) \qquad N_{OAM} = 2M - 1$$

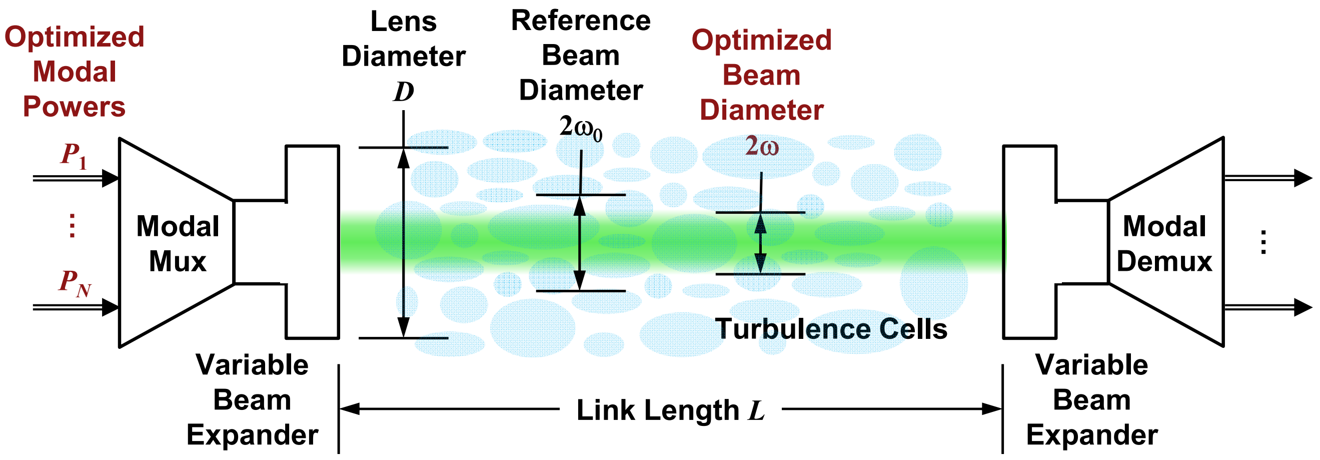
- Given lens diameter D, choose reference beam waist ω_0 using truncation parameter $\tau(M)$:

$$\frac{D}{2\omega_0} = \tau(M) = \sqrt{M}$$

M	N_{LG}	$\tau(M)$
1	1	1.0
4	10	2.0
9	45	3.0

Ensures that higher-order modes are not clipped.

Optimized Link Design (2/2)



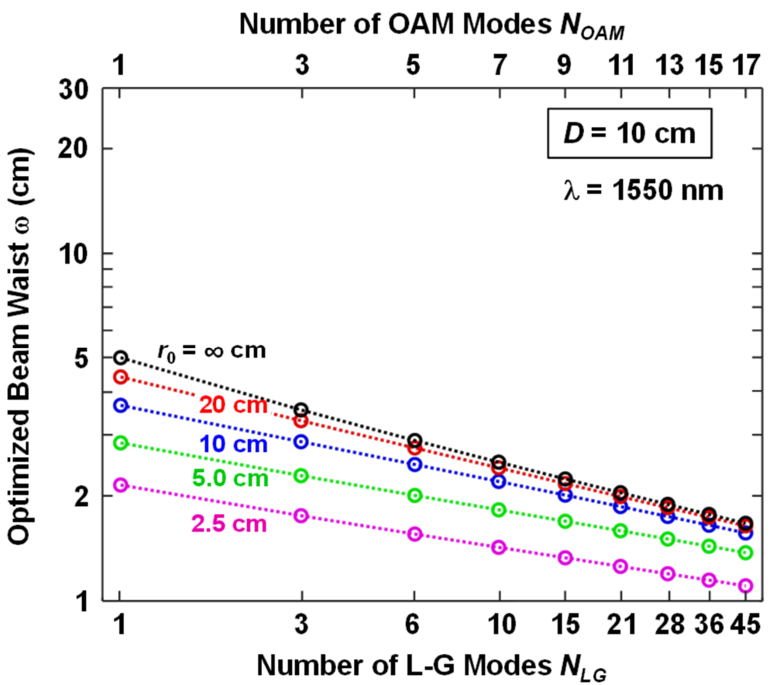
- Given field coherence length $\delta_0 = r_0 / 6.88^{3/5}$ and reference beam waist ω_0, compute optimized beam waist ω:

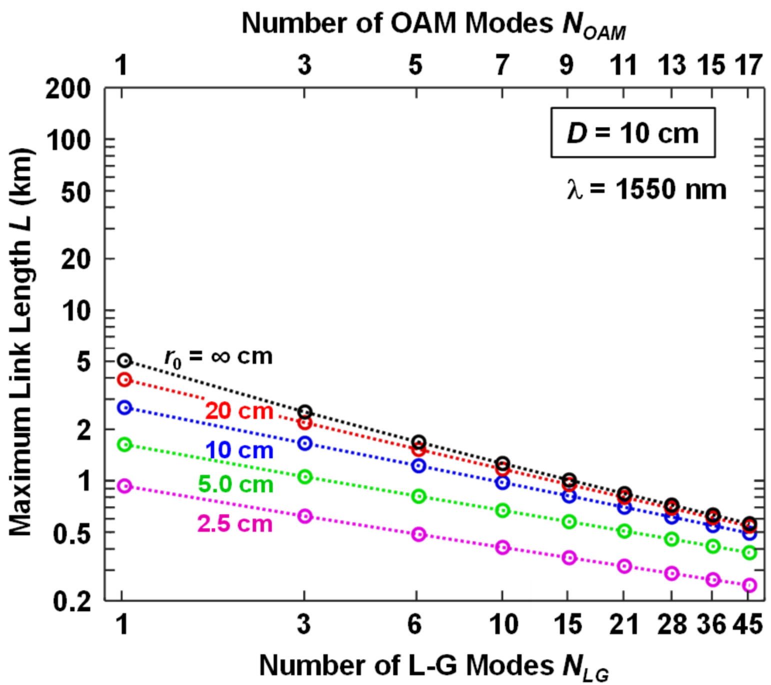
$$\frac{1}{\omega^2} = \frac{1}{\omega_0^2} + \frac{1}{2\delta_0^2} \frac{\sqrt{1+2\left(\omega_0/\delta_0\right)^2}}{1+\left(\omega_0/\delta_0\right)^2}$$

- Estimate maximum link length L as Rayleigh range corresponding to optimized beam waist ω:

$$L \le \frac{\pi\omega^2}{\lambda}$$

Optimized Beam Waist and Maximum Link Length

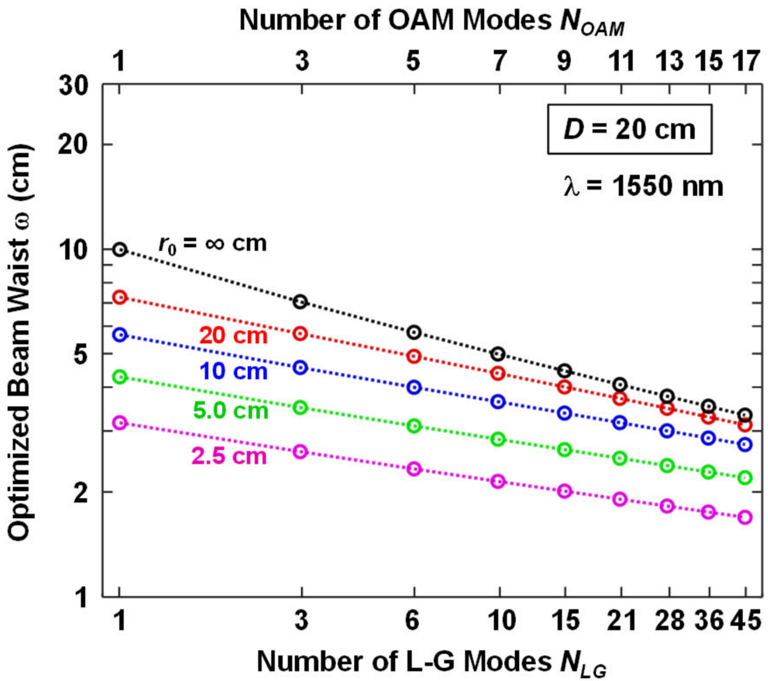


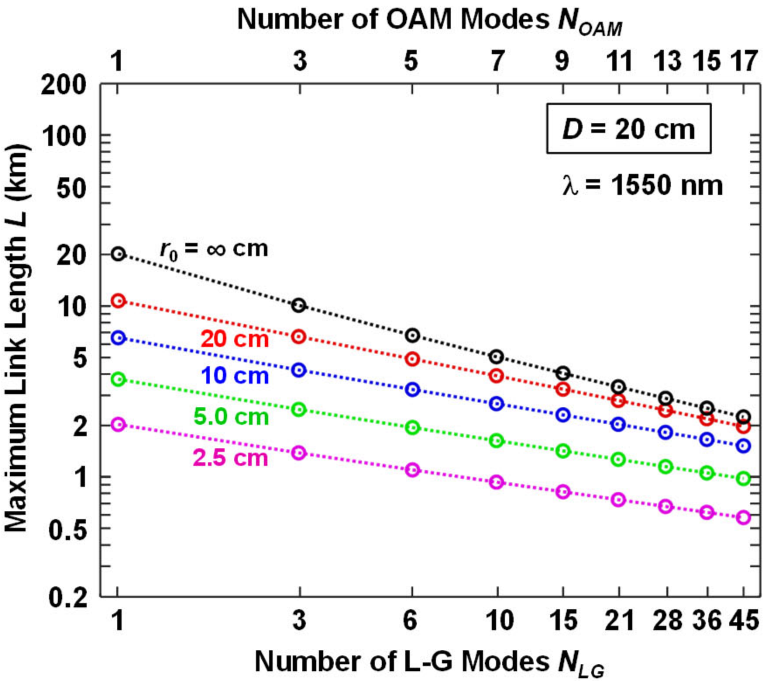


- When $r_0 = \infty$ (no turbulence), maximum L is close to value limited by space-bandwidth product with $N_F = N_{LG}$ (see Symmetric Two-Lens Link):

$$L \leq \frac{\pi D^2}{4\lambda\sqrt{N_{LG}}}$$

Optimized Beam Waist and Maximum Link Length





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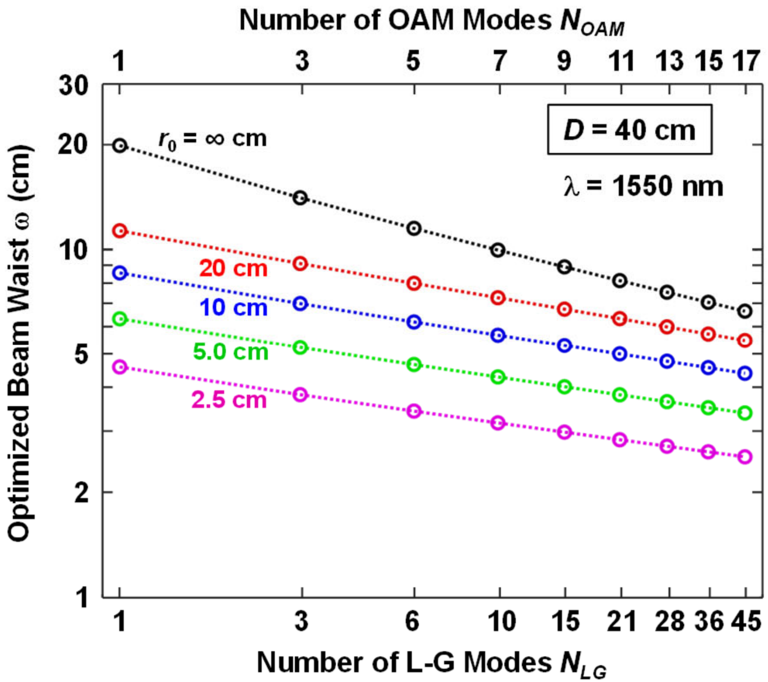
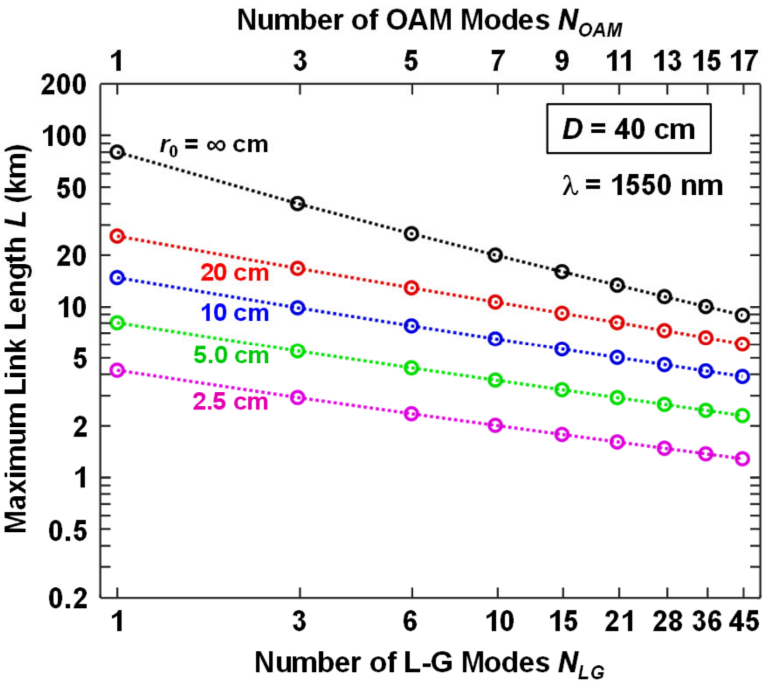
Optimized Beam Waist and Maximum Link Length





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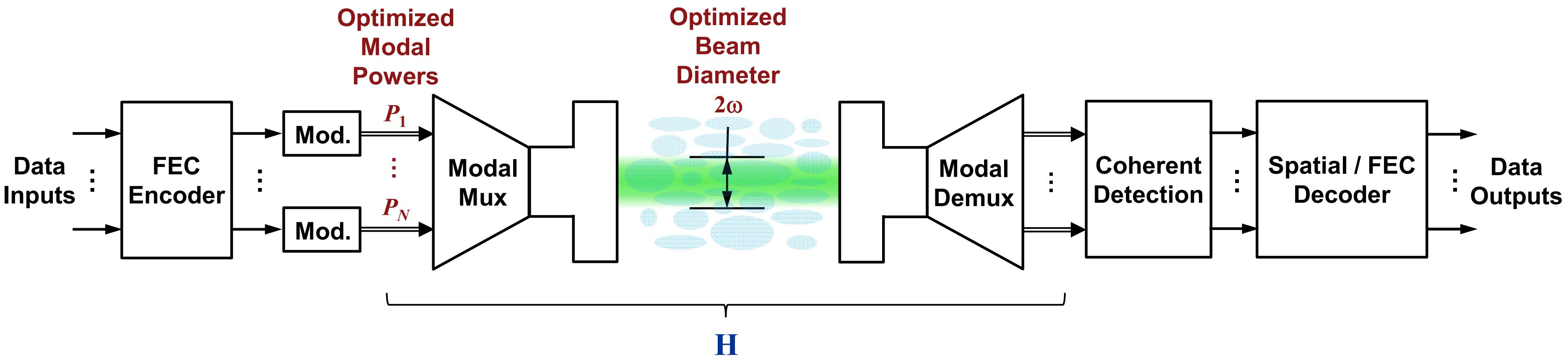
Optimized System Design (1/2)



- Require at least a mode group-selective multiplexer to enable optimized per-group power allocation.
- $\mathbf{H}$ not diagonal and not unitary: multiplexed signals are not orthogonal.
- Spatial / FEC decoder should mitigate non-orthogonality.
  - Better: successive interference cancellation. SIC can approach optimal ML performance.[1, 2]
  - Worse: minimum-mean-square-error equalization. MMSE is impaired by noise enhancement.

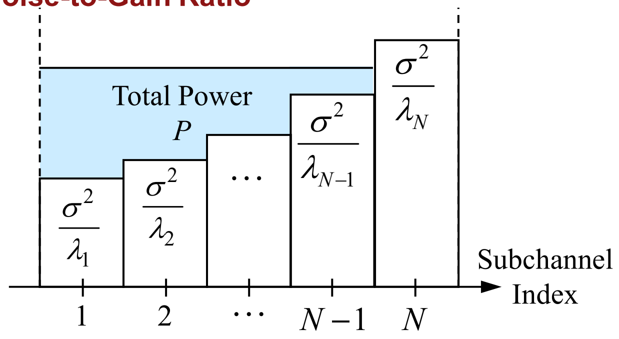
1. P.W. Wolniansky *et al.*, *ISSSE* 1998.
2. E. S. Chou and J. M. Kahn, *JLT* **40** (2022).

Optimized System Design (2/2)

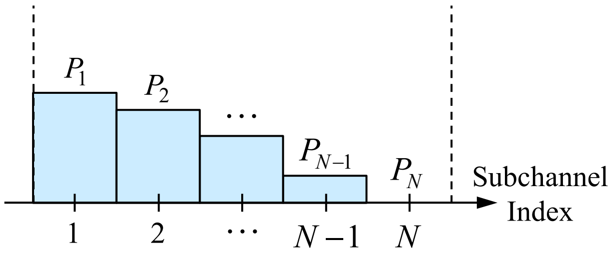
Assume:

- Coherent detection with equal noise power σ^2 per receiver.
- Constraint on total transmit power $P = \sum_{n=1}^{N} P_n$.
- Total SNR is $\gamma = \frac{1}{\sigma^2} \sum_{n=1}^{N} P_n$.
- Transmitter knows turbulence statistics via field correlation length δ_0. Can optimize power allocation $\{P_1,\ldots,P_N\}$ based on turbulence eigenvalues $\{\lambda_1,\ldots,\lambda_N\}$.

Noise-to-Gain Ratio



Optimal Power Allocation



Capacity and Effective Degrees of Freedom

- Let power allocation $\{P_1,\ldots,P_N\}$ be represented by a matrix

$$\mathbf{Q} = \mathrm{diag}\left(P_1 / P,\ldots,P_N / P\right)$$

- Average capacity per unit bandwidth (an expectation over turbulence realizations):[1]

$$SE = E_{\mathbf{H}}\left\{\log_2\left(\det\left(\mathbf{I} + \gamma \mathbf{H}\mathbf{Q}\mathbf{H}^H\right)\right)\right\}$$

- Number of modes effectively conveying information:[2]

$$EDOF = \frac{d}{d\delta} SE\left(2^{\delta}\gamma\right)\Big|_{\delta=0}$$

  - Low SNR: power-limited, EDOF depends weakly on N.
  - High SNR: mode-limited, EDOF approaches N.

1. E. Telatar, *Eur. Trans. Telecommun*. **10** (1999).

2. D.-S. Shiu *et al*., *Trans. on Commun*. **48** (2000).

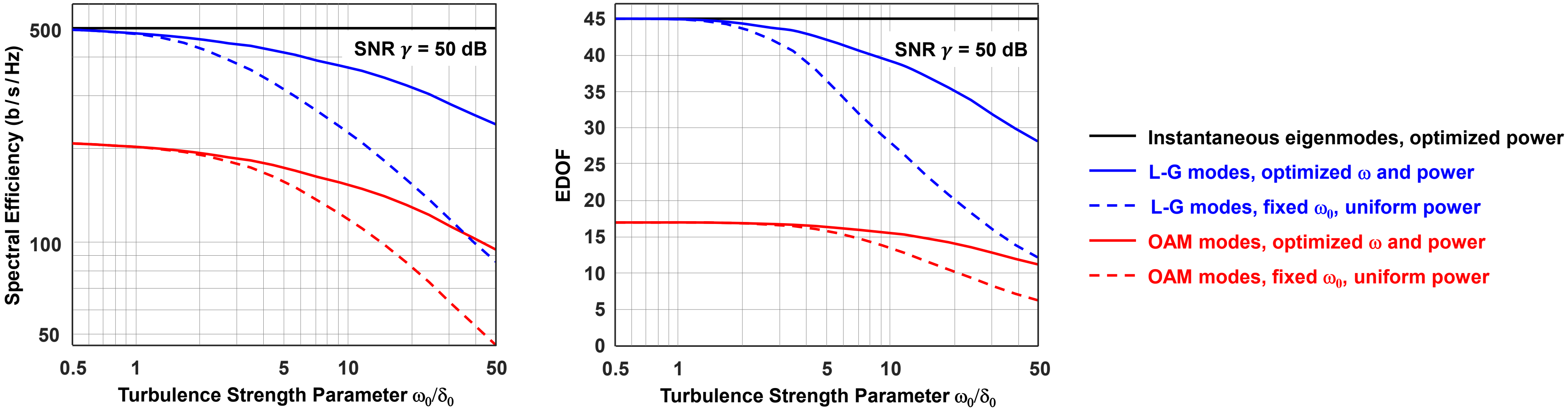
Comparison of Mode Sets in Turbulence (1/3)

- Design link to support $M = 9$ mode groups (see Optimized Link Design).
  - L-G modes: $N_{LG} = \frac{1}{2} M \left(M + 1 \right) = 45$
  - OAM subset: $N_{OAM} = 2M - 1 = 17$

- Monte Carlo simulations
  - Model phase aberration $\Phi(\rho)$ by propagation through successive phase screens.
  - Compute overlap integral between transmit mode j and receive mode i to obtain channel matrix element $\mathbf{H}_{ij}$.
  - Generate 10^4 realizations of channel matrix $\mathbf{H}$ for each scheme.

- Transmission strategies when transmitter has specified knowledge of atmosphere:
  - Perfect (knows $\mathbf{H}$): instantaneous eigenmodes, power allocation based on instantaneous eigenvalues.
  - Statistical (knows δ_0): optimized beam waist ω, power allocation based on statistical eigenvalues λ_n.
  - None: fixed beam waist ω_0, uniform power allocation.

Comparison of Mode Sets in Turbulence (2/3)



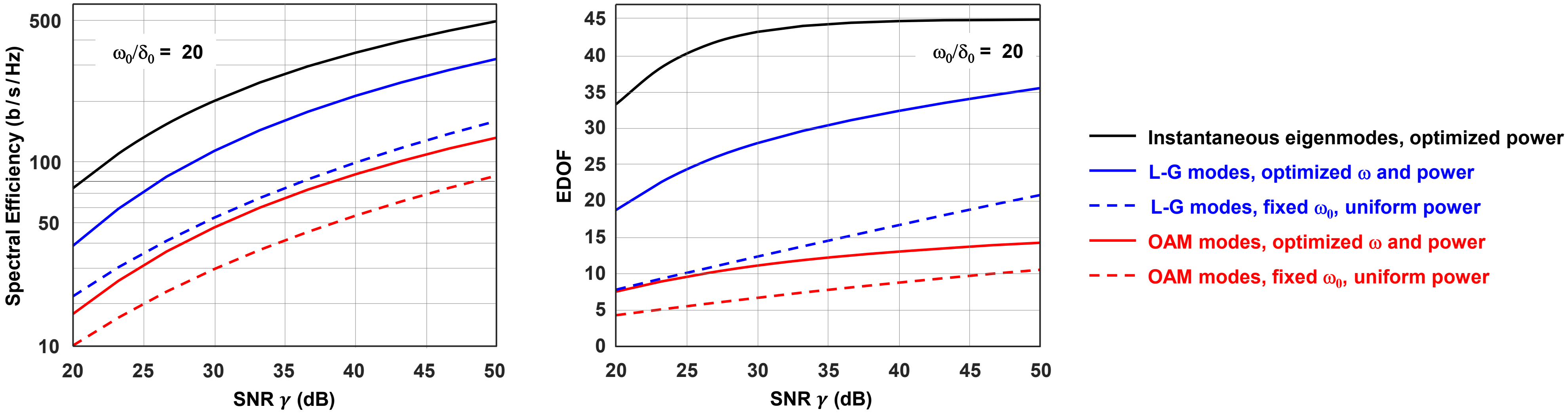
Very weak turbulence

- Instantaneous eigenmodes, optimized L-G modes and fixed L-G modes all offer EDOF = 45.
- Optimized OAM modes and fixed OAM modes offer EDOF = 17.

Moderate to strong turbulence

- Instantaneous eigenmodes offer EDOF = 45.
- Optimized L-G modes outperform fixed L-G modes. Optimized OAM modes outperform fixed OAM modes.

Comparison of Mode Sets in Turbulence (3/3)



At all SNR γ

- Optimized L-G modes outperform fixed L-G modes. Optimized OAM modes outperform fixed OAM modes.

As SNR $\gamma \to \infty$

- EDOF $\to$ 45 for instantaneous eigenmodes (rapidly) and for optimized L-G modes (slowly).
- EDOF $\to$ 17 for optimized OAM modes.
- EDOF are lower for fixed L-G modes and fixed OAM modes than for their optimized counterparts.

Conclusions

- Studied modal multiplexing assuming transmitter knows statistics but not instantaneous state of turbulence.

Optimal modes

- Suffer least degradation by turbulence among all mode sets.
- Are best basis set for transmission, reception and MIMO signal processing when number of modes is constrained.
- Are a set of L-G modes (or H-G modes) with beam waist ω scaled depending on field correlation length δ_0.
- Are eigenmodes of mutual intensity function in receiver plane.
  Eigenvalues λ_n describe robustness to degradation by turbulence.

Optimal transmission scheme

- Uses variable-magnification optics to optimize beam waist ω.
  Optimizes power allocation to eigenmodes depending on eigenvalues λ_n.
- Significantly outperforms schemes using fixed beam waist ω_0 and uniform power allocation.

Adaptive modal signal processing

- Enables compensation of both phase and amplitude fluctuations. Can adapt faster than adaptive optics.

To Learn More

- These tutorial slides.

- A. Belmonte and J. M. Kahn, "Optimal Modes for Spatially Multiplexed Free-Space Communication in Atmospheric Turbulence", *Optics Express* **29** (2021).

- N. Zhao, X. Li, G. Li and J. M. Kahn, "Capacity Limits of Spatially Multiplexed Free-Space Communication", *Nature Photonics* **9** (2015).