Characterizing Mode-Dependent Loss and Gain in Multimode Components

Karthik Choutagunta, *Student Member, IEEE*, Sercan Ö. Arik, Keang-Po Ho, *Senior Member, IEEE*, and Joseph M. Kahn, *Fellow, IEEE*

***Abstract*—Mode-dependent loss and gain (MDL and MDG) of multimode components are fundamental impairments that reduce the capacity of mode-division-multiplexed (MDM) systems. MDL of components is commonly quantified either in terms of the root mean square (rms) or peak-to-peak (P-P) gain and loss variations. It is incorrect to specify only the P-PMDL of components if they are to be used in an MDM system with nonnegligible mode coupling, because the system's overall coupled gains are random variables whose statistics cannot be determined from the P-P MDL values. On the other hand, measurements of the rms MDL of components are sufficient to determine the rms value of the system's overall coupled MDL, regardless of whether the system has weak or strong coupling. We propose novel algorithms based on convex optimization, which can efficiently measure all modal gains of any multimode component using low-cost direct-detection hardware. In particular, we propose an efficient algorithm that produces accurate measurements of all modal gains by estimating a high-dimensional MDL ellipse using a sequence of power measurements.**

***Index Terms*—Convex optimization, direct detection, estimation, mode-dependent loss and gain, mode-division multiplexing.**

I. Introduction

As SINGLE-MODE fiber (SMF) systems approach fundamental capacity limits, mode-division multiplexing (MDM) has the potential to scale the transmission capacity per fiber with low cost [1], [2]. MDM systems typically use multimode fiber (MMF) or coupled-core multicore fiber (MCF) to transmit data on propagating modes. In an MDM system employing D propagating modes (including spatial and polarization degrees of freedom), the total capacity ideally scales in proportion to D [3].

Key to the deployment of MDM systems are multimode components such as multimode or multicore erbium doped fiber amplifiers (for simplicity, these are collectively referred to as MM-EDFAs). These components inevitably introduce mode-dependent loss and gain (collectively referred to as MDL), leading to a decrease in average system capacity, and an increase in system outage probability [4]–[6]. Furthermore, propagating modes couple to each other via intended or unintended perturbations along the fiber. In MDM systems using coherent detection with MIMO signal processing, mode coupling and modal dispersion (MD) can be overcome using equalizers with sufficient memory length, and they pose no fundamental performance limitations [7], [8]. However, MDL leads to a non-unitary channel transfer matrix that can impair system performance [9]–[11].

A multimode component supporting D spatial and polarization modes will have in general D different modal delays and D different modal gains (the latter typically measured in logarithmic or decibel units). For the sake of convenience, a single number can describe MD or MDL either as (i) the root-mean-square (rms) of these D modal delays or gains, or (ii) as the peak-to-peak (P-P) difference of the highest and lowest delays or gains. MDL is commonly quoted in P-P units [11]–[13], stemming from practices developed in the SMF literature, where the polarization-mode dispersion (PMD) or polarization-dependent loss (PDL) of discrete components is specified as a maximum difference. This is a complete characterization of modal gains and delays for $D = 2$ but not for $D > 2$. The effects of MDL on MDM systems can also be described using alternative metrics, such as in [14], where the authors related MDL to a reduction in spectral efficiency.

In this paper, we caution against the common practice of specifying only the P-P MDL of multimode components. As explained in Section II, the statistics of MD and MDL in strongly coupled MDM systems are completely characterized by their end-to-end rms MD and rms MDL, respectively [15]–[17]. Specifying only the P-P MD or MDL of components in a system with mode coupling leads to ambiguity about the statistics of the overall system. Ideally, a measurement technique should provide the full distribution of modal delays and gains as both the rms and P-P metrics can be derived.

There already exist methods for characterizing MDL. A multimode device-under-test (DUT) with D inputs and outputs is characterized by a $D \times D$ generalized Jones transfer matrix. Coherent detection can be employed to estimate the transfer Jones matrix of a DUT, and singular value decomposition (SVD) of the estimated matrix yields modal gains [18]. This process has a high hardware complexity but low computational complexity

Manuscript received January 25, 2018; revised May 21, 2018; accepted June 24, 2018. Date of publication June 27, 2018; date of current version July 19, 2018. This work was supported in part by Huawei Technologies Co., Ltd., and in part by a Stanford Graduate Fellowship. *(Corresponding author: Karthik Choutagunta.)*

K. Choutagunta and J. M. Kahn are with the Department of Electrical Engineering, E. L. Ginzton Laboratory, Stanford University, Stanford, CA 94305 USA (e-mail: kchoutag@stanford.edu; jmk@ee.stanford.edu).

S. Ö. Arik was with the Department of Electrical Engineering, E. L. Ginzton Laboratory, Stanford University, Stanford, CA 94305 USA. He is now with Baidu Silicon Valley Artificial Intelligence Lab, Sunnyvale, CA 94089 USA (e-mail: sercanarik@baidu.com).

K.-P. Ho is with Apple, Cupertino, CA 94014 USA (e-mail: kpho@ieee.org).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/JLT.2018.2851184

0733-8724 © 2018 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission.
See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

and is considered accurate. We wish to develop simple techniques that use direct detection with low hardware complexity. Since MDL characterization can be performed offline, we are willing to tolerate higher computational complexity. To this end, we propose new techniques using direct detection for estimating the modal gains and the input/output eigenmodes of the MDL operator of multimode components.

The remainder of this paper is organized as follows. Section II reviews the modeling of MDL in multimode components and discusses the importance of correctly quantifying MDL in P-P or rms units for weakly and strongly coupled systems. Section III describes novel convex optimization-based algorithms for MDL estimation using low-cost hardware and direct detection. We provide conclusions in Section IV.

II. Modeling Mode-Dependent Loss and Gain

Even though the MDL in MMF is typically small, MDM signals propagating in different modes experience unequal gains when they pass through MM-EDFAs, splices, or other perturbations along the link. In this section, we describe the propagation of modes in the presence of mode-coupling, modal dispersion, and non-negligible MDL. We review the statistics of MDL in the weak- and strong-coupling regimes. We then discuss metrics for MDL characterization and when it is appropriate to use the rms and P-P modal metrics.

A. Brief Review of Mode-Dependent Loss Modeling

Any linear multimode component supporting D modes at angular frequency Ω be compactly represented by a $D \times D$ propagation matrix

$$\mathbf{M}(\Omega) = \exp(\bar{g}/2)\mathbf{V}(\Omega)\,\boldsymbol{\Sigma}(\Omega)\,\mathbf{U}(\Omega)^H \quad (1)$$

that describes the effects of mode-coupling and MDL. In (1), $\mathbf{U}(\Omega)^H$ denotes the matrix Hermitian conjugate of $\mathbf{U}(\Omega)$. The effects of modal dispersion can also be included if the propagation matrix is frequency-dependent.[1] Here, $\bar{g}$ is the mode-averaged gain, and $\mathbf{V}(\Omega)$ and $\mathbf{U}(\Omega)$ are $D \times D$ unitary matrices describing mode coupling inside the component. The matrix $\boldsymbol{\Sigma}(\Omega)$ is a real-valued $D \times D$ diagonal matrix given as

$$\boldsymbol{\Sigma}(\Omega) = \mathrm{diag}\left[\mathrm{e}^{g_1(\Omega)/2}, \mathrm{e}^{g_2(\Omega)/2}, \ldots, \mathrm{e}^{g_D(\Omega)/2}\right], \quad (2)$$

where $g_1(\Omega), \ldots, g_D(\Omega)$ are the modal gains measured in log-power-gain units, and $\mathrm{diag}(\cdot)$ represents a square matrix formed by placing a vector on the main diagonal of a $D \times D$ matrix of zeros. In this paper, we assume that $\bar{g} = 0$ because the mode-averaged MDL does not affect mode-dependent quantities. Since the mode-averaged gain is accounted-for in $\bar{g}$, we can assume that the modal gains are zero mean (in log-power-gain units): $\sum_{i=1}^{D} g_i(\Omega) = 0$. This is equivalent to the condition that the product of the modal gains in linear units is one.

[1] Although this paper focuses on MDL, the concepts regarding P-P vs rms characterization in MDM systems apply also to modal dispersion. MD causes the matrices in (1) to be frequency dependent because the modes can experience different group delays.

Equation (1) is in the form of a singular value decomposition. The columns of $\mathbf{U}(\Omega)$ and $\mathbf{V}(\Omega)$ are often called Schmidt modes[2] [19]. The DUT can be viewed as a mode converter that maps the ith input Schmidt mode $\mathbf{u}_i(\Omega)$to the ith output Schmidt mode $\mathbf{v}_i(\Omega)$ with a frequency-dependent modal power gain $e^{g_i(\Omega)}$ (or $g_i(\Omega)$ in logarithmic units). In the absence of any MDL, all $g_i(\Omega)$ are zero, $\boldsymbol{\Sigma}(\Omega)$ is an identity matrix and $\mathbf{M}(\Omega)$ is a unitary matrix. Since $\mathbf{M}(\Omega)$ is unitary, orthogonal modes remain orthogonal after propagation through the DUT. If MDL is present, then $\mathbf{M}(\Omega)$ is not a unitary matrix, and propagation through the DUT breaks the orthogonality between modes. In the remainder of this paper, we drop the explicit dependence on Ω to simplify notation.

Given a generalized Jones transfer matrix $\mathbf{M}$, an MDL operator is defined as

$$\mathbf{G} = \mathbf{M}^H\mathbf{M}, \quad (3)$$

whose eigenvalues are the modal gains $(g_1, \ldots, g_D)$, ordered as $g_1 \leq \cdots \leq g_D$ (measured in log-power-gain units). The rms MDL in decibel units can be calculated from the modal gains distribution as

$$\mathrm{MDL}_{\mathrm{rms}} = \gamma\sqrt{\frac{1}{D}\sum_{i=1}^{D} g_i^2}, \quad (4)$$

where $\gamma = 10/\ln 10 \approx 4.34$ is a factor to convert from log-power-gain units to decibel units. This is equivalent to the standard deviation, or the square root of the variance, because $\sum g_i = 0$. The P-P MDL in decibel units can be calculated as

$$\mathrm{MDL}_{\text{P-P}} = \gamma\,(g_D - g_1)\,. \quad (5)$$

The MDL operator can be equivalently described as $\mathbf{M}\mathbf{M}^H$ which has the same eigenvalues as (3). The eigenvectors of $\mathbf{G}$ represent the input Schmidt modes, which are an optimal basis of MIMO transmission through the DUT without crosstalk at frequency Ω.

In long-haul MDM systems, modal gains determine the channel capacity of the system [4], [14], [17], [19]–[21]. As the MDL increases, the average capacity decreases, and the variance of capacity increases, increasing the probability of outage. If the end-to-end MDM system is characterized by a generalized Jones transfer matrix $\mathbf{M}_{\mathrm{tot}}$, then its channel capacity per unit frequency (also known as the spectral efficiency) is given by [3]

$$C = \sum_{j=1}^{D}\log_2\left(1 + \frac{\mathrm{SNR}}{1/D\sum_{k=1}^{D}\mathbb{E}\left\{\lambda_k^2\right\}}\cdot\lambda_j^2\right), \quad (6)$$

where the signal-to-noise ratio SNR is defined as the mode-averaged received signal power divided by the mode-averaged received noise power. λ_j are the singular values of $\mathbf{M}_{\mathrm{tot}}$ (or the square-root of the eigenvalues of ${\mathbf{M}_{\mathrm{tot}}}^H\mathbf{M}_{\mathrm{tot}}$), which represent the *coupled* spatial subchannel gains of the overall system. Note that λ_j are different from $e^{g_j/2}$, which are the modal amplitude gains in linear units for the individual components in the

[2] For the sake of clarity, we use the term "Schmidt modes", which result from a singular value decomposition of the DUT's transfer matrix. The input Schmidt modes coincide with the eigenvectors of the MDL operator **G**.

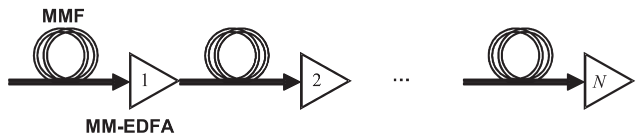

Fig. 1. Typical MDM link with *N* spans of MMF. Each span contains a multimode EDFA, which can introduce MDL.

link. Since the λ_j are random variables in a link with random mode coupling, the capacity *C* is also a random variable whose statistics depend on the statistics of the λ_j.

B. RMS vs P-P MDL Statistics

Most literature on spatial multiplexing today quotes MDL of components in P-P units only. Even though the distinction may seem subtle at first, we show in this section that the incorrect choice of metric can lead to inaccurate characterization of MDM systems, especially as the number of modes increases. The key points regarding P-P and rms metrics are also applicable for modal group delay estimation. We consider an MDM link with *N* spans of MMF, with each span containing a MM-EDFA that can introduce MDL, as shown in Fig. 1. We assume that all *N* MM-EDFAs behave similarly for simplicity.

When an MDM system only contains components whose mode coupling matrices are nearly diagonal, it is said to be weakly coupled. This is a rare case in practice, as polarization modes or degenerate mode groups will be coupled after propagating a short distance due to random perturbations along the link. Nevertheless, if the MDM link in Fig. 1 is assumed to be weakly coupled, then the MDL increases linearly with the number of spans, so the end-to-end P-P MDL may be approximately *N* times that of a single MM-EDFA. Such a characterization is insightful because a high P-P MDL implies that data signals carried on orthogonal modes experience highly unequal gains, which is suboptimal from a transmission standpoint. The system design objective is to minimize the P-P gain. Similarly, the rms MDL of the end-to-end system is approximately *N* times that of a single MM-EDFA. For the case of $D = 2$ modes, the rms and P-P PDL in dB units are related as $\text{PDL}_{\text{P-P}}[\text{dB}] = 2 \times \text{PDL}_{\text{rms}}[\text{dB}]$ because the two gains must sum to 0 (in dB). Such a relationship is not possible for $D > 2$ modes. In the weak coupling regime, the system's coupled gains, P-P and rms MDL all scale linearly in the number of MDL sources. Therefore, if one measures all *D* modal gains, or just the P-P MDL, or just the rms MDL of each component, the values can be added to determine the MDL of the overall weakly coupled system.

Such a scaling does not hold true for systems with non-negligible mode coupling. Most practical MDM systems will have coupling among polarization modes and among degenerate spatial modes in mode groups. In strongly coupled systems, such as long-haul coherent MDM links, the overall channel matrix is the product of many independent transfer matrices, each with random mode coupling. The coupled gains of strongly coupled MDM systems are random variables. In the low-to-moderate MDL range of practical interest, the logarithm of the MDL operator is approximately a zero-trace Gaussian unitary ensemble. The end-to-end MDL has a standard deviation of [17], [22]

$$\sigma_{mdl} = \xi\sqrt{1 + \xi^2/(12\,(1 - D^{-2}))}. \tag{7}$$

Note that (7) only depends on D and the accumulated MDL parameter ξ, which is a factor that relates the uncoupled MDL to the coupled MDL. For example, if a long-haul link contains $N \gg 1$ sources of MDL, each having an rms MDL value of σ_g, then the standard deviation of the accumulated MDL is $\xi = \sqrt{N} \cdot \sigma_g$. The end-to-end MDL depends nonlinearly on ξ. This nonlinear relationship was shown to be analytically correct for $D = 2$ modes and in the limit of very large number of modes ($D = \infty$), and was also numerically verified for multimode fibers with other number of modes [22]. In the limit that the number of modes D is very large, then (7) becomes

$$\sigma_{mdl} = \xi\sqrt{1 + \xi^2/12}, \tag{8}$$

which coincides with (1) of [17]. If one calculates ξ based on the rms MDL values of the individual components inside a MDM link, it is possible to use (7) to determine the overall MDL statistics of the strongly coupled system. For a more detailed treatment of MDL statistics in MDM systems, the reader is encouraged to refer to [17].

The P-P MDL of a component cannot sufficiently characterize the system if the component is to be inserted in a link with strong mode coupling. The P-P MDL of a strongly coupled long-haul system is a random variable. Whenever the fibers connecting adjacent MM-EDFAs are moved or otherwise disturbed, the unitary matrices characterizing the mode coupling change, causing the coupled modal gains of the system to change as well. Due to this randomness, no single measurement of P-P MDL of an MM-EDFA is useful once it is inserted into a long fully coupled system. In the strong-coupling regime, the rms MDL has greater utility than the P-P MDL. It is good practice to always measure the rms MDL of components.

We now show that the discrepancy between P-P and rms MDL metrics becomes worse as D increases to many modes. To illustrate this point, it is easy to identify two distributions of modal gains that have identical P-P MDL but different rms MDL. An example for $D = 4$ modes is a DUT with uniform distribution of gains $\mathbf{g}^{(1)} = (-3, -1, +1, +3)$ and a DUT with uniform distribution of gains $\mathbf{g}^{(2)} = (-3, -3, +3, +3)$, which have rms MDLs of 2.2 dB and 3.0 dB, respectively. In fact, as shown in Fig. 2 for the general case of D modes, if the P-P MDL is fixed at Δg, then the distribution $\mathbf{g}^{(\min)} = (-\Delta g/2, 0, \ldots, 0, \Delta g/2)$ has the lowest possible rms MDL of

$$\sigma_{g,\min} = \Delta g/\sqrt{2D}, \tag{9}$$

where as the distribution $\mathbf{g}^{(\max)} = (-\Delta g/2, -\Delta g/2, \ldots \Delta g/2, \Delta g/2)$ has the highest possible rms MDL of

$$\sigma_{g,\max} = \Delta g/2. \tag{10}$$

The difference between the highest and lowest possible rms MDL for a fixed P-P MDL represents ambiguity in the MDL statistics of the overall system because ξ in (7) is not precisely

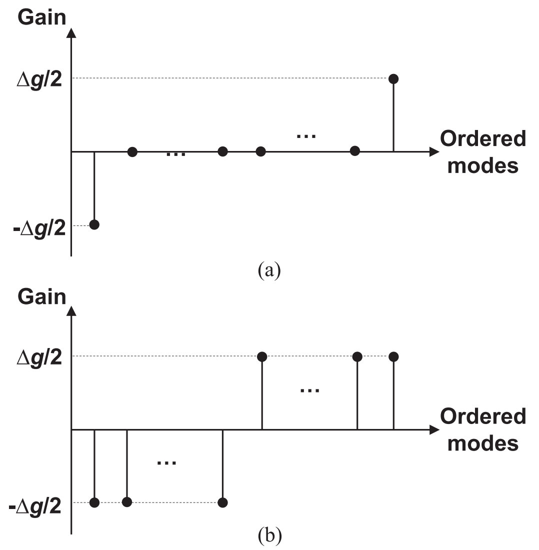

Fig. 2. Distributions of the modal gains with (a) the lowest rms MDL and (b) the highest rms MDL for a fixed p-p MDL spread. The modal gains in (a) are distributed as $\mathbf{g}^{(\min)} = (-\Delta g/2,\, 0,\, \ldots,\, 0,\, \Delta g/2)$. The modal gains in (b) are distributed as $\mathbf{g}^{(\max)} = (-\Delta g/2,\, -\Delta g/2,\, \ldots,\, \Delta g/2,\, \Delta g/2)$.

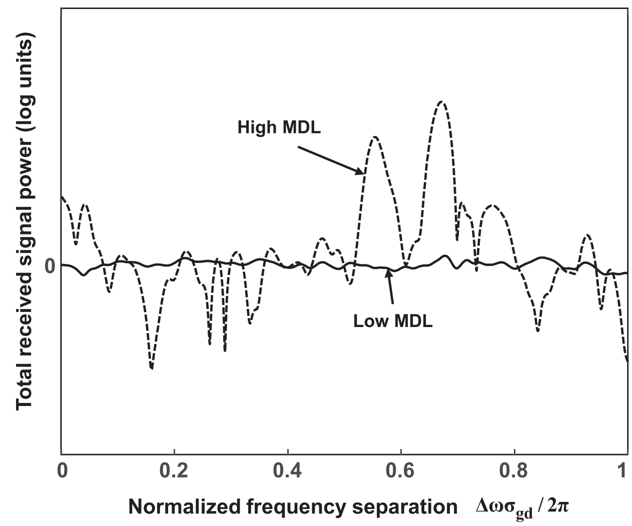

Fig. 3. The MDL of devices with modal dispersion is frequency dependent. A high MDL causes the modal gains of devices to fluctuate more with frequency. σ_{gd} is the rms modal group delay spread.

specified. In SMF systems with $D = 2$ modes, there is no ambiguity because there is only one degree of freedom which causes the minimum and maximum possible rms to coincide. However, for $D > 2$, we can observe from (9) and (10) that the ambiguity increases with a higher number of modes.

C. Goals of MDL Characterization Techniques

Any MDL measurement technique should strive to measure at least both the P-P MDL and the rms MDL of components. However, techniques that can measure the entire distribution of modal gains are preferred because both the P-P and rms metrics can be derived from the distribution.

Input Schmidt modes are also of interest to optical component designers because they provide insight about which combinations of input modes experience the worst-case loss and gain. For component design, output Schmidt modes are usually not as important as the input Schmidt modes, except when assessing the performance of cascaded devices. However, the Schmidt modes do not ultimately matter in a system with full random coupling (such as a long-haul MDM link) because it is not possible to control which combinations of modes enter and exit each component in the system. The Schmidt modes should be characterized when possible but are not as critical as the modal gains.

It is an added benefit if the MDL measurement technique can characterize the crosstalk matrix of components. In systems where mode coupling is not compensated, it is favorable to have low-crosstalk components, corresponding to a nearly diagonal crosstalk matrix.

MDL measurement techniques should ideally have the capability to calibrate for systematic errors. Any built-in MDL in the measurement setup should be considered, so that only the MDL of the DUT is calculated.

Here, we briefly comment on the frequency dependence of MDL. As modeled in (1), a general multimode device with built-in modal dispersion will exhibit frequency-dependent MDL. As shown in Fig. 3, the modal gains of a DUT with high MDL will fluctuate in frequency more than those of a DUT with low MDL due to the interplay between MDL, mode coupling, and modal dispersion [23]. Therefore, the MDL of each component in a link must be characterized independently at each frequency of interest. The techniques discussed in Section III must be repeated at each frequency of interest.

III. Low-Complexity MDL Estimation

If one has access to coherent detection hardware, like setups found in state-of-the-art optical communication laboratories, MDL estimation is trivial because all modal gains can be extracted from the singular value decomposition of the measured transfer matrix. Here we present two novel techniques for MDL estimation that use less complex hardware than coherent detection. We show that convex optimization can be used to accurately estimate all modal gains using direct-detection measurements alone. The tradeoff of lower hardware complexity for higher computational complexity is well justified because MDL estimation is intended solely for characterization purposes.

SubSection III-A presents a method to estimate a high-dimensional MDL ellipsoid to yield all modal gains and input Schmidt modes of any multimode DUT. SubSection III-B presents another algorithm based on phase retrieval that can measure everything that our first technique can, and additionally can estimate the DUT's output Schmidt modes with more hardware. Both methods should be repeated at each frequency of interest because MDL is, in general, a frequency-dependent effect. We compare the performance of both techniques in SubSection III-C.

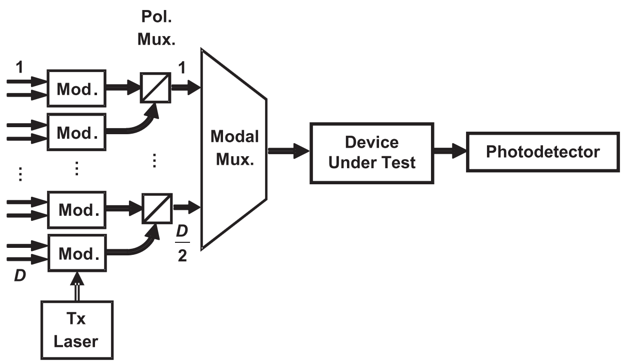

Fig. 4. Hardware setup for the convex optimization-based MDL ellipsoid estimation technique consisting of a laser, low-speed modulators, a multiplexer, a device under test, and a photodetector.

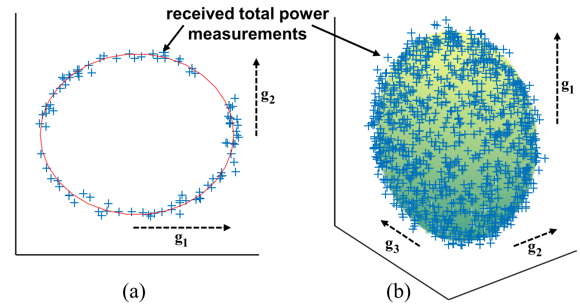

Fig. 5. Estimation of an underlying D-dimensional ellipsoid from noisy power measurements of a randomly transmitted set of D-dimensional vectors. Examples for (a) $D = 2$ modes and (b) $D = 3$ modes are shown. The semi-axis lengths of the estimated ellipsoid are the modal gains.

A. MDL Ellipsoid Estimation Method

Consider the hardware setup shown in Fig. 4. The multiplexer with a transfer matrix $\mathbf{M}_{\text{mux}} \in \mathbb{C}^{D\times D}$ maps an array of D single-mode inputs, represented by a D-dimensional complex vector $\mathbf{x}^{(k)} \in \mathbb{C}^D$ of modal amplitudes, onto all the D spatial modes of a DUT. The measurement process assumes a sequence of N measurement vectors $\mathbf{x}^{(k)}$ are known by the transmitter and launched into the DUT, where the superscript $k = 1, 2, \ldots, N$ denotes the kth vector in the sequence. The real and imaginary components of each element in $\mathbf{x}^{(k)}$ are complex circular Gaussian random variables subject to a total power constraint $|\mathbf{x}^{(k)}|^2 = 1$, where $|.|^2$ represents the squared L_2 norm. This means that the $2D$-dimensional vector created by concatenating the real and imaginary values of $\mathbf{x}^{(k)}$ is uniformly distributed on the $2D$-dimensional hypersphere [24]. Consequently, the marginal distribution of the real and imaginary parts of each element in $\mathbf{x}^{(k)}$ is uniformly distributed in $[-1,\ 1]$.

The multiplexer's output is connected to a DUT which has a transfer matrix $\mathbf{M} = \mathbf{V\Sigma U}^H \in \mathbb{C}^{D\times D}$. $\mathbf{M}$ is the transfer matrix of the unknown DUT, and we wish to estimate all of its D modal gains. The total power received over all modes when the kth measurement vector is launched is a scalar quantity given by

$$d^{(k)} = \left|\mathbf{MM}_{\text{mux}}\mathbf{x}^{(k)}\right|^2 + n^{(k)},\ k = 1, \ldots, N \tag{11}$$

and it can be measured by a single photodetector. Thermal noise $n^{(k)} \in \mathbb{R}$ corrupts each direct detection measurement. Assuming $n^{(k)}$ is additive Gaussian noise, we can frame MDL estimation as the following convex semidefinite program:

$$\min \sum_{k=1}^{N} \left|d^{(k)} - \mathbf{x}^{(k)H}\mathbf{G}\mathbf{x}^{(k)}\right|^2 \quad \text{s.t.} \quad \mathbf{G} \geq 0 \tag{12}$$

where $\mathbf{G}$ is a positive-semidefinite matrix that characterizes the MDL ellipsoid. Note that in the absence of noise ($n^{(k)} = 0$ for all k), the objective function in (12) assumes a minimum value of 0 when $\mathbf{G} = (\mathbf{MM}_{\text{mux}})^H\mathbf{MM}_{\text{mux}}$ because

$$\begin{aligned}\mathbf{x}^{(k)H}\mathbf{G}\mathbf{x}^{(k)} &= \mathbf{x}^{(k)H}(\mathbf{MM}_{\text{mux}})^H\mathbf{MM}_{\text{mux}}\mathbf{x}^{(k)}\\ &= \left(\mathbf{MM}_{\text{mux}}\mathbf{x}^{(k)}\right)^H\mathbf{MM}_{\text{mux}}\mathbf{x}^{(k)}\\ &= \left|\mathbf{MM}_{\text{mux}}\mathbf{x}^{(k)}\right|^2\\ &= d^{(k)}.\end{aligned} \tag{13}$$

When there is thermal noise present, the best value of $\mathbf{G}$ is given by a positive semidefinite matrix that minimizes the squared error terms across the batch of launched measurement vectors.

Once the optimization problem (12) is solved (usually by convex optimization solvers such as CVX [25]), the next step is to remove systematic MDL in the measurement setup described by the matrix $\mathbf{M}_{\text{mux}}$ using the de-embedding substitution

$$\mathbf{G} \leftarrow \left(\mathbf{M}_{\text{mux}}^H\right)^{-1}\mathbf{G}\mathbf{M}_{\text{mux}}^{-1}. \tag{14}$$

The estimated matrix $\mathbf{G}$ is in the form of the MDL operator of the DUT. Hence,

$$\mathbf{G} = \mathbf{M}^H\mathbf{M} = \mathbf{U}\text{diag}\left(e^{g_1}, e^{g_2}, \ldots, e^{g_D}\right)\mathbf{U}^H, \tag{15}$$

and the vector of modal gains can be calculated in decibel units as

$$\mathbf{g} = \gamma \log \text{eig}\left(\mathbf{G}\right) = \left\{g_1, g_2, \ldots, g_D\right\}, \tag{16}$$

where $\text{eig}(\cdot)$ is the vector of eigenvalues of its argument matrix, and $\log(\cdot)$ is the element-wise natural logarithm.

The intuition behind our estimation algorithm is to visualize MDL as being described by an ellipsoid whose semi-axis lengths are given by the modal gains, as shown in Fig. 5. A set of random modes can measure the distance from the origin to the surface of the ellipsoid. By making measurements using a sufficiently large number of randomly chosen modes, it is possible to estimate the length and directions of the principal axes of the ellipsoid. Note that the steps described in (11)–(16) are able to recover all the modal gains of the DUT without ever having to explicitly compute the generalized Jones transfer matrix of the DUT, $\mathbf{M}$, itself! Recovering the modal gains only

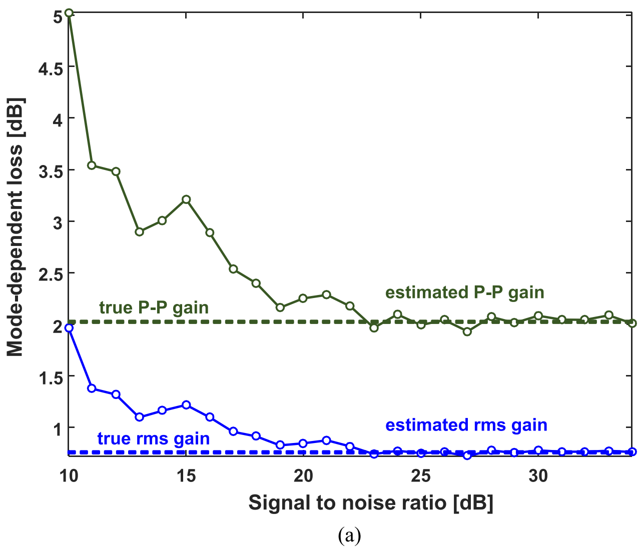

Fig. 6. The MDL ellipsoid estimation algorithm is used to estimate the MDL of a typical DUT supporting $D = 4$ modes using 100 random measurement vectors. The estimation is performed independently at each SNR point using noisy power measurements. (a) Both the estimated P-P and rms MDL values converge to their true values as the measurement SNR increases. (b) All $D = 4$ modal gains can be accurately estimated using the algorithm.

requires estimating $\mathbf{M}^H\mathbf{M}$, which can be accomplished using low-complexity hardware such as a modal multiplexer and a single photodetector.

Fig. 6 shows simulation results for MDL estimation using the MDL ellipsoid method. An example DUT supporting $D = 4$ modes is generated by a creating a 4×4 matrix with random complex-valued entries, and rescaling the matrix to remove mode-averaged loss. The true modal gains of this DUT are $(-1.2, -0.05, 0.45, 0.8)$ dB, determined by computing the eigenvalues of the DUT's MDL operator. MDL estimation is performed using 100 random measurement vectors $\mathbf{x}^{(n)}$ to randomly sample the four-dimensional MDL ellipse. At each SNR test point, the estimation problem in (12) was solved using CVX, a package for specifying and solving convex programs [25]. The algorithm does an excellent job in reconstructing the MDL ellipse of the DUT and recovering the underlying modal gains. Fig. 6(a) shows that P-P gain estimation is less robust to noise than rms gain estimation in the regime of low measurement SNR. This is because the P-P gain estimate using (5) can be corrupted by errors in estimating either the highest modal gain or the lowest modal gain. However, the rms gain calculation using (4) averages estimation errors from all D modes, so it is more robust against measurement noise. Although we have only considered a DUT with four modes in this illustrative example, the algorithm can easily scale to characterize devices with large number of modes. The number of random measurement vectors should increase super-linearly with the number of modes to ensure that the high-dimensional MDL ellipse is adequately sampled.

Both P-P and RMS MDL can be calculated using this ellipsoid reconstruction method because of all the modal gains are known. It is further possible to calculate the input Schmidt modes $\mathbf{U}$ corresponding to each modal gain by performing an eigen decomposition of $\mathbf{G}$. However, the output Schmidt modes $\mathbf{V}$ cannot be retrieved using this method. When an arbitrary optical device with a unitary transfer matrix is inserted between the DUT and the photodetector, the device scrambles the output modes but preserves the total power obtained by the photodetector. Because the measurement technique is invariant to this unknown unitary optical device, output Schmidt modes cannot be calculated, but the estimated modal gains and input Schmidt modes remain the same. Since $\mathbf{M}$ is only known up to a unitary matrix factor, it is likewise not possible to calculate the crosstalk matrix. However, this ellipsoid reconstruction method is especially appealing due to low hardware complexity. The ability to measure all modal gains and the input Schmidt modes is also valuable to component and system designers.

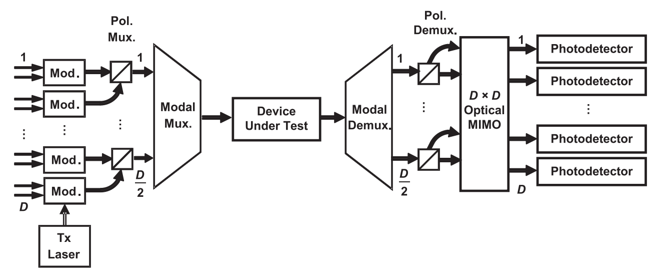

Fig. 7. Hardware setup for phase retrieval showing a laser, low-speed modulators, a multiplexer, a device under test, a demultiplexer, an optical MIMO device, and an array of measurement photodetectors. The optical MIMO device is optional and can be inserted to calibrate for a non-unitary demultiplexer and estimate output Schmidt modes.

B. Phase Retrieval-Based MDL Estimation

Phase retrieval is a method for reconstructing the full optical field from only intensity measurements at the receiver that was recently applied to enable direct-detection mode-division multiplexing over MMFs [26]. The same direct detection technique can be adapted to measure modal gains. As shown in Fig. 7, the hardware setup consists of a laser, a multiplexer with D single-mode inputs and a transfer matrix $\mathbf{M}_{\text{mux}}$, the DUT with unknown transfer matrix $\mathbf{M}$, a demultiplexer with D single-mode outputs and a transfer matrix $\mathbf{M}_{\text{demux}}$, and an array of D photodetectors. An optical MIMO device to perform $D \times D$ modal interferometry can be optionally inserted after the demultiplexer and prior to photodetection. The inclusion of the optical MIMO device allows for measuring Schmidt modes and for removing systematic MDL in the measurement setup, as will be discussed below.

As in the MDL ellipsoid estimation technique, a sequence of N known measurement vectors $\mathbf{x}^{(k)} \in \mathbb{C}^D,\ k = 1, \ldots, N$ are launched into the DUT. For each vector of modal amplitudes $\mathbf{x}^{(k)}$ that is transmitted, each of the D photodetectors measures a power

$$d_i^{(k)} = \left|\mathbf{\Gamma}_i \mathbf{x}^{(k)}\right|^2 + n_i^{(k)},\ i = 1, \ldots, D \tag{17}$$

where $\mathbf{\Gamma} = \mathbf{M}_{\text{demux}}\mathbf{M}\mathbf{M}_{\text{mux}}$ characterizes the combined effect of the multiplexer, demultiplexer and the DUT, $\mathbf{\Gamma}_i$ is the ith row of $\mathbf{\Gamma}$, and $n_i^{(k)}$ are uncorrelated thermal noises. Each row of $\mathbf{\Gamma}$ can be separately estimated using

$$\min\ \mathrm{tr}(\mathbf{G}_i) + \lambda \sum_{k=1}^{N} \left(\mathbf{x}^{(k)H}\mathbf{G}_i\mathbf{x}^{(k)} - d_i^{(k)}\right)^2 \text{ s.t. } \mathbf{G}_i \geq 0, \tag{18}$$

$$\hat{\mathbf{G}}_i = \sum_{j=1}^{D} \hat{\chi}_{i,j}\,\hat{\mathbf{u}}_{i,j}\,\hat{\mathbf{u}}_{i,j}^H, \tag{19}$$

$$\mathbf{\Gamma}_i = \sqrt{\hat{\chi}_{i,1}}\,\hat{\mathbf{u}}_{i,1}^T \tag{20}$$

for $i = 1,\ \ldots\ , D$. As explained in [26], λ is a regularization parameter and each row of $\mathbf{\Gamma}$ is sequentially estimated using PhaseLift [27] by solving (18) for an optimal matrix parameter $\hat{\mathbf{G}}_i$. Principal component analysis is then used to find its rank-one component. Once $\mathbf{\Gamma}$ is estimated from (18)–(20), the systematic MDL arising from the multiplexer and demultiplexer should be factored out. This is only possible in two cases: (i) $\mathbf{M}_{\text{demux}}$ is unitary, or (ii) optical MIMO interferometry is used prior to photodetection. The unitary requirement of $\mathbf{M}_{\text{demux}}$ arises because phase information is lost in direct detection, and $\mathbf{\Gamma}$ can only be estimated up to a diagonal unitary matrix multiplication from the left.

In other words, steps (18)–(20) can actually only provide an estimate

$$\tilde{\mathbf{\Gamma}} = \begin{pmatrix} e^{j\phi_1} & & 0 \\ & \ddots & \\ 0 & & e^{j\phi_D} \end{pmatrix} \mathbf{\Gamma} = \mathrm{diag}(\mathbf{\Phi})\mathbf{\Gamma} \tag{21}$$

where $\mathrm{diag}(\mathbf{\Phi})$ is a diagonal matrix of unknown phase shifts. Since we do not know this matrix exactly, we cannot multiply $\mathbf{\Gamma}$ by the inverses of $\mathbf{M}_{\text{mux}}$ and $\mathbf{M}_{\text{demux}}$ to recover $\mathbf{M}$ and calculate the modal gains. On the other hand, if $\mathbf{M}_{\text{demux}}$ is unitary, then the estimation of modal gains is invariant to the unknown phase shifts, and we can calculate the modal gains as

$$\begin{aligned} \mathbf{g} &= \gamma \log \mathrm{eig}\left(\left(\mathbf{\Gamma}\mathbf{M}_{\text{mux}}^{-1}\right)^H \left(\mathbf{\Gamma}\mathbf{M}_{\text{mux}}^{-1}\right)\right) \\ &= \gamma \log \mathrm{eig}\left(\left(\mathbf{M}_{\text{demux}}\mathbf{M}\right)^H \left(\mathbf{M}_{\text{demux}}\mathbf{M}\right)\right) \\ &= \gamma \log \mathrm{eig}\left(\mathbf{M}^H\mathbf{M}\right). \end{aligned} \tag{22}$$

If $\mathbf{M}_{\text{demux}}$ is not a unitary matrix, then it is impossible to isolate $\mathbf{M}$ unless modal interferometry is performed, as in case (ii).

The operation of modal interferometry in case (ii) using optical MIMO is explained in more detail as follows. To resolve ambiguity in $\mathrm{diag}(\mathbf{\Phi})$ and therefore estimate $\mathbf{\Gamma}$ exactly, we solve another convex semidefinite program after interfering the outputs of demultiplexer with MIMO equalizer masks. The optical MIMO can be physically implemented in integrated optics using a mesh of Mach-Zehnder interferometers [28]. As described in further detail in [26], the estimation problem becomes

$$\min \mathrm{tr}(\mathbf{K}) + \lambda \sum_{n} \sum_{d=1}^{D} \left(\psi_d^{(n)H}\mathbf{K}\psi_d^{(n)} - d_d^{(n)}\right)^2 \text{ s.t. } \mathbf{K} \geq 0 \tag{23}$$

$$\hat{\mathbf{K}} = \sum_{k=1}^{D} \hat{\chi}_k\,\hat{\mathbf{u}}_k\,\hat{\mathbf{u}}_k^H \tag{24}$$

$$\mathbf{\Phi} \leftarrow \sqrt{\hat{\chi}_1}\,\hat{\mathbf{u}}_1^*. \tag{25}$$

Since phase retrieval recovers the DUT's transfer matrix $\mathbf{M}$, it is possible to compute the DUT's crosstalk matrix. Fig. 8. shows the estimation of the modal gains for a DUT with $D = 4$ modes and 100 measurement vectors at varying SNRs. The transfer matrix of the DUT and its true modal gains are determined using the same procedure as in Fig. 6. The regularization parameter is chosen to be $\gamma = 10^{-3}$. As expected, the estimated modal gains converge to the true values as the measurement SNR is increased.

C. Comparison of Proposed MDL Estimation Algorithms

Fig. 9 summarizes the relative performance of the MDL ellipsoid method and the phase retrieval-based MDL estimation method in a variety of settings. We consider the performance of both algorithms to measure MDL for a random ensemble of 30 DUTs with low rms MDL (1.0 dB), medium rms MDL (2.5 dB), and high rms MDL (5.0 dB) using different numbers of measurement vectors ($N = 50 - 150$). Each DUT was numerically simulated by creating a random 4×4 matrix and scaling the modal gains to match the desired rms MDL value. In general, using more measurement vectors to perform MDL estimation reduces the average absolute error in the estimated rms MDL for both techniques because the algorithms reconstruct a more accurate MDL operator for each DUT. For a fixed N, the algorithms perform worse when the DUT's built-in rms MDL is increased. This is to be expected because the high-dimensional MDL landscape becomes highly skewed in regions of highest and lowest MDL, and there is a lower probability that measurement vectors corresponding to these regions are launched. Interestingly, the phase retrieval method is more robust than the MDL ellipsoid method for measuring DUTs with high MDL. We believe this is the case because the phase retrieval measurement makes D times as many measurements as the MDL ellipsoid method and is able to recover more accurate modal gains.

In Fig. 10, we study the relative performance of our proposed algorithms when the number of modes D is varied from 2 to 10. At each test point, we numerically simulate a random en-

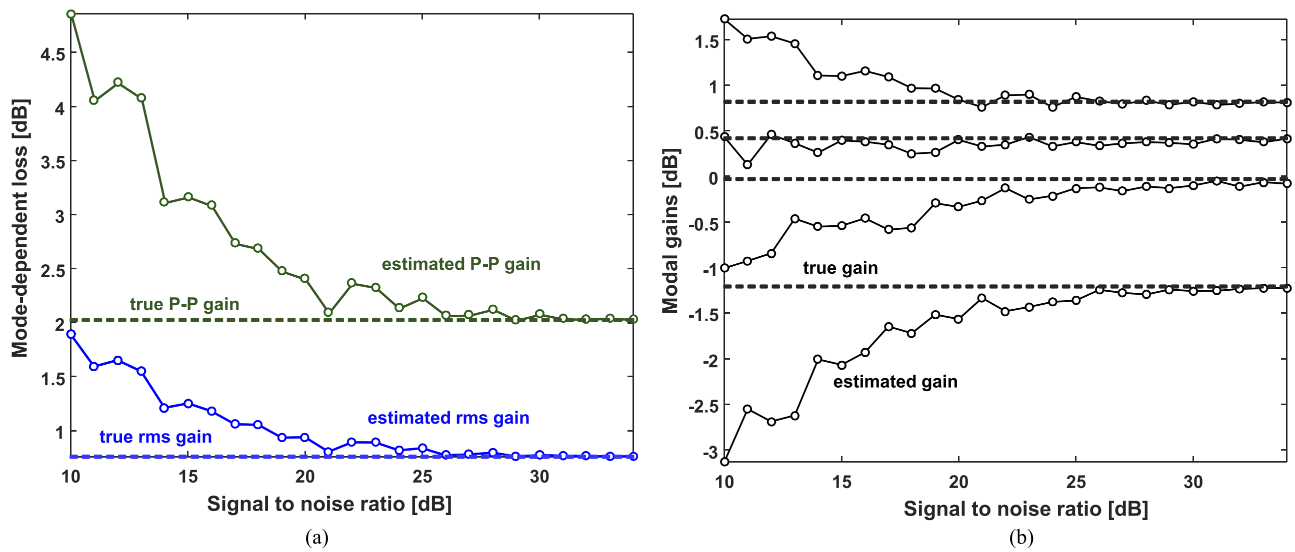

Fig. 8. Phase retrieval is used to estimate the MDL of a typical DUT supporting $D = 4$ modes using 100 random measurement vectors. The estimation is performed independently at each SNR point using noisy power measurements. (a) Both the estimated P-P and rms MDL values converge to their true values as the measurement SNR increases. (b) All $D = 4$ modal gains can be accurately estimated using the algorithm.

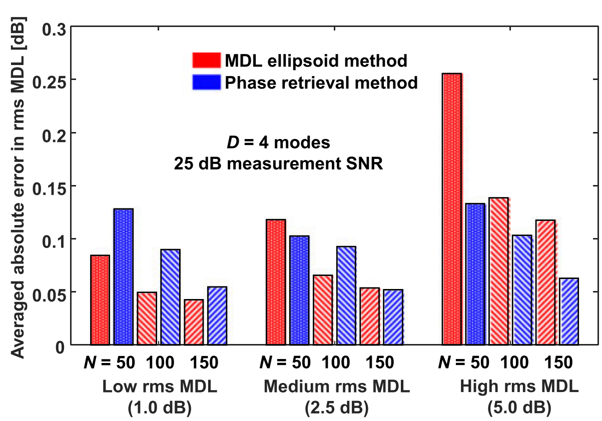

Fig. 9. Accuracy of the MDL ellipsoid method and the phase retrieval method for $D = 4$ modes in the low, medium, and high rms MDL regimes when estimation is performed using $N = 50, 100$ and 150 measurement vectors. At each MDL test point, 30 independent DUTs are realized and their modal gains are scaled to match the desired rms MDL. The average absolute errors between the actual and estimated rms MDL are plotted.

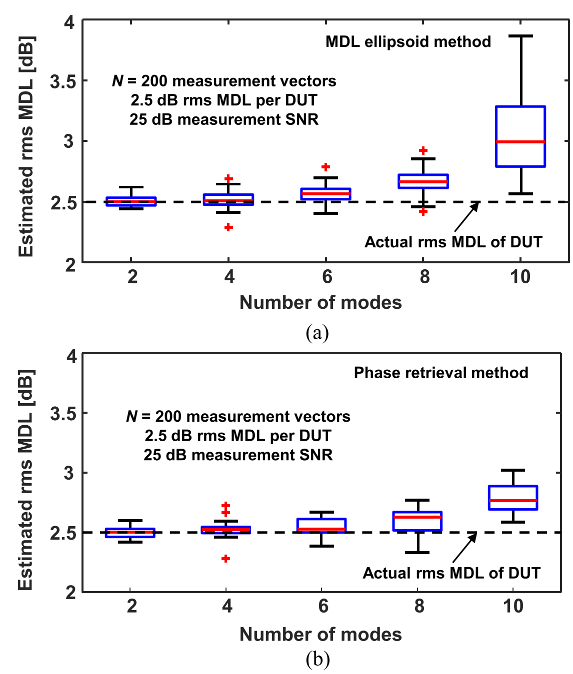

Fig. 10. Box plots of the estimated rms MDL using (a) the MDL ellipsoid method and (b) the phase retrieval method when the number of modes of the DUT is varied from 2 to 10. Both methods used $N = 100$ measurement vectors at a SNR of 25 dB. On each box, the central red mark indicates the distribution's median, and the bottom and top edges of the box indicate the 25th and 75th percentiles, respectively. The whiskers extend to the most extreme data points not considered outliers, and the outliers are plotted individually using red plus signs.

semble of 30 DUTs with actual rms MDL of 2.5 dB. $N = 200$ measurement vectors are used to estimate the rms MDL with the MDL ellipsoid method and phase retrieval method at a measurement SNR of 25 dB. The box plot in Fig. 10(a) shows that the median estimated rms MDL is within 0.1 dB of the actual rms MDL for up to $D = 6$ modes, but quickly becomes inaccurate for $D \geq 8$ modes. This suggests that N must be scaled as D is increased to retain measurement accuracy. Fig. 10(b) shows that the median estimated rms MDL recovered by the phase retrieval method is roughly within 0.2 dB for $D \leq 10$ modes. The phase retrieval method can provide more accurate MDL estimates than the MDL ellipsoid method for the same number of measurement vectors (or equivalently, the same accuracy using fewer measurement vectors), albeit at the cost of slightly higher hardware and computational complexity.

IV. Conclusion

MDL is a fundamental impairment affecting the capacity of MDM systems. It is not only critically important to measure the MDL of components to assess its impact on system performance, it is equally important to use proper metrics to quantify its impact. In the regime of full, random mode coupling, the rms modal gain is more important than the P-P gain, because the former completely characterizes MDL statistics.

In many MDL measurement applications, it may be preferable to use estimation techniques that require low hardware complexity, possibly at the expense of higher computational complexity. Low-complexity techniques that can measure the rms gain, P-P gain, individual modal gains, Schmidt modes, and the crosstalk matrix of components are especially desired. As shown in this paper, it is possible to estimate MDL of components using direct detection methods, forgoing the complex hardware of coherent detection. The novel convex optimization-based algorithms described in Section III are promising MDL estimation methods because they satisfy these requirements.

References

[1] P. J. Winzer, "Energy-efficient optical transport capacity scaling through spatial multiplexing," *IEEE Photon. Technol. Lett.*, vol. 23, no. 13, pp. 851–853, Jul. 2011.

[2] G. Li, N. Bai, N. Zhao, and C. Xia, "Space-division multiplexing: The next frontier in optical communication," *Adv. Opt. Photon.*, vol. 6, pp. 413–487, 2014.

[3] S. Ö. Arık and J. M. Kahn, "Diversity-multiplexing tradeoff in mode-division multiplexing," *Opt. Lett.*, vol. 39, pp. 3258–3261, 2014.

[4] E. Ip, G. Milione, Y.-K. Huang, and T. Wang, "Impact of mode-dependent loss on long-haul transmission systems using few-mode fibers," in *Proc. Opt. Fiber Commun. Conf. Exhib.*, 2016.

[5] P. J. Winzer and G. J. Foschini, "MIMO capacities and outage probabilities in spatially multiplexed optical transport systems," *Opt. Express*, vol. 19, pp. 16680–16696, 2011.

[6] Agilent Technologies, "Measuring polarization dependent loss of passive optical components," Agilent Technologies, Santa Clara, CA, USA, Appl. note, 2008. [Online]. Available: http://literature.cdn.keysight.com/litweb/pdf/5988-1232EN.pdf

[7] S. Ö. Arık, D. Askarov, and J. M. Kahn, "Adaptive frequency-domain equalization in mode-division multiplexing systems," *J. Lightw. Technol.*, vol. 32, no. 10, pp. 1841–1852, May 2014.

[8] S. O. Arik, J. M. Kahn, and K. P. Ho, "MIMO signal processing for mode-division multiplexing: An overview of channel models and signal processing architectures," *IEEE Signal Process. Mag.*, vol. 31, no. 2, pp. 25–34, Mar. 2014.

[9] S. Warm and K. Petermann, "Splice loss requirements in multi-mode fiber mode-division-multiplex transmission links," *Opt. Express*, vol. 21, pp. 519–532, 2013.

[10] T. Duthel, C. R. S. Fludger, J. Geyer, and C. Schulien, "Impact of polarisation dependent loss on coherent POLMUX-NRZ-DQPSK," in *Proc. Conf. Opt. Fiber Commun./Nat. Fiber Opt. Engineers Conf.*, 2008, pp. 1–3.

[11] N. Cvijetic, E. Ip, N. Prasad, M.-J. Li, and T. Wang, "Experimental time and frequency domain MIMO channel matrix characterization versus distance for 6×28 Gbaud QPSK transmission over 40×25 km few mode fiber," in *Proc. Opt. Fiber Commun. Conf. Exhib.*, 2014, pp. 1–3.

[12] R. Ryf *et al.*, "Combined SDM and WDM transmission over 700-km few-mode fiber," in *Proc. Conf. Opt. Fiber Commun./Nat. Fiber Opt. Engineers Conf.*, 2013, pp. 1–3.

[13] N. K. Fontaine and R. Ryf, "Characterization of mode-dependent loss of laser inscribed photonic lanterns for space division multiplexing systems," in *Proc. 18th OptoElectron. Commun. Conf. held jointly with Int. Conf. Photon. Switching*, 2013, pp. 1–2.

[14] A. Andrusier, M. Shtaif, C. Antonelli, and A. Mecozzi, "Assessing the effects of mode-dependent loss in space-division multiplexed systems," *J. Lightw. Technol.*, vol. 32, no. 7, pp. 1317–1322, Apr. 2014.

[15] M. B. Shemirani, W. Mao, R. A. Panicker, and J. M. Kahn, "Principal modes in graded-index multimode fiber in presence of spatial- and polarization-mode coupling," *J. Lightw. Technol.*, vol. 27, no. 10, pp. 1248–1261, May 2009.

[16] K. P. Ho and J. M. Kahn, "Statistics of group delays in multimode fiber with strong mode coupling," *J. Lightw. Technol.*, vol. 29, no. 21, pp. 3119–3128, Nov. 2011.

[17] K.-P. Ho and J. M. Kahn, "Mode-dependent loss and gain: Statistics and effect on mode-division multiplexing," *Opt. Express*, vol. 19, pp. 16612–16635, 2011.

[18] R. Ryf *et al.*, "Mode-division multiplexing over 96 km of few-mode fiber using coherent 6×6 MIMO processing," *J. Lightw. Technol.*, vol. 30, no. 4, pp. 521–531, Feb. 2012.

[19] K. P. Ho and J. M. Kahn, "Linear propagation effects in mode-division multiplexing systems," *J. Lightw. Technol.*, vol. 32, no. 4, pp. 614–628, Feb. 2014.

[20] P. J. Winzer, H. Chen, R. Ryf, K. Guan, and S. Randel, "Mode-dependent loss, gain, and noise in MIMO-SDM systems," in *Proc. Eur. Conf. Opt. Commun.*, 2014, pp. 1–3.

[21] C. Antonelli, A. Mecozzi, M. Shtaif, and P. J. Winzer, "Modeling and performance metrics of MIMO-SDM systems with different amplification schemes in the presence of mode-dependent loss," *Opt. Express*, vol. 23, pp. 2203–2219, 2015.

[22] K. P. Ho, "Exact model for mode-dependent gains and losses in multimode fiber," *J. Lightw. Technol.*, vol. 30, no. 23, pp. 3603–3609, Dec. 2012.

[23] K. P. Ho and J. M. Kahn, "Frequency diversity in mode-division multiplexing systems," *J. Lightw. Technol.*, vol. 29, no. 24, pp. 3719–3726, Dec. 2011.

[24] M. E. Muller, "A note on a method for generating points uniformly on N-dimensional spheres," *Commun. Assoc. Comput. Mach.*, vol. 2, pp. 19–20, 1959.

[25] M. Grant and S. Boyd (2014). *CVX: Matlab Software for Disciplined Convex Programming, Version 2.1*.

[26] S. Ö. Arik and J. M. Kahn, "Direct-detection mode-division multiplexing in modal basis using phase retrieval," *Opt. Lett.*, vol. 41, pp. 4265–4268, 2016.

[27] E. J. Candès, T. Strohmer, and V. Voroninski, "PhaseLift: Exact and stable signal recovery from magnitude measurements via convex programming," *Commun. Pure Appl. Math.*, vol. 66, pp. 1241–1274, 2013.

[28] D. A. B. Miller, "Self-aligning universal beam coupler," *Opt. Express*, vol. 21, pp. 6360–6370, 2013.

Karthik Choutagunta (S'11) received the B.S. degree from The University of Texas at Austin, Austin, TX, USA, in 2014, and the M.S. degree in electrical engineering from Stanford University, Stanford, CA, USA, in 2016, where he is currently working toward the Ph.D. degree. He worked with Coherent Logix, Austin, TX, USA, in summer 2012, with Intel Corp., Austin, TX, USA, from 2012 to 2014, with Apple, Inc., Cupertino, CA, USA, in fall 2015 and summer 2016, and with Facebook, Inc., Menlo Park, CA, USA, in summer 2017. His main current research interests include various topics in signal processing, artificial intelligence, and optimization.

Sercan Ö. Arik received the B.S. degree from Bilkent University, Ankara, Turkey, in 2011, and the M.S. and Ph.D. degrees in electrical engineering from Stanford University, Stanford, CA, USA, in 2013 and 2016, respectively. He is currently a Research Scientist with Baidu Silicon Valley Artificial Intelligence Lab, Sunnyvale, CA, USA. He has co-authored more than 25 journal and conference papers. His main current research interests include applications of signal processing, machine learning, optimization, and statistics.

Keang-Po Ho, biography not available at the time of publication.

Joseph M. Kahn (M'90–SM'98–F'00) received the A.B., M.A., and Ph.D. degrees in physics from the University of California at Berkeley (U.C. Berkeley), Berkeley, CA, USA, in 1981, 1983, and 1986, respectively. From 1987 to 1990, he was with AT&T Bell Laboratories, Crawford Hill Laboratory, Holmdel, NJ, USA. He demonstrated multi-Gbit/s coherent optical fiber transmission systems, setting world records for receiver sensitivity. From 1990 to 2003, he was with the Faculty of the Department of Electrical Engineering and Computer Sciences, U.C. Berkeley, performing research on optical and wireless communications. Since 2003, he has been a Professor of electrical engineering at Stanford University, Stanford, CA, USA, where he is the Head of the Optical Communications Group. His current research interests include fiber-based imaging, spatial multiplexing, rate-adaptive and spectrally efficient modulation and coding methods, coherent detection and associated digital signal processing algorithms, digital compensation of fiber nonlinearity, and free-space systems. He was the recipient of the National Science Foundation Presidential Young Investigator Award in 1991. From 1993 to 2000, he served as a Technical Editor for IEEE PERSONAL COMMUNICATIONS MAGAZINE. Since 2009, he has been an Associate Editor for IEEE/OSA JOURNAL OF OPTICAL COMMUNICATIONS AND NETWORKING. In 2000, he helped found StrataLight Communications, where he served as a Chief Scientist from 2000 to 2003. StrataLight was acquired by Opnext, Inc., in 2009.