Low-Loss Fiber Bragg Grating Mode Scramblers Exploiting Propagation Constant Engineering

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Outline

- Multimode fiber design by refractive index optimization
- Integrated spatial-division-multiplexed links
- Mode scrambler design
- Mode scrambler performance
- Impact of fabrication errors
- Conclusions

Research supported by Ciena Corporation

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Gradient-Descent Refractive Index Optimization

- Assume N guided spatial modes. Taylor series expansion of propagation constants about $\omega = \omega_0$:

$$\beta^{(i)}(\omega) \approx \beta_0^{(i)} + \beta_1^{(i)}(\omega - \omega_0) + \tfrac{1}{2}\beta_2^{(i)}(\omega - \omega_0)^2 + \cdots \quad \text{for } i = 1, \ldots, N$$

propagation constant · group delay · chromatic dispersion

Propagation constant optimization

- Desired propagation constants: $\beta_{0,\,\text{des}}^{(1)}, \ldots, \beta_{0,\,\text{des}}^{N}$
- Objective function: $J = \sum_{i=1}^{N} \left(\beta_0^{(i)} - \beta_{0,\,\text{des}}^{(i)} \right)^2$
- Gradient descent-based refractive index optimization update:

$$n(x,y) \leftarrow n(x,y) - \eta' \frac{\partial J}{\partial n(x,y)}$$

becomes

$$n(x,y) \leftarrow n(x,y) \times \left[1 - \eta \sum_{i=1}^{N} \left(\frac{\beta_0^{(i)} - \beta_{0,\,\text{des}}^{(i)}}{\beta_0^{(i)}} \right) \left\| \psi^{(i)}(x,y) \right\|^2 \right]$$

Index update at each (x,y) is sum of modal intensities $\|\psi^{(i)}(x,y)\|^2$ weighted by relative errors $(\beta_0^{(i)} - \beta_{0,\text{des}}^{(i)}) / \beta_0^{(i)}$.

- Use mode solver to compute mode fields $\psi^{(i)}(x,y)$ and propagation constants $\beta_0^{(i)}$.
- Often assume axially symmetric index profile $n(x,y) = n(r)$.

Group delay or chromatic dispersion optimization

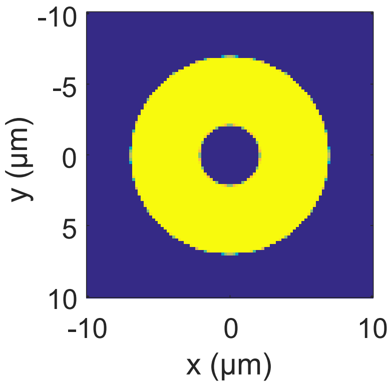
- See paper.

K. Choutagunta and J. M. Kahn, *J. Lightw. Technol.* **39** (2021).

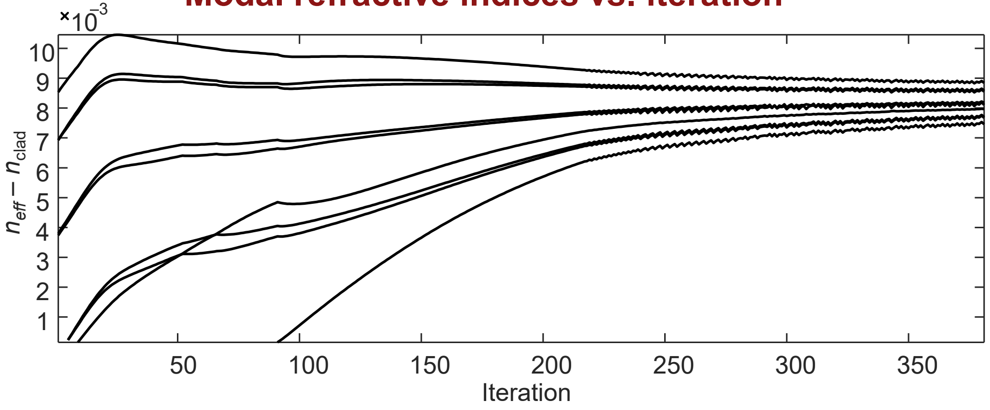
Example: Ring-Core to Coupled Multi-Core Transformation

- Start with 5-mode ring-core fiber. Optimize index profile to push propagation constants closer together.
- Fiber transformed into 9-core coupled multi-core fiber, which has 9 modes with roughly equal propagation constants.
- 8 of the 9 cores coincide with locations where the ring-core fiber modes have high intensities.

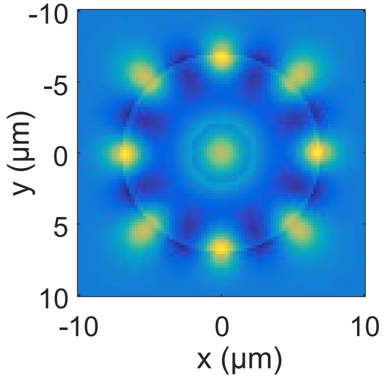
Initial index profile



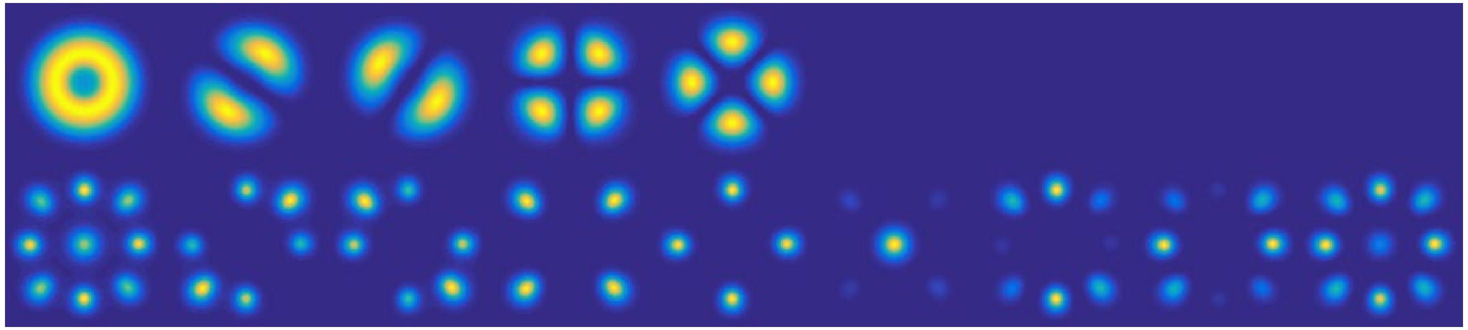
Modal refractive indices vs. iteration



Final index profile



5 initial modes



9 final modes

K. Choutagunta and J. M. Kahn, *J. Lightw. Technol.* **39** (2021).

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Spatial-Division Multiplexing in Submarine Cables

- In cables longer than ~5000 km, the electrical power available for the optical amplifiers is limited by:
  - Resistance of copper conductors: 1 Ω/km.
  - Feed voltage: $\pm$18 kV.

- Given a fixed total power, to maximize the cable capacity, one should:
  - Increase the number of parallel spatial channels S (typically single-mode fibers).
  - Transmit less power and data in each channel.

- Follows from Shannon's capacity formula

$$C = 2 \cdot S \cdot W \cdot \log_2 \left(1 + \frac{P_{signal} / S}{P_{noise}} \right) \quad \left(\frac{\text{b}}{\text{s}} \right),$$

which is linear in S and logarithmic in $1/S$.

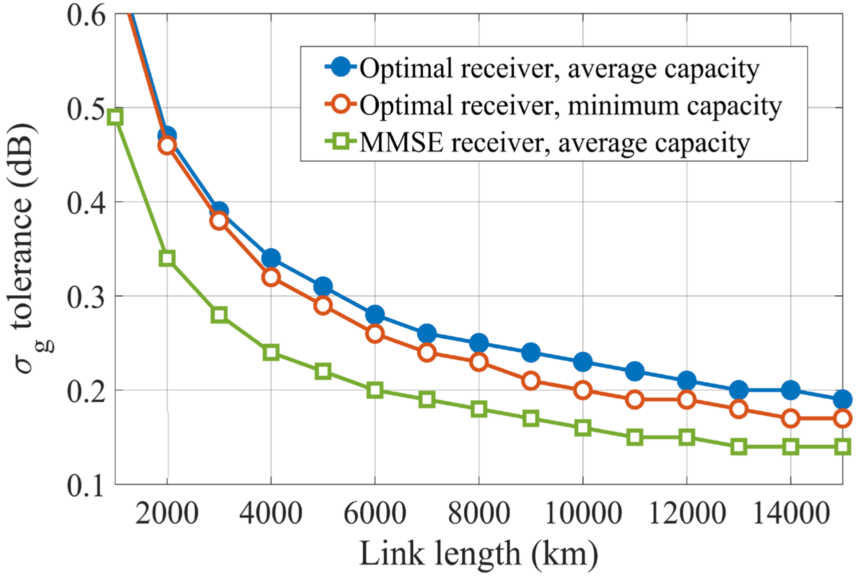
S	number of spatial channels
C	channel capacity (b/s)
W	bandwidth (Hz)
P_{signal}	total signal power over S spatial channels (W)
P_{noise}	noise power per spatial channel (W)

SDM Fibers and Amplifiers for Submarine Links

Economical scaling

- Should replace separate single-mode fibers by integrated multi-mode or multi-core fibers.
- To obtain a clear integration advantage, want inline amplifiers that:
  - Do not require fan out / fan in.
  - Use number of pump modes < number of signal modes.

For 1-dB capacity penalty, require MDG STD
$\sigma_g \approx 0.1 - 0.2$ dB per span, assuming 7 spatial modes [1].



Critical requirements

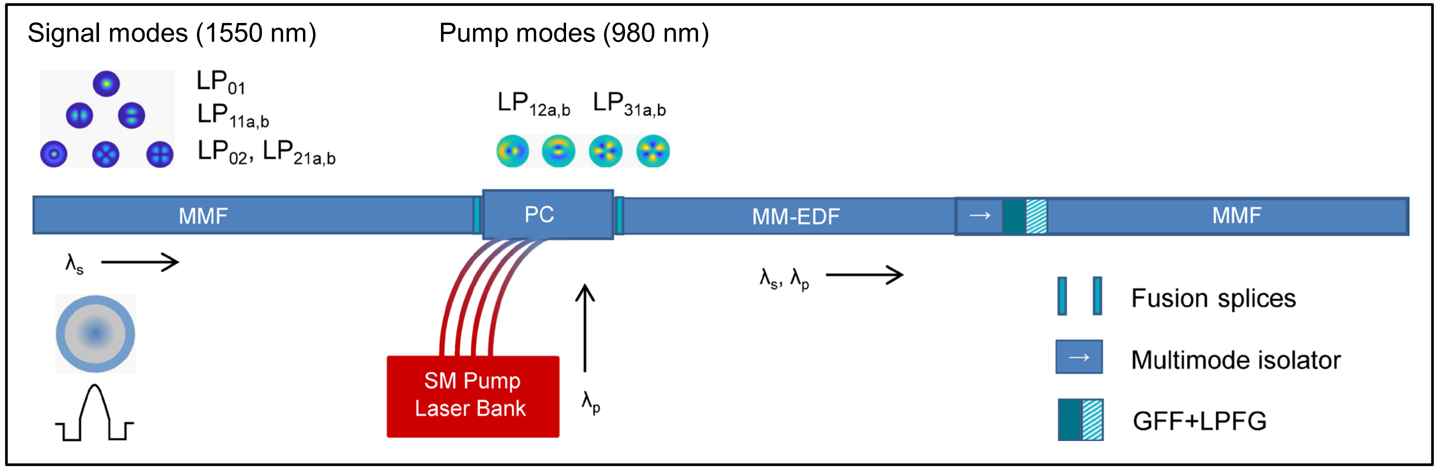
- Fibers: ultra-low loss, low group delay spread.
- Amplifiers: low noise, high efficiency, low mode-dependent gain.
- Full random coupling between all modes in each span:
  - Can occur in fibers (e.g., coupled-core multi-core fibers provide full strong coupling).
  - Can be enhanced by mode scramblers (typical multi-mode fibers provide strong intra-group but weak inter-group coupling).

1. D. A. A. Mello, *et al.*, *J. Lightw. Technol.* **38** (2020).

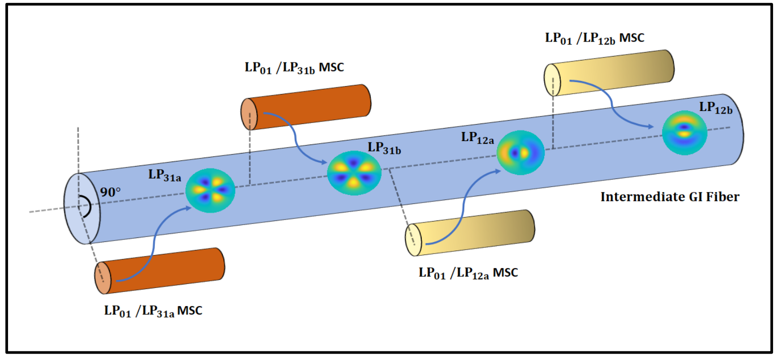
Scalable Multimode Amplifiers Designed

- Amplifies 6 signal modes (replacing 6 single-mode amplifiers) using only 4 pump modes.
- Promises ~30% conversion efficiency and ~0.1 dB mode-dependent gain STD.

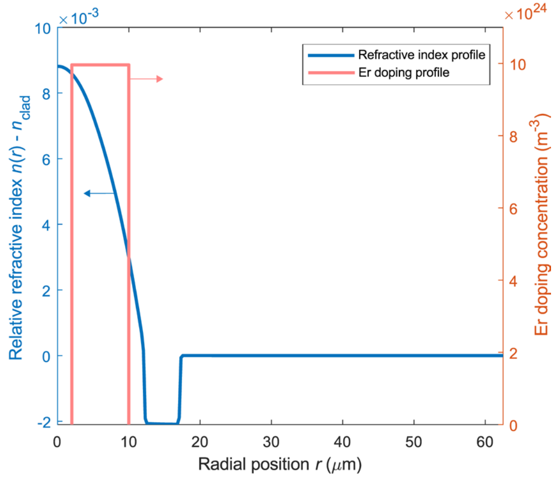
Erbium-doped fiber amplifier subsystem



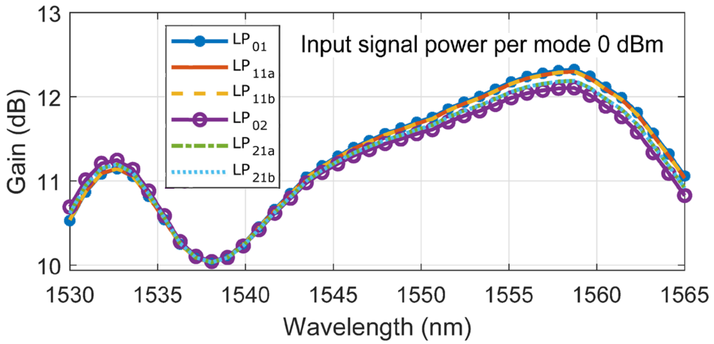
Mode- and wavelength-selective pump coupler



Index and Er doping profiles



Mode- and wavelength-dependent gain



H. Srinivas, *et al.*, *J. Lightw. Technol.* **41** (2023).

Remaining Challenges

- 6-mode fiber with low group delay spread
  - Currently designing pairs of graded-index fibers for group delay compensation.
- 6-mode mode scrambler providing strong scrambling and MDL STD $\sigma_g < 0.1$ dB.
  - This work.

Previous Mode Scramblers

Approach	Number of Spatial Modes	Strong Scrambling	Mode-Dependent Loss STD (dB)	Mode-Average Loss (dB)	Expt. Demo.	Reference
Chirp-optimized long-period grating in graded-index fiber	6	Y	0.36	0.45	N	D. Askarov and J. M. Kahn, *J. Lightw. Technol.* **33** (2015).
Uniform long-period grating in graded-index fiber supporting more modes	15	N	0.06	0.05	N	H. Liu *et al.*, *Scientific Reports* **8** (2018).
CO_2-laser-inscribed long-period grating in step-index fiber spliced to graded-index fiber	6	Y	4	0.5	Y	Y. Zhao *et al.*, *Optics Letters* **43** (2018).
MPLC-based demultiplexer-multiplexer combination	10	Y	1	0.8	Y	J. Li, *et al.*, *Proc. of ECOC* (2018).
3D-printed mechanical long-period grating	3	Y	0.1	0.2	Y	X. Huang, *et al.*, *Photon. Technol. Lett.* **34** (2022).
MPLC-based demultiplexer-multiplexer combination	15	Y	2	2	Y	M. V. D. Hout, *et al.*, *J. Lightw. Technol.* **42** (2024).
Uniform long-period grating in free-form-optimized graded-index fiber	6	Y	0.011	0.027	N	This work.

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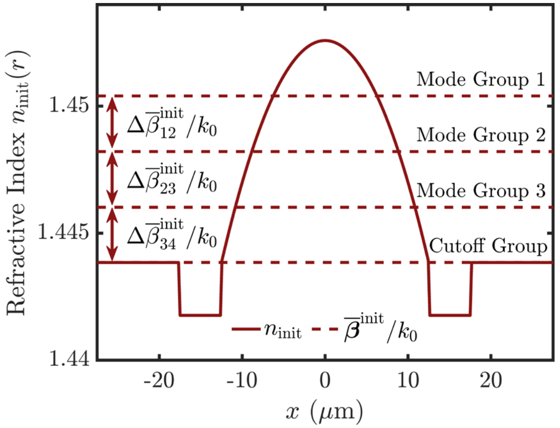
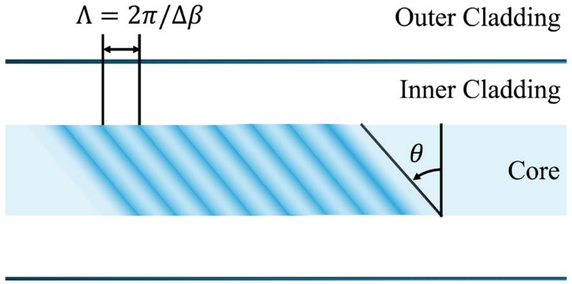
General Design Approach

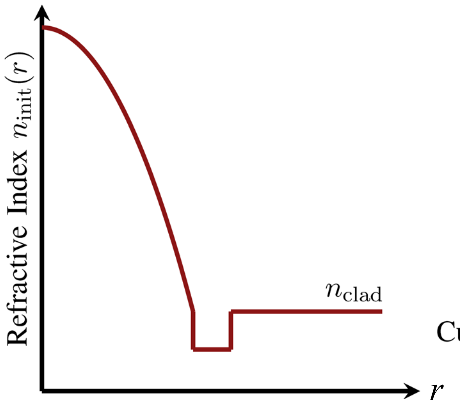
- Given an initial 6-mode GI fiber, to couple mode groups 1, 2, 3, want to choose a tilted grating with period $\Lambda \approx \dfrac{2\pi}{\Delta\overline{\beta}_{12}^{\text{init}}} \approx \dfrac{2\pi}{\Delta\overline{\beta}_{23}^{\text{init}}}$.
- Problem: since $\Delta\overline{\beta}_{34}^{\text{init}} \approx \Delta\overline{\beta}_{12}^{\text{init}} \approx \Delta\overline{\beta}_{23}^{\text{init}}$ the grating will also couple to cutoff mode group 4.
- Solution: modify to a desired 6-mode GI profile such that $\Delta\overline{\beta}_{12}^{\text{des}} \approx \Delta\overline{\beta}_{23}^{\text{des}}$ but $\Delta\overline{\beta}_{34}^{\text{des}} \gg \Delta\overline{\beta}_{12}^{\text{des}}, \Delta\overline{\beta}_{23}^{\text{des}}$.

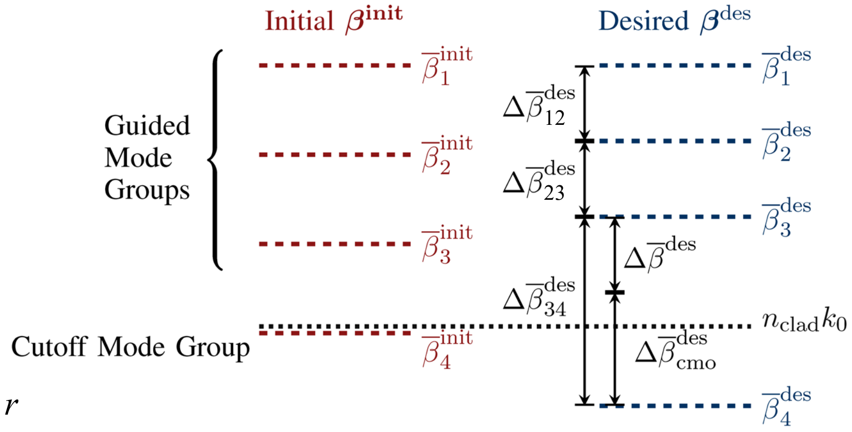
Initial 6-mode graded-index fiber



Bragg grating







O. Krutko, *et al.*, to appear in *J. Lightw. Technol.* (2024).

Transverse Refractive Index Profile Optimization

- To find an axially symmetric transverse index profile $n(r)$, define an objective function

$$J = \sum_{i \in M_{\text{guided}}} \left(\beta_i - \beta_i^{\text{des}}\right)^2 + w_0 \sum_{i \in M_{\text{cutoff}}} \left(\text{Re}\left\{\beta_i - \beta_i^{\text{des}}\right\}\right)^2$$

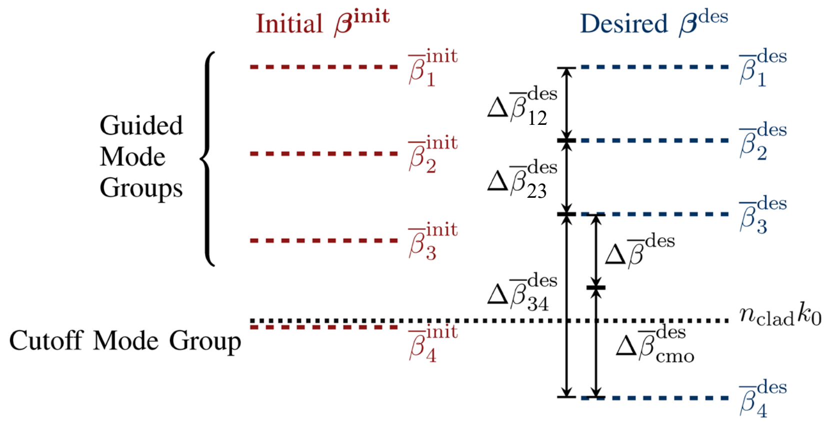
and iteratively update $n(r)$ using

$$n(r) \leftarrow n(r) - \mu \frac{\partial J}{\partial n(x,y)}.$$

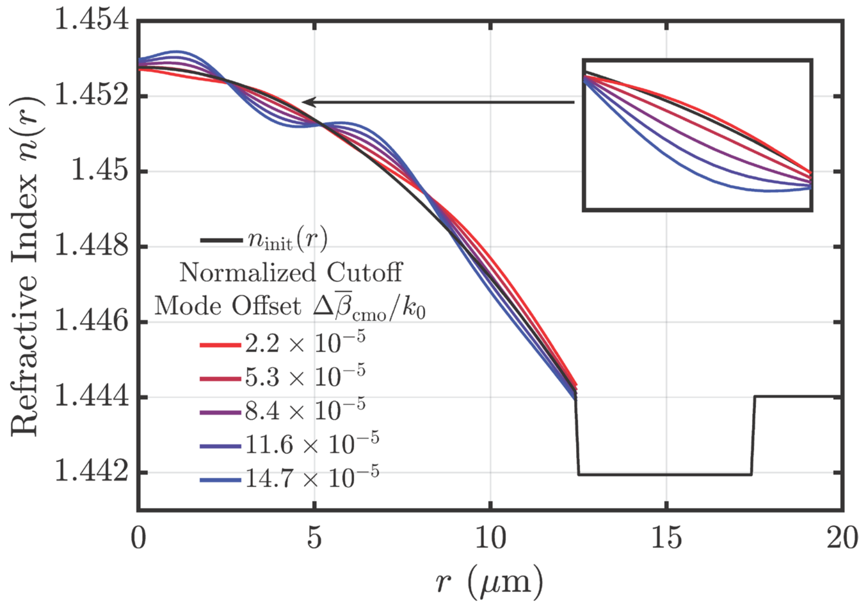
- After each iteration, smooth $n(r)$ using a Gaussian filter.

- Define a cutoff mode offset:

$$\Delta\overline{\beta}_{\text{cmo}}^{\text{des}} = \Delta\overline{\beta}_{34}^{\text{des}} - \frac{1}{2}\left(\Delta\overline{\beta}_{12}^{\text{des}} + \Delta\overline{\beta}_{23}^{\text{des}}\right).$$



Optimized 6-mode graded-index fiber



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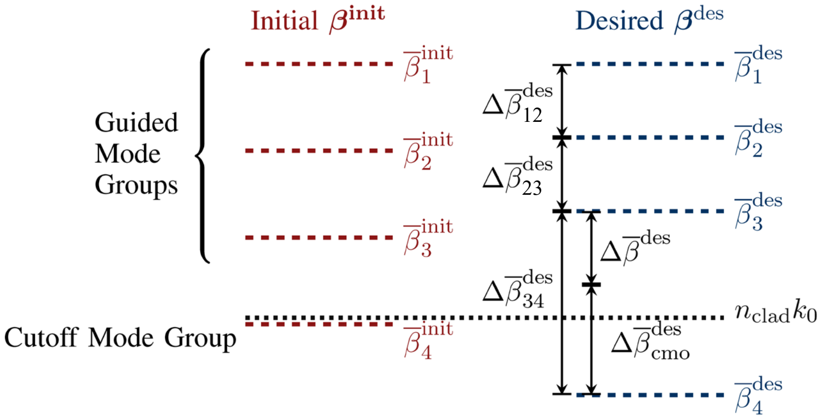
Understanding Transverse Refractive Index Optimization

Early iterations (0 to 60)

- Index updates increase $\overline{\beta}_2$ and $\overline{\beta}_3$ to make $\Delta\overline{\beta}_{12}$ and $\Delta\overline{\beta}_{23}$ equal, but smaller than their initial values $\Delta\overline{\beta}_{12}^{\text{init}}$ and $\Delta\overline{\beta}_{23}^{\text{init}}$.

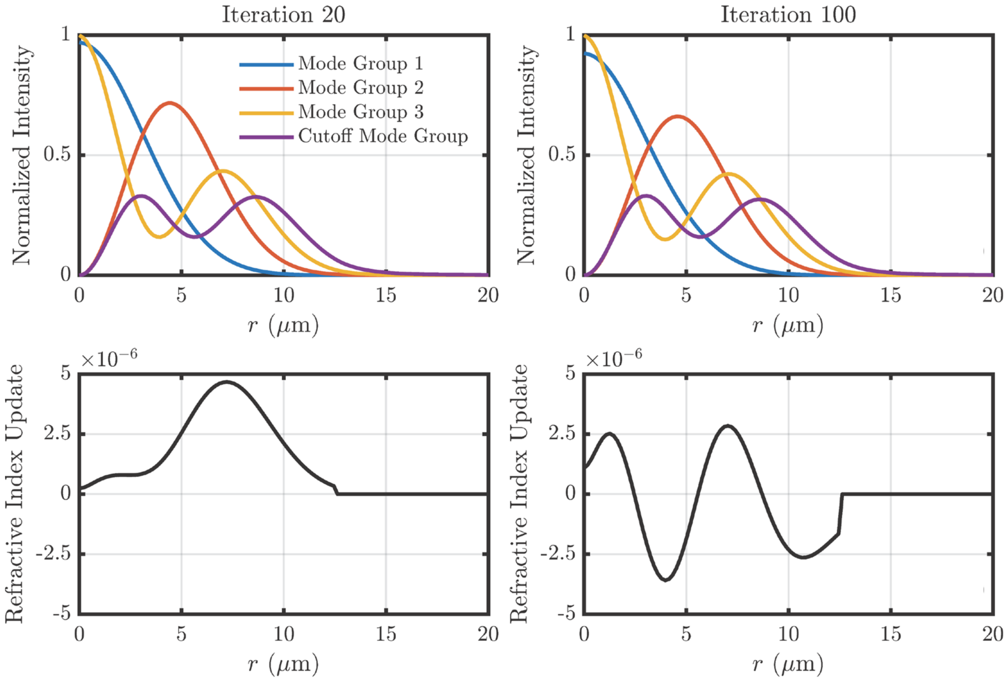
Late iterations (60 and up)

- Index updates decrease $\overline{\beta}_4$ to make $\Delta\overline{\beta}_{34}$ larger than $\Delta\overline{\beta}_{12}$ and $\Delta\overline{\beta}_{23}$.



Mode group intensity profiles

Index profile updates



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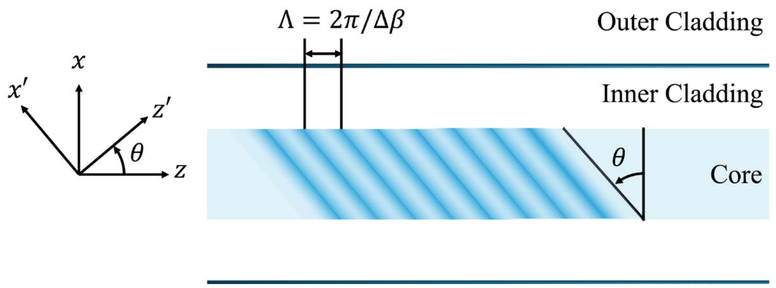
Longitudinal Refractive Index Profile Optimization



- Grating index modulation modeled as uniform and confined to core:

$$\Delta n_{\text{grating}}(x,y,z) = 2\chi\left(1+\cos\left(\frac{2\pi}{\Lambda\cos\theta}z'\right)\right)$$

- Objective function:

$$\sum_{\lambda}\left[w_1\Phi\left(\frac{\sigma_{\text{GD},\lambda}(K)}{\sqrt{K}\sigma_{\text{GD},\lambda}(1)}\right)+w_2\Phi\left(\frac{\sigma_{\text{MDL},\lambda}(K)}{\sqrt{K}\sigma_{\text{MDL},\lambda}(1)}\right)+w_3\sigma_{\text{MDL},\lambda}^{\text{MS}}+w_4\alpha_{\text{MAL},\lambda}^{\text{MS}}\right]$$

- The function

$$\Phi(x)=\begin{cases}0 & x\le 1\\ 50(x-1)^2 & x>1\end{cases}$$

penalizes designs that do not yield accumulation of GD and MDL with $\sqrt{K}$.

- Scrambler modeled using coupled mode theory at 3 wavelengths λ = 1530, 1550, 1565 nm.
- Scrambler MDL STD $\sigma_{\text{MDL},\lambda}^{\text{MS}}$ and MAL $\alpha_{\text{MAL},\lambda}^{\text{MS}}$ evaluated as explained below.
- Link GD STD $\sigma_{\text{GD},\lambda}(K)$ and $\sigma_{\text{MDL},\lambda}(K)$ can be evaluated using multi-section model (see below) or using closed-form expressions involving scrambler transfer matrix $\mathbf{R}$ [2].

Grid search optimization parameters

χ	modulation depth	coarse grid
Λ	grating period	fine grid
L	grating length	fine grid
θ	grating tilt angle	fixed at 85º

1. O. Krutko, *et al.*, to appear in *J. Lightw. Technol.* (2024).
2. A. Vijay, *et al.*, to appear in *J. Lightw. Technol.* (2024).

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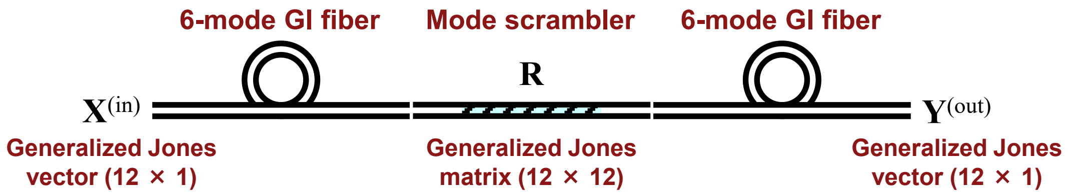
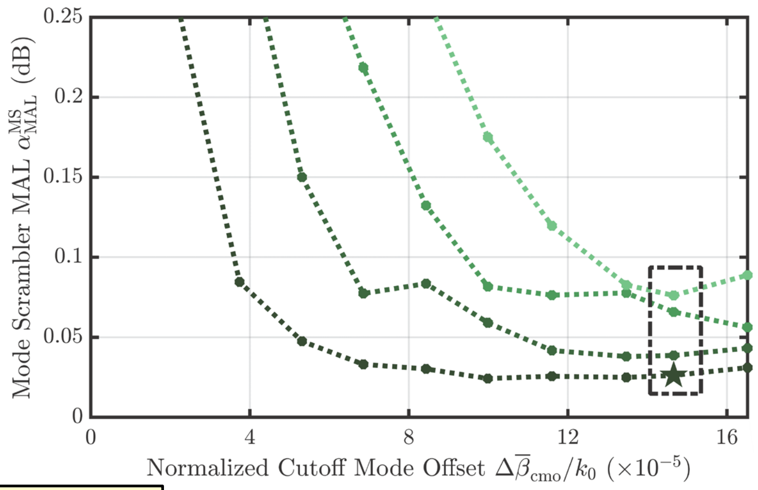
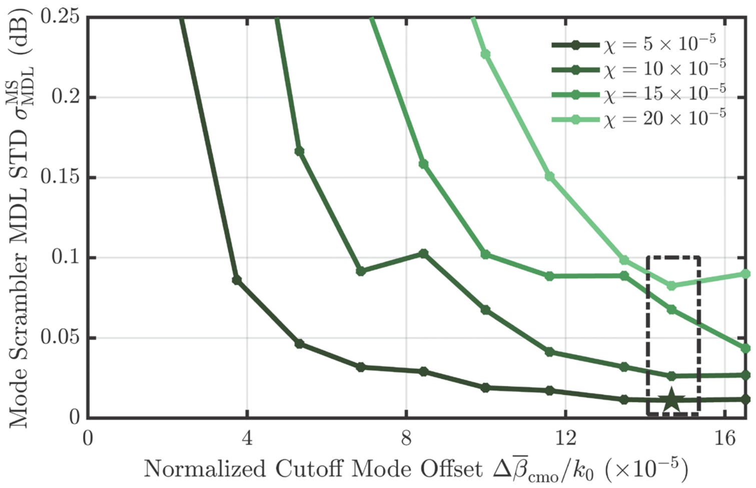
Mode Scrambler Losses



- Scrambler generalized Jones matrix **R** models:
  - Coupling between guided modes.
  - Coupling to unguided modes.
  - Splicing to transmission fiber.





- Scrambler modal gain operator:

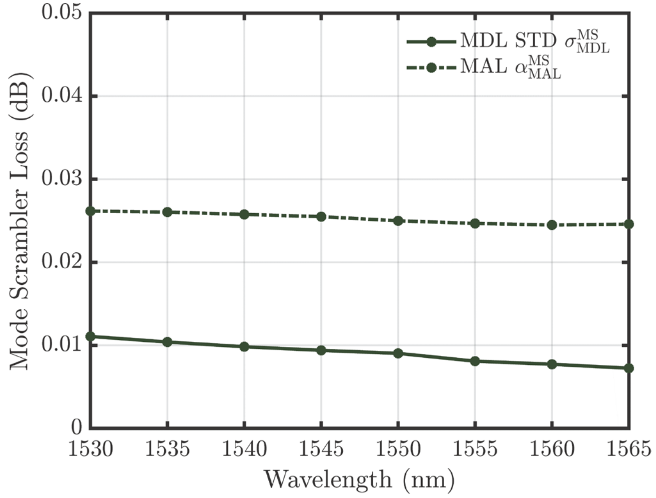
$$\mathbf{F}_{\mathrm{MS}} = \mathbf{R}\mathbf{R}^{H}$$

- Singular value decomposition:

$$\mathbf{F}_{\mathrm{MS}} = \mathbf{V}_{\mathrm{MS}}\mathbf{\Lambda}_{\mathrm{MS}}\mathbf{V}_{\mathrm{MS}}^{H} \qquad \mathbf{\Lambda}_{\mathrm{MS}} = \mathrm{diag}\left\{e^{g_{\mathrm{MS},1}},\ldots,e^{g_{\mathrm{MS},12}}\right\}$$

- Mode-averaged loss and STD of mode-dependent loss:

$$\alpha_{\mathrm{MAL}}^{\mathrm{MS}} = \frac{1}{12}\sum_{i=1}^{12} g_{\mathrm{MS},i} \qquad \sigma_{\mathrm{MDL}}^{\mathrm{MS}} = \sqrt{\frac{1}{12}\sum_{i=1}^{12}\left(g_{\mathrm{MS},i} - \alpha_{\mathrm{MAL}}^{\mathrm{MS}}\right)^2}$$



- Choose cutoff mode offset as shown to minimize losses.
- For modulation depth $\chi = 5 \times 10^{-5}$, $\alpha_{\mathrm{MAL}} < 0.027$ dB and $\sigma_{\mathrm{MDL}} < 0.011$ dB over C band.

O. Krutko, *et al.*, to appear in *J. Lightw. Technol.* (2024).

Mode Scrambler Effectiveness

- MMF transfer matrices (unitary, intra-group coupling only): $\mathbf{M}_1(\omega),\ldots,\mathbf{M}_K(\omega)$

  EDFA transfer matrices (non-unitary, diagonal): $\mathbf{A}$

- Link transfer matrix: $\mathbf{M}_{\text{tot}}(\omega)=\mathbf{M}_K(\omega)\mathbf{AR}\ldots\mathbf{M}_2(\omega)\mathbf{ARM}_1$

- Link group delay operator (eigenvalues are coupled GDs $\tau_{\text{tot},i}$):

$$\mathbf{G}_{\text{tot}}(\omega)=j\frac{\partial\mathbf{M}_{\text{tot}}}{\partial\omega}\mathbf{M}_{\text{tot}}^{-1}(\omega)$$

  Link coupled GD STD: $\sigma_{\text{GD}}^{\text{tot}}(K)=\sqrt{\frac{1}{12}E\left\{\left\|\boldsymbol{\tau}_{\text{tot}}\right\|^2\right\}}$

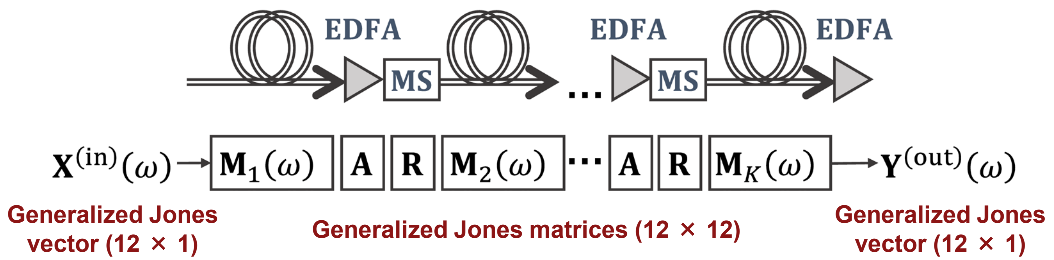
- Link modal gain operator: $\mathbf{F}_{\text{tot}}=\mathbf{M}_{\text{tot}}\mathbf{M}_{\text{tot}}^H$

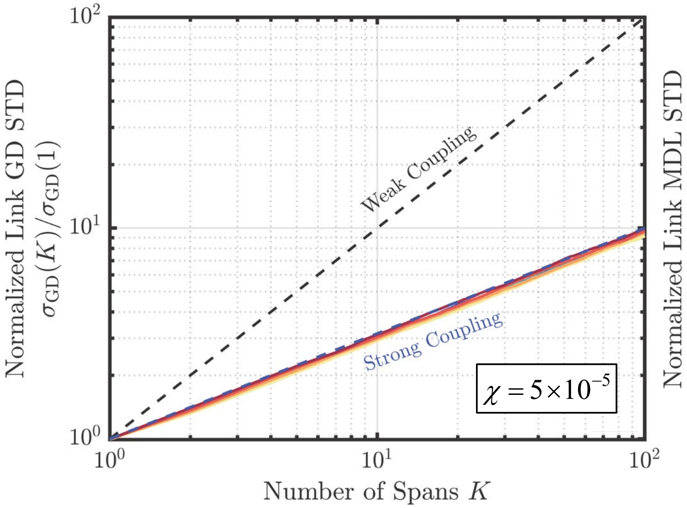
  Singular value decomposition:

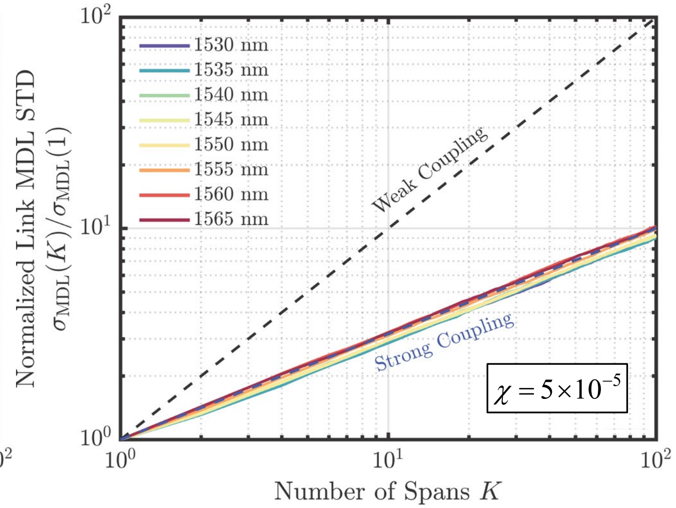
$$\mathbf{F}_{\text{tot}}=\mathbf{V}_{\text{tot}}\boldsymbol{\Lambda}_{\text{tot}}\mathbf{V}_{\text{tot}}^H \qquad \boldsymbol{\Lambda}_{\text{tot}}=\text{diag}\left\{e^{g_{\text{tot},1}},\ldots,e^{g_{\text{tot},12}}\right\}$$

  Link coupled MDG/MDL STD: $\sigma_{\text{MDL}}^{\text{tot}}(K)=\sqrt{\frac{1}{12}E\left\{\left\|\mathbf{g}_{\text{tot}}\right\|^2\right\}}$

- Link GD STD and MDG/MDL STD both scale with $\sqrt{K}$ over C band.







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Impact of Transverse Index Profile Errors on Losses

- Generate spatial frequency-limited refractive index errors [2]:

$$\delta n(r) = \sum_{i=1}^{N} \left[A_i \cos(2\pi f_i r + \phi_i) + B_i \sin(2\pi f_i r + \theta_i) \right]$$

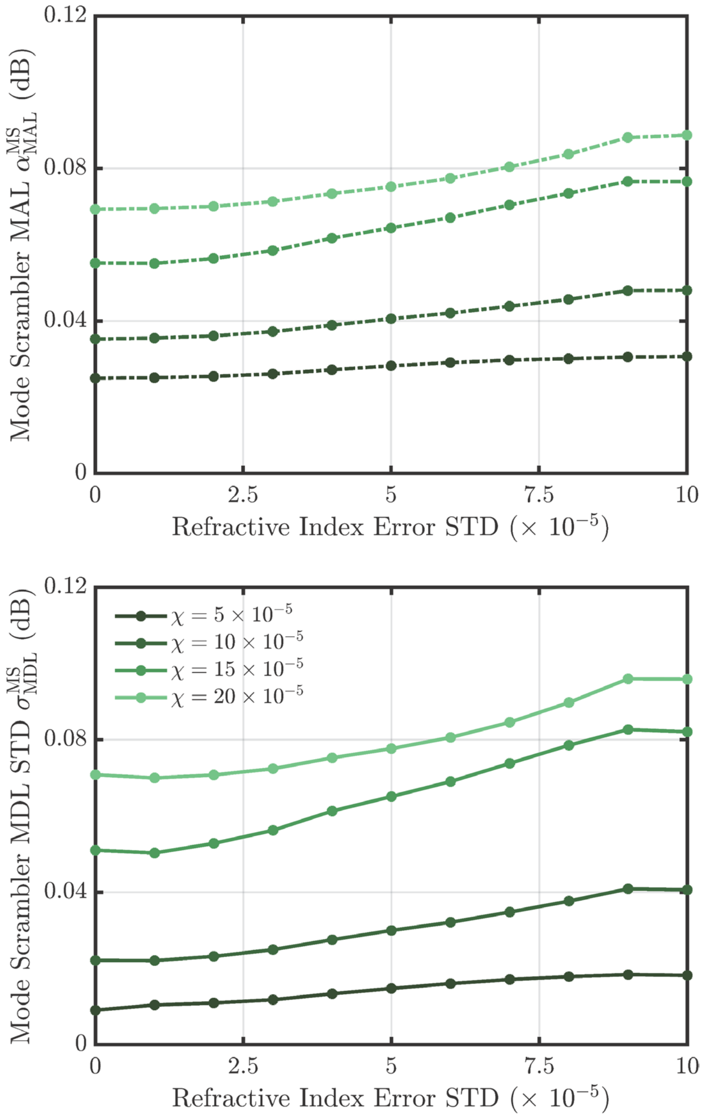
N = 5

f_i equally spaced from 5×10^{-4} m^{-1} to 25×10^{-4} m^{-1}

A_i, B_i Gaussian distributed

ϕ_i, θ_i uniformly distributed

- Scrambler MAL and MDL are less impacted by transverse index profile errors when the modulation depth χ is low.



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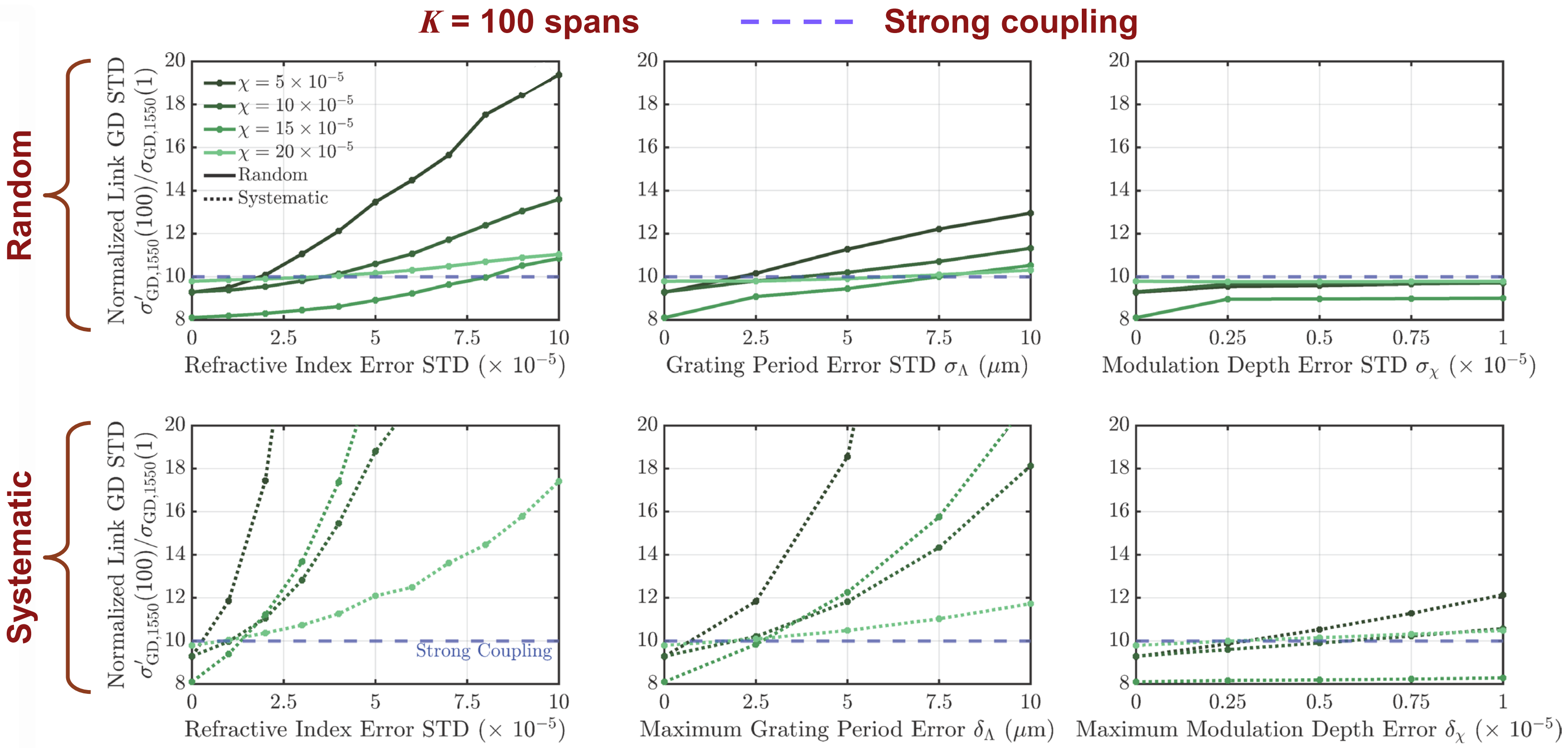
2. K. Choutagunta and J. M. Kahn, *J. Lightw. Technol.* **39** (2021).

Impact of Transverse or Longitudinal Index Errors on Effectiveness

Scrambler effectiveness impacted	
more	**less**
▪ by transverse index errors ▪ by grating period errors	▪ by modulation depth errors
▪ by systematic errors	▪ by random errors
▪ when modulation depth χ is low	▪ when modulation depth χ is high



- Depending on fabrication tolerances, higher modulation depths may be preferred, despite higher losses.

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- Multimode fibers and amplifiers can yield an integrated solution for SDM systems, but require:
  - Fibers with low group delay spread.
  - Amplifiers with low mode-dependent gain.
  - Strong mode coupling provided by fibers or scramblers.
- To date, effective scramblers for $N > 3$ spatial modes have had too much mode-dependent and mode-averaged loss.
- To obtain an effective, low-loss long-period Bragg grating scrambler for 6-mode graded-index fiber, we have:
  1. Used gradient-descent optimization of the transverse index profile $n(r)$ to obtain equal spacing between propagation constants of guided modes, but larger spacing to unguided modes.
  2. Used grid-search optimization of the longitudinal grating parameters.
- We obtained a strong scrambler with MDL STD < 0.011 dB and MAL < 0.027 dB over the C band.
- Choice of longitudinal grating modulation depth:
  - Low: less MDL and MAL, but effectiveness impacted more by fabrication errors.
  - High: more MDL and MAL, but effectiveness impacted less by fabrication errors.

Thank you!

To learn more:

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