Accurate Bit-Error-Ratio Computation in Nonlinear CRZ-OOK and CRZ-DPSK Systems

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***Abstract*—An efficient, accurate method to compute the bit-error ratio in nonlinear chirped return-to-zero on–off keying (CRZ-OOK) and CRZ differential phase-shift-keying (DPSK) systems is presented. The accumulated optical noise at the receiver is modeled as additive nonwhite Gaussian noise. At high signal powers, phase adjustment is employed to maintain the validity of the Gaussian noise model. Results are presented for 10-Gb/s single-channel CRZ-OOK and CRZ-DPSK systems propagating over 8000 km.**

***Index Terms*—Error analysis, fiber nonlinearity, optical fiber communication, optical noise.**

I. Introduction

ANALYTICAL computation of the bit-error ratio (BER) in fiber communication systems using optical amplifiers has been intensively studied in the last decade [1]–[8]. Most work [1]–[5] has neglected fiber nonlinearity. Under this assumption, amplified spontaneous emission (ASE) noise from optical amplifiers retains a white Gaussian distribution during propagation, and the accumulated optical noise at the receiver can be modeled as additive white Gaussian noise (AWGN). Two main classes of BER computation techniques have been proposed for the linear propagation regime: the eigenfunction expansion (EFE) method [1], [2] and the Karhunen–Loéve series expansion method [3]–[5]. Both classes of techniques can yield highly accurate BER results for systems employing on–off keying (OOK) and differential phase-shift-keying (DPSK), arbitrary pulse shaping and arbitrary optical–electrical filtering.

Long-haul systems, however, generally operate in a nonlinear regime in which interaction between signal and ASE via fiber nonlinearity invalidates the AWGN noise model, causing both classes of BER computation techniques to fail. In order to more accurately model noise in nonlinear OOK systems, [6] and [7] proposed to remove phase jitter from individual pulses after propagation through each fiber span, and to model the remaining optical noise as additive Gaussian noise. This method has been proved to be applicable to soliton systems [6] and to an OOK system using the chirped return-to-zero (CRZ) format [7]. We note that phase jitter removal has been justified by the observation that a direct detection optical receiver is not phase-sensitive [6], [7]; therefore, this method may not be applicable to DPSK systems or to OOK systems using narrow-band optical filters. An alternative technique for computing the BER in nonlinear systems modeled optical noise as Gaussian-distributed and obtained fairly accurate results [8], but considered only one type of nonlinear interaction, i.e., parametric amplification.

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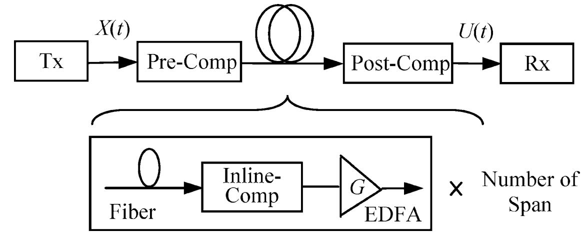

Fig. 1. Schematic of typical CRZ-OOK or CRZ-DPSK system.

An accurate, general model for accumulated optical noise in long-haul systems has yet to be identified. In this letter, we show that in systems using the CRZ format, one may model accumulated optical noise as Gaussian-distributed and compute the BER fairly accurately using the EFE method. Intuitively, this approach works because in CRZ systems, pulses are highly dispersed over most of the link [9], and the nonlinear interaction between signal and ASE is sufficiently weak that optical noise remains nearly Gaussian-distributed. At high signal powers, when the noise tends to deviate from a Gaussian distribution, we perform a phase adjustment in the BER calculation so that the Gaussian noise model remains approximately valid.

II. System Model and BER Calculation

A schematic diagram of a CRZ system using OOK or DPSK modulation format is shown in Fig. 1.

We assume the transmitted information is a pseudorandom bit sequence (PRBS) with a period long enough (e.g., 2^4-1) to take account of intersymbol interference. The modulated signal field can be expressed as

$$X(t) = \sum_n I_n p(t - nT) \tag{1}$$

where $I_n \in \{0, 1\}$ in OOK systems and $I_n \in \{e^{j0}, e^{j\pi}\}$ in (binary) DPSK systems, $p(t)$ is the elementary pulse shape, and T is the bit interval. For CRZ systems, the elementary pulse usually has a 33% duty ratio and takes the form [7]

$$p(t) = \sqrt{3MP_{\text{ave}}}\cos\left[\frac{\pi}{2}\sin\left(\frac{\pi t}{T}\right)\right]\exp\left[iA\pi\cos\left(\frac{2\pi t}{T}\right)\right] \tag{2}$$

where P_{ave} is the average power, $M = 2$ for OOK, $M = 1$ for DPSK, A is the chirp parameter.

Strong precompensation is typically used in CRZ systems to spread the pulses. As shown in Fig. 1, the precompensated signal is launched into the fiber link, which generally consists of many spans. In each span, an inline compensator compensates the chromatic dispersion of the preceding fiber, and an erbium-doped fiber amplifier (EDFA) compensates the loss. The ASE noise from EDFAs is the dominant noise source in the system.

At the end of the fiber link, optical pulses are compressed by a postcompensator and passed to a receiver. For OOK, a receiver uses a bandpass optical filter and a photodetector, while a DPSK receiver uses an optical bandpass filter, a Mach–Zehnder interferometer and balanced photodetector. In both receivers, the photodetector is followed by a lowpass electrical filter and sampling and decision circuits.

We assume the optical field at the postcompensator output $U(t)$ has a nonwhite Gaussian distribution. Using the EFE method, we can find a set of orthonormal basis functions $\{\phi_m\}$ and express the decision sample in an OOK or DPSK receiver as

$$y(t) \approx \sum_{m=1}^{M} \lambda_m \left|r_m(t)\right|^2 \tag{3}$$

where r_m is the projection of $U(t)$ onto the basis function ϕ_m and λ_m is the weighting factor sorted in descending order of absolute value. As the ϕ_m have been called modes [1], we refer to r_m as the (complex) amplitude of the mth mode. In matrix form

$$y(t) \approx \vec{r}^{*T} \Lambda \vec{r} \tag{4}$$

where $\vec{r} = \{r_1, r_2, \ldots, r_M\}^T$, $\Lambda = \text{diag}\{\lambda_m\}$. As we assume $U(t)$ is nonwhite Gaussian noise, $\vec{r}$ is a Gaussian vector with interdependent components r_m. In order to find the moment-generating function (MGF) of $y(t)$ to compute the BER, we need to know C_r, the covariance matrix of $\vec{r}$, which we estimate by ensemble-averaging the data obtained in a number of Monte Carlo simulations. In these simulations, we propagate signal and noise together and use the split-step Fourier method to solve the nonlinear Schrödinger equation. After estimating C_r, we obtain the MGF following [6] and compute the BER using the saddle point integration method presented in [4].

The accuracy of the estimate of C_r and of the BER is determined by N, the number of Monte Carlo realizations used to estimate C_r. By observing the variation of BER for different random initial seeds, as in [6], we find the BER converges when $N \geq 4000$. Therefore, we choose $N = 4000$ realizations to estimate C_r and compute the BER. Although $N = 4000$ is a substantial number, the transmitted PRBS can be as short as $2^4 - 1$, so the Monte Carlo simulations can be performed in less than 20 min on a personal computer.

When signal power is very high, we observe that the distribution of $\vec{r}$ may deviate substantially from Gaussian. Fig. 2(a) shows the distribution of the first mode amplitude r_1, for the 14th bit in the PRBS 101 011 001 000 111 when the signal power is 2/3 mW in a CRZ-OOK system (details of the system setup are given in Section III). In this case, we can transform the distribution of r_1 to be approximately Gaussian by adjusting the phase of r_1. Observing the spiral-shaped distribution of r_1, we perform the phase adjustment following a simple algorithm:

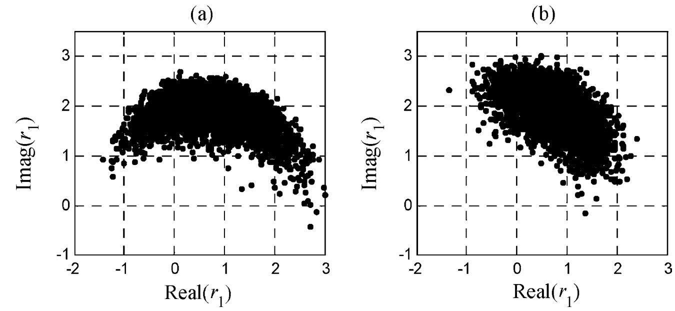

Fig. 2. Effect of phase adjustment on the first mode amplitude r_1 for CRZ-OOK at an average signal power of 2/3 mW. (a) Before phase adjustment. (b) After phase adjustment.

$$r_1 = r_1 \exp\left(j\alpha|r_1|^2\right) \tag{5}$$

where α is a positive number that depends on the peak pulse power. This algorithm is similar to the phase compensation method employed in [10]. We note that this phase adjustment does not change the distribution of the decision variable $y(t)$ because of the squaring operation in (3). Instead, phase adjustment is used to insure that the assumption of Gaussian noise remains approximately valid. Fig. 2(b) illustrates the effect of the phase adjustment. It is observed that the distribution of the first mode amplitude approaches a two-dimensional Gaussian after phase adjustment. In Section III, we describe the improvement in BER accuracy obtained by using phase adjustment.

Phase adjustment is necessary only for r_1 in OOK systems or for r_1 and r_2 in DPSK systems, because other modes make much smaller contributions to the decision samples [1], and they have nearly Gaussian distributions without phase adjustment. The phase adjustment for r_2 in DPSK systems follows a similar algorithm: $r_2 = r_2 \exp(j\alpha|r_2|^2)$.

In the remainder of this letter, for the sake of brevity, we refer to the entire BER calculation technique described above (including the estimation of C_r) as the EFE method.

III. Results

We have used the EFE method to calculate the BER for single-channel CRZ-OOK and CRZ-DPSK systems operating at a bit rate $R = 10$ Gb/s. In both systems, the fiber link comprises 100 spans, each of 80-km length, for a total length of 8000 km. In each span, we use 80 km of LEAF fiber with dispersion and loss parameters 3.5 ps/nm/km and 0.2 dB/km. The inline compensation corresponds to about 92% of the dispersion of an 80-km span of LEAF fiber. The precompensators and postcompensators each provide -1176-ps/nm dispersion, 4.2 times the dispersion in an 80-km span of LEAF fiber. The nonlinear parameter γ for LEAF fiber is taken to be $1.5\ \text{W}^{-1} \cdot \text{km}^{-1}$. Each EDFA is taken to have a noise figure of 4.5 dB.

The elementary pulse shape takes the form (2) with a chirp parameter $A = 0.4$. In the EFE method, the transmitted signal is a

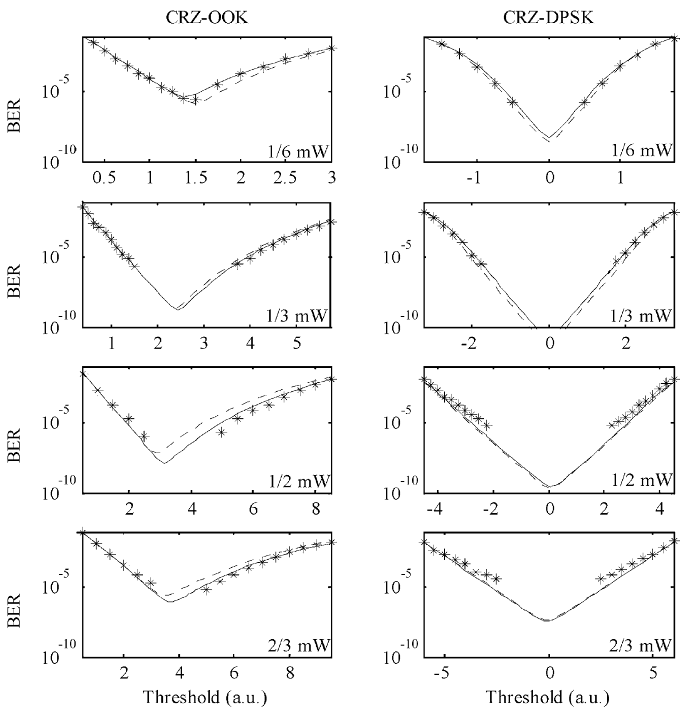

Fig. 3. Comparison of BERs obtained with EFE method and Monte Carlo simulation for different average signal powers in CRZ-OOK and CRZ-DPSK systems. Dashed lines: EFE without phase adjustment. Solid lines: EFE with phase adjustment. Stars: Monte Carlo simulation. Left column: CRZ-OOK. Right column: CRZ-DPSK.

$2^4 - 1$ bit PRBS. When Monte Carlo simulation is performed to verify the results of the EFE method, we use $2^{10} - 1$ bit PRBS in each trial and perform more than 10^5 trials to estimate the BER. At the receiver, the optical bandpass filter is a Gaussian filter with a bandwidth (full width at -3 dB) of $2.5R$ and the electrical lowpass filter is a fifth-order Bessel filter with a bandwidth (-3 dB) of $0.75R$.

Fig. 3 compares BERs obtained using the EFE method and Monte Carlo simulation for CRZ-OOK and CRZ-DPSK. They are plotted as BER versus decision threshold. Dashed and solid lines indicate BERs computed without and with phase adjustment, respectively. For each modulation format, four different average signal powers are chosen.

Referring to Fig. 3, when the decision thresholds are optimized, the average signal power that minimizes the BER is 1/3 mW for both CRZ-OOK and CRZ-DPSK. At this optimal power, the EFE method and Monte Carlo simulations agree with each other (even without phase adjustment), verifying the accuracy of the EFE method for CRZ systems.

When the signal power is lower than the optimum value, the EFE method still yields accurate BERs for both CRZ-OOK and CRZ-DPSK systems. This is expected, because as signal power is reduced, nonlinear effects become weaker, and the distribution of accumulated optical noise becomes closer to Gaussian, insuring that the EFE method will yield accurate BER results.

When the signal power is higher than the optimum value, if phase adjustment is not employed, the BERs obtained using the EFE method begin to deviate from those of the Monte Carlo method. However, once phase adjustment is utilized, the accuracy of EFE method is improved significantly for CRZ-OOK, and the EFE method largely agrees with Monte Carlo simulation. Even at lower power levels, phase adjustment improves the accuracy of the EFE method slightly for CRZ-OOK.

In CRZ-DPSK systems, when the signal power is above the optimal value, BERs obtained by the EFE method deviate from those of the Monte Carlo method, with or without phase adjustment. The phase adjustment algorithm (5) is ineffective for DPSK because at high powers, the distribution of the mode amplitudes differs from the spiral shape shown in Fig. 2(a). We are attempting to identify phase adjustment schemes that are applicable to DPSK systems.

IV. Conclusion

We have shown that a nonwhite Gaussian noise model and an EFE method can be used to accurately compute BERs in CRZ-OOK and CRZ-DPSK systems for signal powers at or below the optimal value. If phase adjustment is utilized, accurate BERs are obtained for CRZ-OOK even at signal powers above the optimal value. Phase adjustment is largely ineffective for CRZ-DPSK. Although we have only verified the accuracy of this BER computation method in CRZ systems, we expect it to be accurate in other long-haul systems, provided that optical pulses are highly dispersed in most of the fiber link, as they are in CRZ systems.

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