

# Variable attenuator for Gaussian laser beams

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A method of attenuating laser beams is described, which is continuously variable, gives an output beam of constant Gaussian shape, and is adaptable to a wide range of laser types and power levels. The theory of the method is discussed, and the performance of a prototype system, used with a CO laser and having a dynamic range of  $>10^5$ , is presented.

## I. Introduction

Much use is made of lasers in studying nonlinear optical interactions with matter because of the high power and spectral brightness available from laser beams. However, central to any investigation of a power-dependent effect is a means of controlling that power. In addition, so that the experimental results can more readily be understood, it is important that the power can be varied without altering any other characteristic of the laser beam, such as direction, polarization, and beam or pulse shape. This can be difficult to achieve since most methods of attenuation will affect one or more of these or other beam properties, and any method chosen must be, of course, able to withstand the high intensities of the laser beam. To make the laser into a comprehensive tool for the study of power-dependent optical effects, it is also obviously desirable that the laser attenuator system should not be restricted to discrete steps of attenuation and should be capable of providing sufficient dynamic range to bridge the gap in output power between the different laser systems (e.g., mode-locked, Q-switched, and cw) and also between laser and conventional spectroscopy.

In this paper, we present a method of attenuating laser beams, which is capable of providing continuously variable attenuation over many orders of magnitude from a high maximum transmission while preserving the shape and all other geometric properties of the beam substantially constant. It is based on the combination of two technologies, spatial filtering and multilayer dielectric mirrors. Both of these are well developed and understood, and so the method should be usable with

most laser systems up to the very high powers which can be handled by dielectric coatings. Additionally, the use of a reflecting attenuator has the advantage that the unwanted power need not be dissipated anywhere inside the system, but can easily be dumped externally; this is particularly important in handling high cw powers.

The system is designed for use with Gaussian laser beams and is intended to give a Gaussian output, this being by far the most common and easily handled laser beam form.<sup>1</sup> It can also be used with imperfect, near Gaussian beams, however, because the spatial filter used in the system will remove the imperfections, leaving a filtered Gaussian output.

The theory of spatial filtering as it relates to Gaussian beams is summarized in Sec. II, and the central concept of the method, the combination of the spatial filter with the attenuator, is discussed. Section III analyzes the effect of attenuating a Gaussian beam with an attenuator whose transmission varies across the width of the beam, showing how continuously variable attenuation can be achieved. In Sec. IV, the practical design of the attenuating dielectric mirror is described, experimental results for a complete prototype system used with a CO laser are given in Sec. V, and these results are discussed in Sec. VI.

## II. Spatial Filtering of Gaussian Beams

When a lens is illuminated by a collimated beam of monochromatic coherent light of wavelength  $\lambda$  (such as is usually obtained from a laser), the amplitude distribution in the back focal plane of the lens (see Fig. 1) is approximately the Fourier transform of the distribution in the front focal plane, i.e.,

$$A(u,v) \propto \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} a(x',y') \exp \left[ -\frac{2\pi i}{\lambda f} (ux' + vy') \right] dx' dy',$$

where  $A(u,v)$  is proportional to the electric field amplitude in the back focal plane and  $a(x',y')$  to that in the front focal plane.

Lasers are often operated in the so-called TEM<sub>00</sub>

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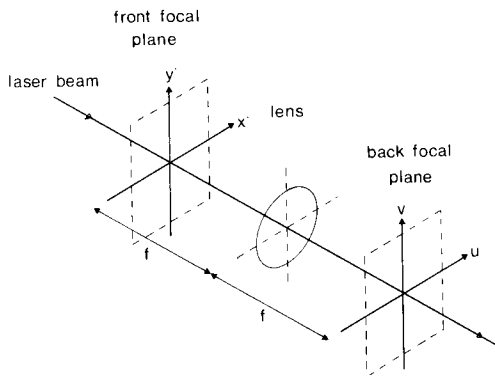


Fig. 1. Coordinate axes for the spatial filter.

mode, which means that the electric field distribution in the output beam is given by the cylindrically symmetric expression

$$E(x',y') \propto \exp\left(-\frac{x'^2 + y'^2}{w^2}\right),$$

when the beam is collimated. Here  $w$  is the radius of the beam when  $E$  has fallen to  $1/e$  of its peak value. This is the Gaussian profile and has the important property that its Fourier transform is also a Gaussian. In the situation depicted in Fig. 1, when the lens is illuminated by this laser beam, the Gaussian profile in the back focal plane can be shown to have  $1/e$  radius  $w_0$  given by  $w_0 = (\lambda f)/(\pi w)$ . In practice, it is usually the case that the form is not a perfect  $TEM_{00}$  mode and can be considered as the sum of the ideal Gaussian and another irregular function. This irregular function, however, will not transform like the Gaussian, and often the transform process operates in such a way that the irregular function separates out. By placing a pinhole of appropriate size (i.e., radius  $R \gg w_0$ ) in the back focal plane, the majority of the Gaussian passes through the pinhole, while the irregular part is not transmitted. In this way, many imperfections can be removed from the original beam.

For example, consider a Gaussian profile on which is superimposed a cosinusoidal variation:

$$E(r) \propto A \exp\left(-\frac{r^2}{w^2}\right) + B \cos 2\pi s x.$$

It is easily shown that the cosine part will transform to lines  $2r_2 = 2\lambda f s$  apart in the back focal plane, which will not be transmitted by the pinhole (as long as  $r_1 > R$ , the radius of the pinhole), and the disturbance on the Gaussian beam will be spatially filtered out.

The discussion so far has considered a laser beam which is originally imperfect. However, the analysis is identical for disturbances introduced into the laser beam by optical components inserted before the front focal plane, and the analysis and effects are similar for disturbances created inside the spatial filter between the lens and the pinhole, or between the front focal plane and the lens (provided the disturbances are slowly

varying compared with the wavelength, a condition which is easily satisfied in practice).

This fact is important here because many otherwise satisfactory forms of attenuation cannot normally be used with lasers due to the effects they have on the beam profile. For example, interference filters or other partial reflectors often introduce interference fringes due to multiple reflections inside the filter substrate, and semidiffuse scatterers or absorbers tend to produce sharp spikes in the profile due also to constructive interference. Consequently, by inserting the attenuator either before or (which is more convenient) between the lens and the pinhole, the spatial filter performs the double purpose of minimizing the effect of the beam shape disturbances introduced by the attenuator and removing imperfections in the original near- $TEM_{00}$  laser beam. Therefore, the use of the combination of the spatial filter with an attenuator makes it possible to preserve a constant Gaussian beam shape even with an attenuator, which, used on its own, could not.

### III. Variable Attenuation of Gaussian Beams

The use of Gaussian profile beams has been considered so far because most lasers operate in approximately such modes, and spatial filtering of them is relatively easily accomplished with a lens and a simple pinhole. Gaussian beams have of course the additional experimental advantages, that their propagation is well understood and relatively simple.<sup>1</sup> (The intensity profile retains a Gaussian form at all times, and its parameters obey elementary mathematical relations.) However, there is one further property of the Gaussian beam which can be used to considerable advantage here.

If a beam is incident on an attenuator whose transmission varies across the width of the beam, the form of the beam will in general be altered because one side of the profile is attenuated more than the other. However, in the special case where the transmission varies exponentially across the width of the attenuator and where the beam is Gaussian, the beam shape is preserved regardless of how much the attenuation changes across the beam width. The only effect is to move the profile slightly sideways. (Even in the case where the transmission does not vary in a precisely exponential manner, the additional effects introduced are not in practice very troublesome, provided that the form of the transmission function is relatively smooth.) This property enables us to use an attenuator whose transmission varies significantly across the beamwidth, thereby being able to compress several orders of magnitude of attenuation variation into a relatively small mechanical movement, without significant effects on the beam profile. The theory of this effect is discussed in detail below.

Suppose that the incident profile  $E(x,y)$  and the transmission of the attenuator  $T(x)$  are given by

$$E(x,y) = A \exp\left(-\frac{x^2 + y^2}{w^2}\right); T(x) = B \exp(-\alpha x).$$

Then the resulting attenuated beam is the product of these two:

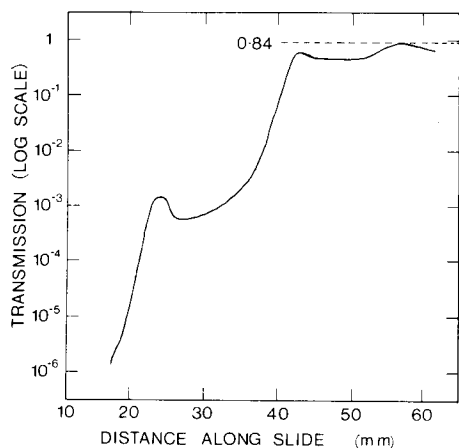


Fig. 2. Transmission of the CO laser prototype system as a function of attenuator slide position.

$$E'(x,y) = E(x,y) \cdot T(x) = AB \exp \left[ - \left( \frac{x^2 + y^2}{w^2} \right) - \alpha x \right],$$

$$E'(x,y) = AB \exp \left( \frac{\alpha^2 w^2}{4} \right) \exp \left[ - \frac{\left( x + \frac{\alpha w^2}{2} \right)^2 + y^2}{w^2} \right].$$

This profile  $E'(x,y)$  corresponds to a Gaussian of the same width  $w$  as the original  $E(x,y)$  with its center at  $x = -\alpha w^2/2, y = 0$  [instead of  $x = 0, y = 0$  for  $E(x,y)$ ] attenuated by a factor  $B$  [which is simply the transmission at the center of  $E(x,y)$ ] multiplied by the width-dependent correction  $\exp(\alpha^2 w^2/4)$ . In practice both the shift  $\alpha w^2/2$  and the width-dependent correction are negligible. If, for example,  $\alpha = 2.3 \text{ cm}^{-1}$  (equivalent to 1 decade change in transmission per centimeter across the width of the attenuator) and  $w = 1 \text{ mm}$ , the shift is  $\sim 115 \mu\text{m}$  (which is less than  $\sim 6\%$  of the beamwidth), and the width-dependent correction is  $\sim 1.3\%$ . The shift shows itself in practice as a slight (and constant) angular deviation of the beam through the pinhole due to the action of the lens. (This angular deviation is changed back into a simple shift by the collimating lens normally used after the spatial filter.) The consequence of this analysis is that the attenuation can vary considerably across the width of the beam (in the above example this variation is about 50%) without significantly disturbing the profile.

This property relies on an exponential change of transmission with position on the attenuator. In practice this may be difficult to achieve exactly, and the transmission curve might resemble that shown in Fig. 2. Here, the curve is no longer linear on a  $\ln T$  vs position graph, but now contains higher order terms. In general, therefore, if we expand mathematically round about position  $d$ , where the incident Gaussian  $E(x,y)$  is centered, we will obtain

$$\ln T(x) = \ln T(0) + \frac{xd}{dx} \ln T_{x=0} + \frac{x^2}{2} \frac{d^2}{dx^2} \ln T_{x=0} + \frac{x^3}{6} \frac{d^3}{dx^3} \ln T_{x=0} + \dots,$$

so that  $\ln T(x) = \ln T(0) + \alpha x + \beta x^2 + \gamma x^3 + \dots$ , i.e.,  $T(x) = T(0) \exp(\alpha x + \beta x^2 + \gamma x^3 + \dots)$ .

The term  $\exp(\alpha x)$  is the term linear in  $x$ , already analyzed. The next term,  $\exp(\beta x^2)$ , can be shown, by a similar analysis, to alter the effective width of the Gaussian to  $w' = w/(1 + w^2\beta)^{1/2}$ . This turns out to be not too restrictive. If, for example, an alteration of 1% in the width can be tolerated, the rate of change of  $\alpha$  [i.e.,  $[d^2/(dx^2)] \ln T$ ] should not exceed 0.01  $(4/w^2)$ , i.e., for  $w = 1 \text{ mm}$  the slope can change from  $\alpha = 0$  to  $\alpha = 2.3 \text{ cm}^{-1}$  (1 decade/cm) in as little as 0.6 cm. The higher order terms [ $\exp(\gamma x^3) + \dots$ ] do distort the beam in a manner which cannot easily be classified and may not be easily dealt with by the spatial filter, and so these should be eliminated as far as possible. In general, therefore, points of sudden inflection in the plot of  $\ln T$  against position should be avoided, as such points imply strong higher order terms in the expression. This effectively means that there should, not surprisingly, be no sudden bumps in the curve. However, the conclusion from this analysis is that, in practice, a variation of attenuation with position, which is not exactly exponential, can be tolerated and its side effects kept within bounds by careful design.

This discussion has assumed that the profile incident on the attenuator is perfectly Gaussian. This may well not be the case when dealing with previously unfiltered laser beams. In such a case, the Gaussian component will behave as discussed above, and, although the irregular component will be modified in form by the attenuator, this will only be to another irregular function, which will be filtered as before.

#### IV. Attenuator Design

The quality of the beam profile obtained from the spatial filter is not greatly dependent on the type of attenuator used, and good results have been obtained with, for example, both roughened and polished slices of dielectric (in this case silicon) as well as more sophisticated multilayer-dielectric mirrors. This latter technology, however, offers considerable scope for tailoring the shape of the transmission function with position across the attenuator, enabling the special property of the Gaussian profile discussed above to be fully exploited. It also has the advantages of being a well developed technology in the 5–10- $\mu\text{m}$  region of the ir spectrum for which the prototype attenuators were required (to match the output wavelength range of the CO and CO<sub>2</sub> lasers), and at most other laser wavelengths as well due to its extensive use in laser mirrors, which, of course, also behave as filters.

The aim in the design of the attenuating device is to produce a transmission which alters monotonically and in a relatively smooth fashion as the device is moved across the beam, so that the output beam can be maintained as a substantially constant Gaussian shape as

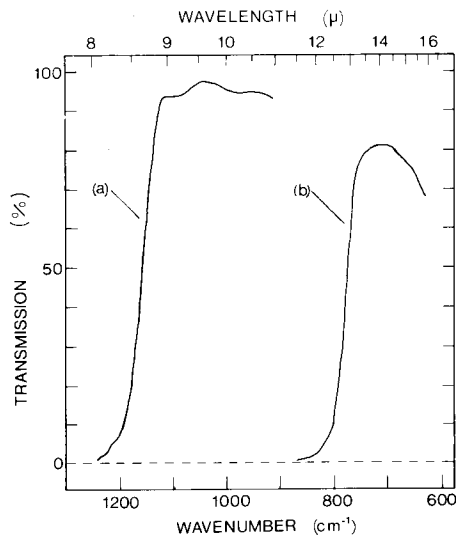


Fig. 3. Spectral transmission characteristics of a wedge-coated filter/attenuator designed for a CO<sub>2</sub> laser (a) at the thin end and (b) at the thick end of the coating.

discussed above. The method used here is to deposit a long-wavelength pass multilayer-dielectric interference filter onto a substrate, but to vary monotonically the thickness of the deposition across the width of the substrates (i.e., the filter coating is wedged). Thus the pass wavelength of the filter alters across its width.

By suitable design of the filter and its deposition, the laser wavelength of interest can be transmitted at one end of the filter (because it lies above the pass wavelength at this end) and almost totally reflected at the other (because it lies beneath the pass wavelength at that end). Between these two extremes the laser beam is partly transmitted. The device therefore behaves like a partially reflecting mirror whose transmission varies across its width, and if the wedging and the spectral edge are smooth and monotonic, so is the variation of transmission.

The multilayer filters used in the prototypes are deposited with an approximately linear thickness wedge along a 63-mm strip of germanium. The filter comprises thirteen fractional-thickness layers of PbTe with ZnS, designed using the Tchebycheff equal ripple polynomial method<sup>2</sup> to give high maximum transmission over a broad wavelength range and an edge which is smooth and monotonic to below  $10^{-3}$ . The substrate is antireflection coated on its reverse side.

A wedge was generated by the well known technique of the offset stationary source set up for a distribution ratio of  $1.5 \times$  thickness over the full extent of the strip. For this ratio, transmission at the design laser wavelength is comfortably below  $10^{-3}$  at one end of the strip and comfortably into the passband at the other end. Figure 3 shows typical spectral performance for such a wedge-coated substrate, here designed for use with a CO<sub>2</sub> laser. As can be seen from this figure, the Tche-

bycheff profile<sup>2</sup> is retained under these deliberately nonuniform conditions of deposition, and this is ascribed to good thickness control in the manufacture.

It is, of course, relatively simple to ensure that beam direction is maintained regardless of attenuation with this type of attenuator by polishing the substrate plane-parallel, and attenuation is totally independent of and has no effect on polarization if the attenuator is operated at normal incidence.

## V. Experimental Performance

Figure 4 shows the mechanical arrangement used in the prototype described here, which was designed for an Edinburgh Instruments PL3 cw CO laser. The lenses are zinc selenide (with antireflection coating), and the attenuator is of the type described above, although the filter design is also an early prototype and is not as smooth as that shown in Fig. 3. A collimating lens is used in addition after the pinhole and is mounted in a focusing action. This makes the whole unit into a telescope and allows adjustment of the divergence angle of the output, a facility which is useful in practice. The attenuator is mounted on a slide at right angles to the optic axis driven by a rack and pinion gear and indexed with a multiturn dial.

Figure 2 shows typical transmission data for the system as a function of attenuator position for the prototype attenuator at 5.3- $\mu\text{m}$  wavelength. This does have some bumps in it, but these have not affected the beam profile in practice. The design figure for the maximum attenuation was a conservative  $10^3$ , but this has been easily surpassed in practice, the data here showing relative transmission lower than  $10^{-5}$ . Similar performance is obtained throughout the wavelength range of the laser (5.15–6.00  $\mu\text{m}$ ), both in the form of the transmission function and the quality of the beam profiles. The performance of the system in terms of beam intensity profiles is shown in Fig. 5. [The intensity is proportional to the square of the electric field, and so the intensity profile of an electric field Gaussian  $E(r)$

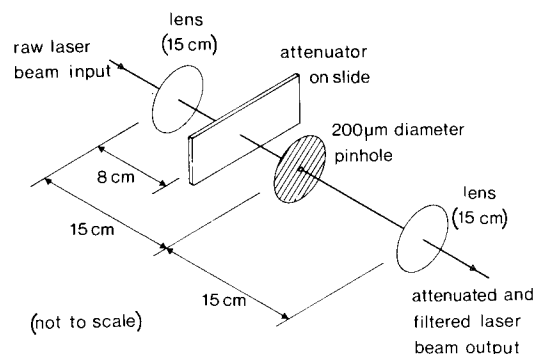


Fig. 4. Mechanical arrangement in the prototype designed for a CO laser.

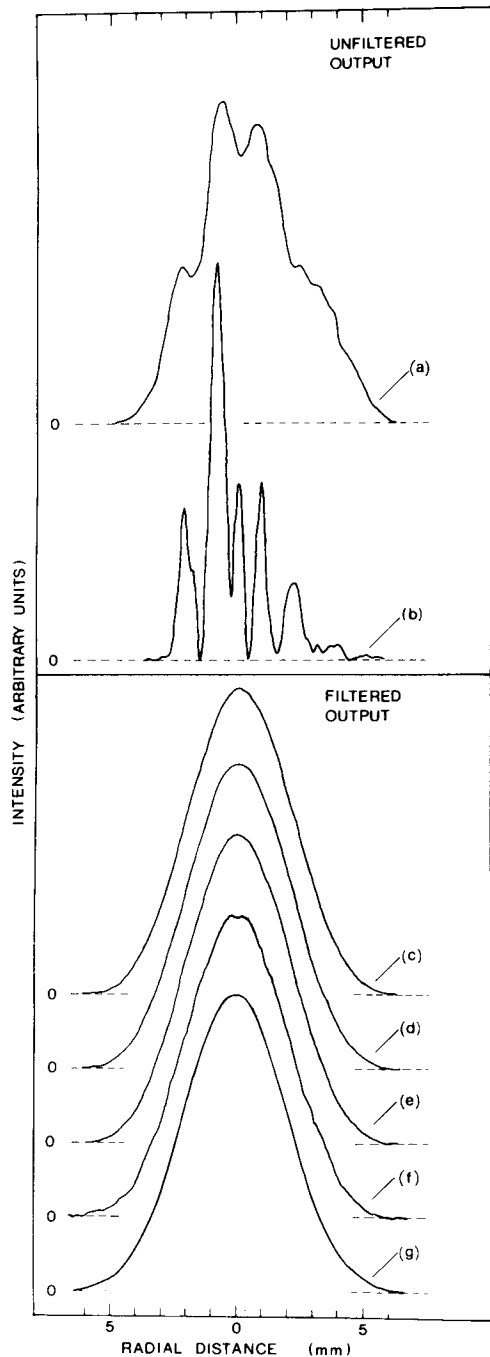


Fig. 5. Beam intensity profiles taken at the position of the output lens (a) in the absence of both the slide and the filtering pinhole (i.e., the raw laser output profile); (b) with the attenuator slide inserted but still without the filtering pinhole; (c) with the complete attenuator/spatial filtering system at 650-mW output power; (d) as (c) but 65 mW; (e) as (c) but 6.5 mW; (f) as (c) but with 650  $\mu$ W. (The resolution here is limited by detector noise.) Curve (g) is a mathematical Gaussian of the same width. (The heights of all traces have been adjusted for comparison.)

$\propto \exp(-r^2/w^2)$  is another Gaussian  $I(r) \propto \exp(-2r^2/w^2)$  of  $1/e$  radius  $w/\sqrt{2}$ .] These were taken by scanning a pyroelectric detector (with a 100- $\mu$ m pinhole fixed in front of it for sampling the beam) across the beam after

the collimating lens. Figure 5(a) shows the profile in the absence of both the attenuator and the spatial filtering pinhole and corresponds to the raw output profile of the laser at  $\sim 800$  mW. (The laser has deliberately not been adjusted for the best mode form.) The attenuator is inserted for Fig. 5(b), and the additional disturbance created can be clearly seen. Figures 5(c)–5(f) show typical profiles obtained, with the attenuator and the filtering pinhole, at different attenuator positions given total powers of 650 mW, 65 mW, 6.5 mW, and 650  $\mu$ W, respectively. The resolution in Fig. 5(f) is limited by detector noise. The heights of all traces in Fig. 5 have been adjusted for comparison purposes. Figure 5(g) shows an ideal Gaussian of the same  $1/e$  width and height.

These data have deliberately been taken with both poor laser mode quality and an imperfect filter design. However, it can be seen that not only are the profiles practically identical in shape over a range of at least 3 decades of power, but also they are near to the perfect Gaussian form. As the laser mode form is improved, the maximum transmission is increased.

## VI. Conclusions

The experimental results show that the system removes the imperfections introduced into the laser beam by the attenuator (and those present in the original beam) to leave a beam profile which conforms very closely to the ideal Gaussian. Despite the fact that the prototype attenuator does not have the ideal form of transmission function, the shape of the output profile does not alter significantly with the attenuator setting over more than 3 decades of power, the continuously variable attenuation range extends over more than 5 decades, and the maximum transmission is high. The over-all conclusion from the practical operation of this prototype is that the spatial filter and attenuator system described here provide a simple and reliable means of controlling the power and profile of a laser beam. Because the technologies involved are well proven, we expect this system to be immediately applicable to nearly all laser types and powers.

This system is protected by British Provisional Patent, Application 20147/77.

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