sons. Firstly, it demonstrates that injection locking is not a necessary condition for effective timing extraction. It also demonstrates that timing extraction can be accompanied by a change in the wavelength of the synchronized clock signal. Such a wavelength change for an optical clock could find applications in hybrid optical switching systems based on time and wavelength switching. These aspects of timing extraction will be considered in a future publication.

Conclusion

We have demonstrated that it is possible to extract a timing signal from a random data signal at either the bit rate frequency or at half the bit-rate frequency, using the same twin section self-pulsating laser diode. It is possible to switch between these two timing extraction states by altering only the dc biases applied to the self-pulsating laser diode. We have also proposed that injection locking of the self-pulsating laser diode is not a necessary condition for timing extraction.

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Optical Frequency Comb Generator Using Phase Modulation in Amplified Circulating Loop

Keang-Po Ho and Joseph M. Kahn

Abstract—**We propose a method to generate a comb of precisely spaced optical frequencies over a terahertz span. This scheme uses a sinusoidally driven phase modulator and an optical amplifier placed within an optical fiber loop, so that the modulation is enhanced by multiple passes through the loop. By maintaining the loop round-trip gain slightly below unity, a comb of tens to hundreds of frequencies can be generated. If the loop input is derived from a laser locked to an absolute frequency reference, then each of the output frequencies will have an absolute accuracy approaching that of the input.**

I. Introduction

DENSE optical frequency-division multiplexing (OFDM) is a promising method to exploit the vast bandwidth of single-mode optical fiber. A key issue in dense OFDM implementation is the absolute frequency accuracy of each individual channel. Atomic or molecular resonances [1] are commonly used for absolute laser frequency stabilization, but they are not readily available at regular intervals. Wavelength comparison using Fabry–Perot interferometers or fiber-ring resonators [2] provides a regularly spaced frequency comb, but may be limited in precision by uncertainty in resonator length. A promising means to create a precisely spaced comb of frequencies is through large-index angle modulation [3]–[5]. However, the number of generable frequencies is limited by the difficulty in achieving large-index phase or frequency modulation at the required gigahertz modulation frequencies. In addition, the modulation index must be chosen carefully to avoid the zeroes of Bessel functions [6].

In this letter, we propose an alternate method for generating a comb of precisely spaced reference frequencies. The basic concept is to place a phase modulator within a circulating loop, so that the phase modulation is enhanced by multiple passes of light through the modulator. The loop includes an optical amplifier to very nearly

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compensate for the round-trip loss. A similar configuration using an acoustooptic modulator has already been proposed [6]. However, because of the limited frequency shift produced by acoustooptic modulators, that technique yields a frequency spacing that is too small for many practical applications.

As shown in Fig. 1, the loop input, E_i, is derived from a laser that is stabilized to an atomic or molecular line at frequency ω_0, e.g., by using the optogalvanic effect. The loop includes a phase modulator, which is driven by a sinusoid at modulation frequency ω_m. After the modulator, an optical amplifier boosts the signal to nearly compensate for the loop losses. For maximum efficiency, the loop roundtrip delay T must satisfy two conditions. The first condition is that $\omega_m T = 2p\pi$, where p is an integer, i.e., the loop resonance frequency must be an integral subharmonic of the modulation frequency. Since most OFDM systems will require a fixed ω_m, this will most often be satisfied by careful fabrication of the loop to the proper length. The second condition on T is that $\omega_0 T = 2q\pi$, where q is a large integer, i.e., the loop resonance frequency must also be an integral subharmonic of the input optical frequency. To satisfy this condition in practice, it will usually be necessary to adjust T under closed-loop control. Thus, a portion of the loop output, E_o, is detected by a photodiode to generate an error signal for locking purposes.

II. Analysis

The loop input is a continuous-wave, time-dependent electric field $E_i(t) = \hat{E}_i e^{-j\omega_o t}$, while the output signal is $E_o(t) = \hat{E}_o e^{-j\omega_o t}$. (Actual electric fields are given by the real part of these quantities.) The electric fields at the other two ports of the coupler are $E_{l1}(t)$ and $E_{l2}(t)$ respectively. At time t, the relation between various electric fields is governed by the following set of equations:

$$E_o(t) = \sqrt{1-\gamma}\left(\sqrt{1-k}\,E_i(t) + \sqrt{k}\,E_{l2}(t)e^{-j\pi/2}\right)$$
$$E_{l1}(t) = \sqrt{1-\gamma}\left(\sqrt{k}\,E_i(t)e^{-j\pi/2} + \sqrt{1-k}\,E_{l2}(t)\right),$$
$$E_{l2}(t) = \sqrt{\alpha}\,e^{-j\beta\sin(\omega_m t)}E_{l1}(t-T) \quad (1)$$

where β is the single-pass peak phase shift of the phase modulator, α is the round-trip loop power transmission coefficient, which results from the amplifier gain minus the modulator insertion loss, k and γ are the power coupling coefficient and the fractional power loss of the coupler, respectively. After N circulations of the light through the loop, the output electric field is given by

$$E_o(t) = \sqrt{(1-\gamma)(1-k)}\,E_i(t) - \sqrt{(1-\gamma)k}\sum_{n=1}^{N} r^n e^{-j\beta F_n(\omega_m t)}E_i(t-nT). \quad (2)$$

The parameter $r = \sqrt{\alpha(1-\gamma)(1-k)}$ is the total round-trip loop gain, which must be less than unity to make the series convergent and render our analysis valid. The total modulator phase shift for n circulations is given by the expression $\beta F_n(\omega_m t) = \beta\sum_{i=1}^{n}\sin\,\omega_m(t-iT)$. We shall tailor the loop length such that the modulator is in resonance with the loop, i.e., $\omega_m T = 2p\pi$. Under these conditions, $F_n(\omega_m t) = n\sin\,\omega_m t$. After making some simplifications and letting N approach infinity, with $\rho = re^{j\omega_o T}$, we obtain the loop output electric field:

$$E_o(t) = \sqrt{(1-\gamma)(1-k)}\,E_i(t) - \sqrt{(1-\gamma)k}\,\frac{\rho e^{-j\beta\sin\omega_m t}}{1-\rho e^{-j\beta\sin\omega_m t}}E_i(t). \quad (3)$$

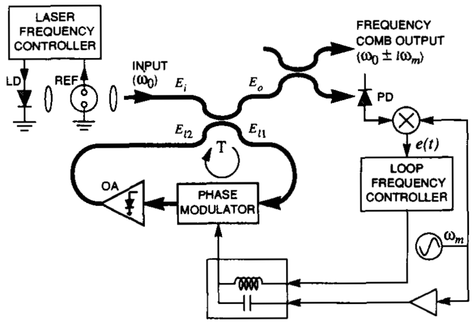

Fig. 1. Schematic of the optical frequency comb generator based upon phase modulation within an amplified circulating loop. The input laser is frequency-locked to an absolute atomic reference, e.g., using the optogalvanic effect, as shown here. LD: laser diode, REF: atomic reference cell, OA: optical amplifier, PD: photodiode.

For a loop without the phase modulator, i.e., $\beta = 0$, the steady-state resonance condition is given by $\omega_o T = 2q\pi$ [7]. We want to operate the loop in this condition because the loop will retain the largest possible fraction of the input light, thus maximizing the phase modulation achieved. Considering the loop losses, if an optical amplifier were not used, then this resonance would also maximize the fraction of light subjected to loop losses, actually yielding the minimum transmission condition. With an optical amplifier employed to nearly compensate the loop losses, this resonance again corresponds to maximum transmission.

We now address the issue of locking the round-trip delay T to resonate with the input light, $\omega_0 T = 2q\pi$. By detecting a portion of the loop output and multiplying it by a sinusoid at frequency ω_m, we can obtain the necessary error signal without dithering the frequency of the input light. Consider the error signal thus obtained, $e(t) = I_o(t) \times \sin(\omega_m t)$, where $I_o(t)$ is the detected photocurrent. Fig. 2 shows the time-averaged (low-pass filtered) $e(t)$ as a function of the round-trip phase difference $\omega_o T$, for the particular case $\beta = \pi/2$. When $\omega_o T = 2q\pi$, the qth resonance of the loop matches the frequency of the injected light; the error signal will be zero and have negative slope. We can change the dc bias on the phase modulator to maintain this resonant condition. The loop frequency controller can be similar in design to a frequency-locking circuit used in a heterodyne optical receiver [8].

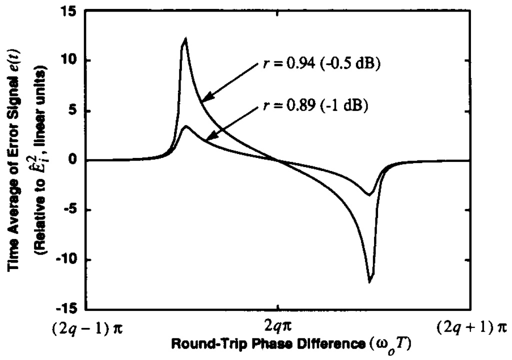

Fig. 2. Time-averaged error signal versus round-trip phase difference of the injected light with $\beta = \pi/2$, $k = 1/2$, $\omega_m T = 2p\pi$, $\gamma = 0$. Loop will lock at the stable nulls, at which $\omega_0 T = 2q\pi$, and not at the unstable nulls $\omega_0 T = (2q \pm 1)\pi$. When $\omega_0 T = 2q\pi$, the qth resonance of the loop is locked to the frequency of the injected light. The magnitude of $e(t)$ assumes unit coupling of the loop output $E(t)$ to a photodetector of unit responsivity, and a sinusoidal oscillator of unit amplitude.

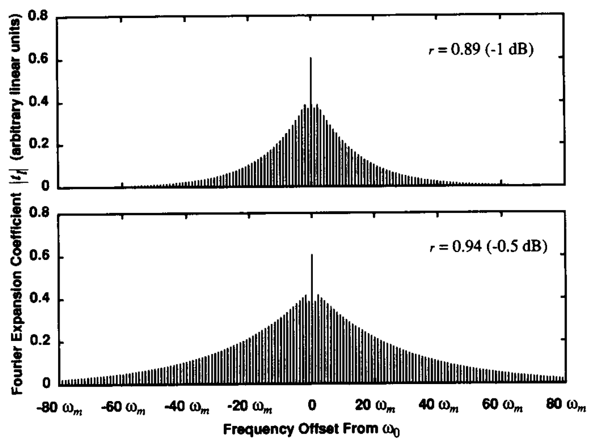

Fig. 3. Fourier expansion coefficients of the output light, $|t_l|$, at their respective frequencies for $\beta = \pi/2$, $k = 0.5$, $\omega_m T = 4\pi$, $\gamma = 0$, at two different loop gains.

The loop should not lock at the antiresonant points, $\omega_0 T = (2q \pm 1)\pi$, since the error signal has positive slope at these unstable nulls. However, if β is chosen to be larger than π, then there arise additional stable nulls in the error signal, at round-trip phase differences not satisfying resonance. In this case, we must extract the error signal using an odd harmonic of ω_m, i.e., $e(t) = I_o(t) \times \sin(m\omega_m t)$ with m an odd integer. Due to the symmetry of $I_o(t)$, m cannot be an even integer, since this would always yield a zero time-averaged error signal. When β lies in the range between π and 2π, the choice $m = 3$ will yield a nonzero time-averaged error signal that has only one stable null. For β larger than 2π, we have not determined a means to generate an error signal having only one stable null. This does not appear to represent a major drawback, because most practical systems will operate with β smaller than 2π, due to the large drive voltage requirement of typical phase modulators.

The electric field described by (3) represents a pulse train having period T, or some integral submultiple thereof. The lth component of the Fourier expansion coefficient of $E_o(t)$ is

$$t_l = \sqrt{(1-\gamma)(1-k)}\,\hat{E}_i \delta_{l,0} - \sqrt{(1-\gamma)k}\,\frac{\rho \hat{E}_i}{2\pi}\int_{-\pi}^{\pi} d\theta \frac{e^{-il\theta - i\beta\sin\theta}}{1-\rho e^{-i\beta\sin\theta}}. \quad (4)$$

Fig. 3 presents the magnitude of the expansion coefficients, $|t_l|$, at their respective frequencies for $\omega_o T = 2q\pi$, and for two different values of round-trip gain r. As r approaches unity, the number of comb frequencies will increase dramatically. However, the number of comb frequencies also depends on the accuracy with which the loop is locked in resonance with the input light. As an approximation, consider the factor from (4), $1/(1 - \rho e^{-j\beta\sin\theta})$, which, as a function of θ, is a pulse peaked at $\theta = 0$. By analogy with the analysis of the finesse of a ring resonator [9], the half-maximum of the pulse is given by the expression $\theta_{HM} \approx (1-r)/(\beta\sqrt{r})$. The number of comb frequencies is then approximately equal to $N_{HM} \approx 2/\theta_{HM}$. For $r = 0.89\,(-1\text{ dB})$, $N_{HM} \approx 27$. Comparing this estimate with Fig. 3, we see that the number of usable comb frequencies is underestimated because this estimate counts only frequencies within the half-maximum. If the locking circuit yields a steady-state phase error of θ_e, then the corresponding number of frequencies is approximately equal to $N_{HM} \approx 2/(\theta_{HM} + \theta_e)$. To date, a ring resonator with optical amplifier has achieved a finesse as high as about 500 [9], corresponding to $N_{HM} = 500$ for $\beta = \pi/2$. This suggests that our proposed scheme could generate a comb with more than 500 frequencies, which would represent a great improvement over other methods based upon angle modulation [4]–[6].

As shown in the Appendix, amplifier spontaneous noise will not significantly impair operation of the proposed comb generator.

III. Conclusion

The proposed precision optical frequency comb generator, based upon phase modulation within an amplified circulating loop, addresses an essential need in dense OFDM communication systems. Our analysis suggests that one can obtain hundreds of frequencies, spaced at regular intervals of several GHz, with frequency spacing controlled by a precise radio-frequency oscillator. Moreover, if the loop input is derived from a laser locked to an absolute frequency reference, then each of the output frequencies can have an absolute accuracy approaching that of the input.

Appendix
Effect of Optical Amplifier Noise

Let $n_a(t)$ denote the optical amplifier noise added to the loop in each circulation. Over the bandwidth of interest, $n_a(t)$ can be assumed to be white so that $\langle n_a(t+\tau)n_a^*(t)\rangle = R_0\,\delta(\tau)$. Also let $n_l(t)$ denote the noise in the loop. The relation between these noises is $n_l(t) = re^{-j\beta\sin(\omega_m t)}n_l(t-T) + n_a(t)$. Thus, we obtain:

$$n_l(t) = \sum_{n=0}^{\infty} r^n e^{-jn\beta\sin(\omega_m t)} n_a(t-nT). \tag{5}$$

The autocorrelation function of $n_l(t)$ is:

$$R(t+\tau,t) = \langle n_l(t+\tau)n_{l*}(t)\rangle$$
$$= R_0 \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty} r^{k_1+k_2} e^{-jk_1\beta\sin\omega_m(t+\tau)+j\beta k_2\sin\omega_m t} \cdot \delta(\tau-(k_1-k_2)T). \tag{6}$$

Note that in each term, $t+\tau$ can be replaced by t because $\tau = (k_1-k_2)T$ in each term and $\omega_m T = 2p\pi$. The summation can first be done over k_1+k_2 and then over k_1-k_2. After a change of variables we get:

$$R(t+\tau,t) = \frac{R_0}{1-r^2}\sum_{n=-\infty}^{\infty} r^{|n|} e^{-jn\beta\sin\omega_m t}\delta(\tau-nT). \tag{7}$$

Since the random process $n_l(t)$ is cyclostationary with period T, its average autocorrelation function [10] is $\bar{R}(\tau) = 1/T\int_{-T/2}^{T/2} R(t+\tau,t)\,dt$. Taking the Fourier transform of $\bar{R}(\tau)$, the power spectrum of $n_l(t)$ is

$$\Phi_l(\omega) = \frac{R_0}{2\pi}\int_{-\pi}^{\pi}\frac{d\theta}{|1-re^{j(\beta\sin\theta+\omega T)}|^2}. \tag{8}$$

The expression (8) has a simple physical interpretation: it is analogous to the average over θ of the transfer function of a Fabry–Perot interferometer that has a round-trip phase-shift $\beta\sin\theta+\omega T$. Fig. 4 presents $\Phi_l(\omega)$ as a function of ω for $r = 0.94$ (-0.5 dB) and for several values of β.

Comparison of (1) and (3) yields $E_{l2}(t) = \rho e^{-j\beta\sin\omega_m t}/(1-\rho e^{-j\beta\sin\omega_m t})E_i(t)$. Comparing the expression for $E_{l2}(t)$ with (8), we obtain $\overline{E_{l2}^2}/r^2\hat{E}_i^2 = \Phi(\omega_0)/R_0$, where $\overline{E_{l2}^2} = 1/T\int_{-T/2}^{T/2}|E_{l2}(t)|^2\,dt$ is the average total signal power in the loop. Therefore, recirculation through the loop enhances the noise density at ω_0 (and at $\omega_0 + 2\pi s/T$, where s is an integer) by the same factor as it enhances the average total signal power.

In application of this comb generator, a heterodyne automatic frequency control (AFC) loop [8], [11] may be used to frequency-lock a slave laser to one of the comb frequencies. The variance of closed-loop frequency error is [11]:

$$\sigma_e^2 = \frac{\Delta\nu}{2\pi}(B_{IF}-\pi B_L) + K\ln\left(\frac{B_{IF}}{2B_L}\right) \quad \text{if} \quad \frac{\Delta\nu}{\pi} \gg \frac{B_L^2}{B_{IF}\,\mathrm{SNR}_{IF}} = \frac{B_L}{\mathrm{SNR}_L}. \tag{9}$$

Here, $\Delta\nu$ is the total linewidth of comb signal and slave laser, B_{IF} is the (double-sided) bandwidth of the intermediate-frequency filter preceding the AFC loop, B_L is the bandwidth of the AFC loop, K is the flicker noise coefficient, and SNR_{IF} and SNR_L are the signal-to-noise ratios within the bandwidths of B_{IF} and B_L, respectively. For typical values B_L of order 100 kHz and $\Delta\nu$ of the order of several MHz, (9) is valid even when $\mathrm{SNR}_L = 1$ (0 dB). The dominant noise in our system is the beat noise from the optical amplifier (either signal-spontaneous or spontaneous-spontaneous noise may dominate, depending on operating conditions). Typical semiconductor laser amplifiers have a total beat noise less than 10^{-19} A^2/Hz [12]. Considering the example of Fig. 3 with $r = 0.94$ (-0.5 dB), assume that the comb output is split into 160 branches, and that each slave laser has a power -10 dBm. Circulation through the loop will enhance the noise power, but after splitting, the noise striking each detector will be much less than 10^{-19} A^2/Hz. Assuming the comb output has a total average signal power of -10 dBm, then at each detector the weakest of the 160 comb frequencies will have a power of -65 dBm, leading to $\mathrm{SNR}_L > 10$. We conclude that amplifier beat noise will not impair frequency locking. We note that the noise peaks, which may lie between the comb frequencies (see Fig. 4), may have greater power than the weaker comb frequencies. A properly designed AFC loop can distinguish the noise peaks, with typical width of hundreds of MHz, from the comb frequencies, which have typical widths of several MHz.

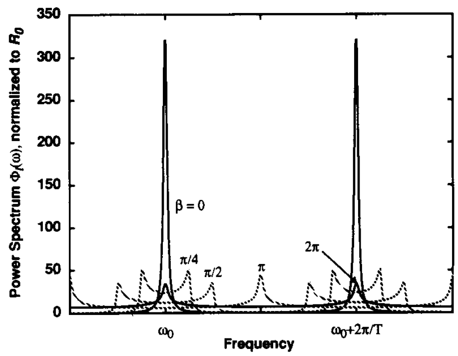

Fig. 4. Power spectrum of noise in the circulating loop with $r = 0.94$ (-0.5 dB) for various modulation indices and $\omega_m T = 2p\pi$. Note that over the bandwidth of interest, $\Phi_l(\omega)$ is periodic with period $2\pi/T$.

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A Novel Analog Optical Link with High Dynamic Range

R. F. Kalman, J. C. Fan, and L. G. Kazovsky

Abstract—**We propose and analyze a new analog optical link. This link, based on phase modulation using a novel external modulator, has high linearity and suppresses the impact of laser relative intensity noise. The link increases the spurious-free dynamic range by 21 dB as compared to a conventional externally modulated AM link for 10 mW of received optical power, relative intensity noise of -130 dB/Hz, and a 15 GHz intermediate frequency.**

I. Introduction

ANALOG optical links have been the subject of considerable attention for a variety of applications including antenna remoting and cable television distribution systems [1]. Amplitude modulated (AM) direct-detection (DD) links have received the majority of attention. Both directly modulated links and externally modulated links have been investigated [2], [3]. While directly modulated links have exhibited good performance, the use of external modulation allows the optical source and the modulator to be independently optimized [2].

The dominant type of external amplitude modulator used to date is the Mach–Zehnder (MZ) modulator, which exhibits an intrinsically nonlinear amplitude modulation characteristic. This nonlinearity limits the spurious-free dynamic range (SFDR), an important measure of link performance [4]. In this letter, we put forth a new analog optical link based on phase modulation (PM) using a novel external modulator. The modulator creates a frequency-upshifted term which is heterodyned with the phase-modulated signal at the detector. We analyze the spurious-free dynamic range (SFDR) of this link and compare it to that of an AM DD link. We show that the new PM link exhibits an improvement in SFDR of 7 dB to more than 20 dB over an AM DD link for realistic parameters.

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II. System Description

The proposed link is shown in Fig. 1. The transmitter consists of a CW laser and a novel three-leg modulator. One leg of the modulator is driven by the signal. The other two legs are driven by quadrature CW RF signals at a frequency ω_{IF}, optically phase shifted by $\pi/2$ from each other. After traversing an optical fiber, the signal is detected at the receiver. Following the photodiode, the signal is amplified and limited. It is then put through a delay-line filter, an envelope detector, and an integrator; these three components function in tandem as a phase demodulator. We refer to the entire system as an heterodyne interferometric phase modulated (HIPM) link.

The optical signals of the second and third legs of the modulator mix with the signal at the detector and result in a series of single-sideband signals at multiples of ω_{IF}. The second and third legs are thus roughly equivalent to a single-sideband frequency shifter.

III. Analysis

The optical fields contributed by the three arms of the modulator are given at the detector by

$$e_1(t) = \sqrt{\epsilon_1 P[1 + n_R(t)]}\, \exp i\left[\omega_0 t + \varphi_\Delta x(t) + \varphi_p(t)\right] \quad (1)$$

$$e_2(t) = \sqrt{\epsilon_2 P[1 + n_R(t)]}\, \exp i \cdot\left[\omega_0 t + \beta_s \sin \omega_{\mathrm{IF}} t - \frac{\pi}{2} + \varphi_p(t)\right] \quad (2)$$

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