Performance of Electrical Equalizers in Optically Amplified OOK and DPSK Systems

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Abstract—We study the performance of feed-forward equalizers (FFEs) and decision-feedback equalizers (DFEs) in compensating chromatic dispersion and first-order polarization-mode dispersion (PMD) in optically amplified nonreturn-to-zero ON–OFF keying (NRZ-OOK) and NRZ differential phase-shift-keying (DPSK) systems at 10 Gb/s. We use Monte Carlo simulation of the least-mean-square algorithm to estimate equalizer tap weights. Once stationary tap weights are obtained, we compute the bit-error ratio using an analytical method whose accuracy has been verified by simulation. We find that FFE and DFE offer the greatest improvement when combating first-order PMD with NRZ-OOK, reducing power penalties by about 45%. With NRZ-DPSK, the equalizers offer quite limited improvement.

Index Terms—Chromatic dispersion (CD), electrical equalizer, optical amplifier, optical fiber communication, polarization-mode dispersion (PMD).

I. INTRODUCTION

OPTICAL fiber communication systems are subject to intersymbol interference caused by chromatic dispersion (CD) and polarization-mode dispersion (PMD). As both CD and PMD originate in the optical domain, the most effective compensation schemes use optical equalization. Nonetheless, electrical equalization schemes are also being widely considered because they offer several potential advantages, including compactness, flexibility, and low cost.

Research on electrical equalizers for optical systems began in the early 1990s [1] and intensified around 2000 [2]–[5], in part, because of advances in high-speed integrated-circuits technology. Most work has investigated experimentally the performance of feed-forward equalizers (FFEs), decision-feedback equalizers (DFEs), or the combination of FFEs and DFEs. By contrast, precise theoretical or numerical analysis of these equalizers has seldom been presented, especially for systems using optical amplifiers. In optically amplified systems, the noise distribution at the receiver is known to be non-Gaussian, following a noncentral chi-square (χ^2) distribution. In the presence of non-Gaussian noise, the computation of optimal equalizer tap weights and the evaluation of the bit-error ratio (BER) have been difficult.

In this letter, we study the performance of optically amplified systems using ON–OFF keying (OOK) and differential

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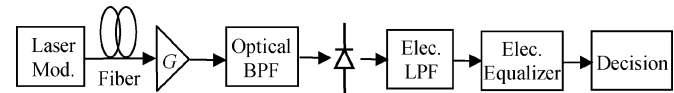
Fig. 1. Fiber communication system using OOK and electrical equalization.

phase-shift-keying (DPSK) modulation with FFE and DFE. Using Monte Carlo simulation, the tap weights are adapted using the least-mean-square (LMS) algorithm until they reach stationary values. The equalizers are then modeled as linear time-invariant electrical filters, and the BER is computed using analytical methods that have been developed recently.

To our knowledge, this is the first time that the performance analysis of electrical equalizers in optically amplified DPSK systems has been described. We note that [5] presented the performance of equalizers in optically amplified OOK systems, but did not describe in detail the methods used to compute BER. Recently, Weiss described the performance of maximum-likelihood sequence detection in optically amplified OOK systems, whose BER lower-bounds that of any electrical equalizer [6].

II. TAP WEIGHT ESTIMATION AND BER CALCULATION

A schematic of an OOK system using an electrical equalizer is shown in Fig. 1. The design of a DPSK system is similar to Fig. 1, except that following the optical bandpass filter, there is an optical interferometer with a one-bit delay, and the single photodetector is replaced by a balanced photodetector.

In Fig. 1, the fiber is assumed to be a lossless linear channel with CD and first-order PMD. First-order PMD can be characterized using the principal states model [7]. Under this model, the fiber is described as a one-input two-output device, with each output corresponding to an output principal state of polarization (PSP). In the optical field domain, the transfer function describing fiber PMD is given by

$$H_f(\omega) = \begin{bmatrix} \sqrt{\gamma} \exp\left[j\omega\left(\frac{-\tau}{2}\right)\right] \\ \sqrt{1-\gamma} \exp\left[j\omega\left(\frac{\tau}{2}\right)\right] \end{bmatrix} \qquad (1)$$

where γ is the power splitting ratio, indicating how the input light power is projected onto the two input PSPs. τ represents the differential group delay between the two PSPs.

Incorporating CD effects, the fiber transfer function becomes

$$H_f(\omega) = \begin{bmatrix} \sqrt{\gamma} \exp\left[j\omega\left(\frac{-\tau}{2}\right) + \frac{jDL\omega^2\lambda^2}{(4\pi c)}\right] \\ \sqrt{1-\gamma} \exp\left[j\omega\left(\frac{\tau}{2}\right) + \frac{jDL\omega^2\lambda^2}{(4\pi c)}\right] \end{bmatrix} \qquad (2)$$

where D is the fiber CD parameter, L is the fiber length, and λ is the signal wavelength, which is assumed to be 1550 nm.

At the fiber output, the received signal is amplified by a lumped optical amplifier. We assume that its amplified spon-

Fig. 2. Structure of FFE + DFE using LMS adaptation.

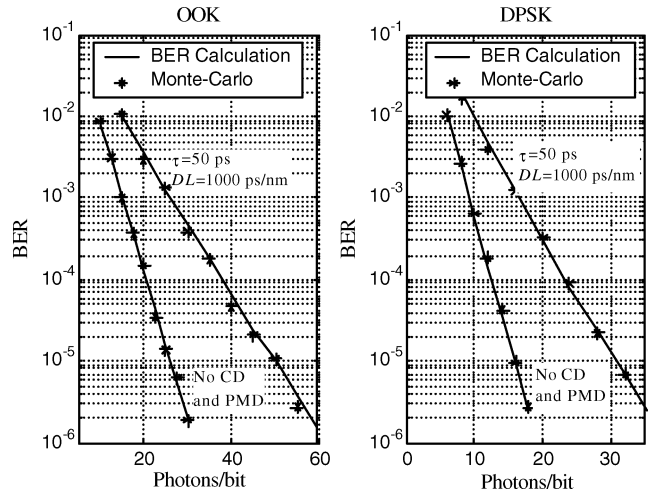Fig. 3. Comparison of BERs obtained by calculation method and Monte Carlo simulation. The horizontal axis is the number of photons per bit of the received optical signal. The bit rate is 10 Gb/s.

taneous emission dominates over other noises at the receiver. Both an optical bandpass filter and an electrical lowpass filter are used to reduce noise. An electrical equalizer is inserted between the electrical lowpass filter and the decision circuit. Fig. 2 shows the structure of the FFE + DFE adapted using the LMS algorithm. We assume that the equalizer taps are spaced by T, the bit duration. In this letter, we consider two equalizer designs: 1) FFE only and 2) FFE and DFE.

We represent the tap weights of the FFE and DFE by a vector $\vec{C} = [c_0, c_1, \ldots, c_M, d_1, \ldots, d_N]$. The LMS adaptation algorithm updates $\vec{C}$ at each interval of T following [8]

$$\vec{C}^{(k+1)} = \vec{C}^{(k)} + \Delta \varepsilon_k \vec{V}^{(k)} \qquad (3)$$

where $\vec{C}^{(k)}$ is the tap weight vector at time kT, Δ is a scale factor that controls the rate of adaptation, ε_k is the difference between decision sample y_k and decision output I_k at time kT, and $\vec{V}^{(k)} = [v_k, v_{k-1}, \ldots, v_{k-(M-1)}, I_{k-1}, \ldots, I_{k-N}]$, where v_{k-m} is the input signal at time $(k-m)T$, $m = 0, \ldots, M-1$ and I_{k-n} is the decision made at time $(k-n)T$, $n = 1, \ldots, N$.

Assuming the tap weights converge to a stationary value $\vec{C}_s$, we estimate $\vec{C}_s$ by Monte Carlo simulation with the LMS algorithm. A pseudorandom bit sequence is encoded to yield a transmitted signal. The signal is distorted by CD and first-order PMD, and is subject to additive noise from the optical amplifier. The FFE and DFE tap weights are adapted using (3). The choice of Δ is a tradeoff between adaptation speed and excess mean-square-error (MSE) [8]. Because PMD fluctuates on a time scale of 10 ms [7], which corresponds to 10^8 bit periods in a 10-Gb/s system, it is not difficult to find a suitable value of Δ. For example, we can choose Δ such that the adaptation time is as short as 1 μs at 10 Gb/s, while still guaranteeing that excess MSE degrades the Q factor [9] by less than 0.1 dB. The 1-μs adaptation time implies that the pseudorandom bit sequence used for tap weight estimation can be as short as $2^{13} - 1$.

After estimating the stationary tap weights $\vec{C}_s$, we model the FFE and DFE as time-invariant electrical filters with tap weights given by $\vec{C}_s$. To model the DFE as a linear filter, we assume that the input to the DFE is the transmitted bit sequence (after proper delay) instead of the decision output sequence $\{I_k\}$. This assumption corresponds to neglecting error propagation, and is frequently employed in the analysis of DFE. It has negligible

impact on BER estimation provided the system operates at low BER [8]. Then, we use the methods described in [10] and [11] to precisely calculate the BER for OOK and DPSK systems in the presence of CD and PMD. Details of implementing these BER calculation methods have been described for OOK in [10] and for DPSK in [11].

We have verified the accuracy of our approach using standard Monte Carlo simulation. Fig. 3 compares the BERs obtained by the BER calculation technique and by standard Monte Carlo simulation for 10-Gb/s OOK and DPSK systems using rectangular nonreturn-to-zero (NRZ) pulses. In both systems, the electrical equalizer is a three-tap FFE + two-tap DFE. The optical bandpass filter is Gaussian, and has a 3-dB bandwidth of 16 GHz for OOK and 18 GHz for DPSK. The electrical lowpass filter is fifth-order Bessel, and has a 3-dB cutoff of 6.5 GHz for OOK and 5.5 GHz for DPSK. These filter bandwidths are chosen to minimize the BER in the absence of CD and PMD. We assume the optical amplifier has a noise figure of 3 dB. Fig. 3 shows that the calculation method agrees with Monte Carlo simulation very well in estimating BER in the absence and presence of dispersion.

III. PERFORMANCE OF ELECTRICAL EQUALIZERS

Using the BER calculation method described in Section II, we have evaluated CD and first-order PMD power penalties at the fixed BER of 10^{-9} in NRZ-OOK and NRZ-DPSK systems. Two types of electrical equalizers are used: three-tap FFE + two-tap DFE and five-tap FFE. All the other system parameters (filter types, filter bandwidths, etc.) are the same as in Section II.

Fig. 4 presents the calculated power penalties. All four subfigures show that there is no substantial performance difference between FFE + DFE and FFE in compensating for CD and PMD. Since the FFE + DFE performs slightly better in most cases, in the following discussion, we focus on the FFE + DFE and refer to it simply as "the equalizer".

With either OOK or DPSK, the equalizer is more effective in reducing the PMD power penalty than in reducing the CD power penalty. This is expected, as first-order PMD causes linear distortion in the electrical domain, while CD causes nonlinear dis-
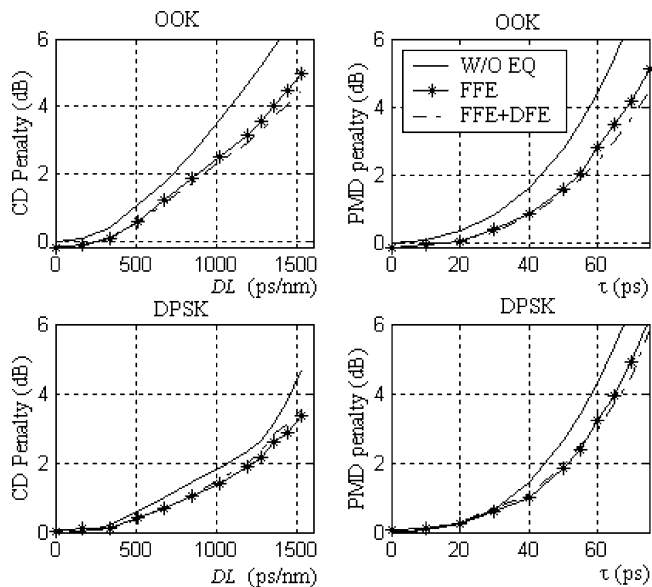
Fig. 4. Power penalties caused by CD and first-order PMD in NRZ-OOK and NRZ-DPSK systems at a bit rate of 10 Gb/s.

tortion in the electrical domain. Electrical equalizers such as FFE or FFE + DFE are generally more effective in combating linear distortion than nonlinear distortion [1].

Fig. 4 also shows that the equalizer is more effective with OOK than with DPSK. For example, the equalizer reduces the PMD power penalty by about 45% for OOK, but by only 25% for DPSK. The relative ineffectiveness of the equalizer for DPSK is due to the presence of an optical interferometer in the DPSK receiver. The interferometer combines the optical signal with a T-delayed copy, introducing strong correlation between samples with T spacing, such as the $v_k, v_{(k-1)}, \ldots, v_{(k-M-1)}$ defined in Section II. When the $v_k, v_{(k-1)}, \ldots, v_{(k-M-1)}$ are combined by the equalizer, the resulting decision sample y_k has a much higher variance than those of $v_k, v_{(k-1)}, \ldots, v_{(k-M-1)}$, which can be easily verified by calculation or Monte Carlo simulation. This variance enhancement significantly degrades the performance of electrical equalization in DPSK systems.

We also evaluated CD and PMD penalties when the tap weights of FFE or FFE + DFE are tuned to minimize the calculated BER. We found that the BER-minimization algorithm performs only slightly better than the LMS algorithm. For example, in a NRZ-OOK system using FFE + DFE, when the LMS algorithm is used, the PMD penalty at $\tau = 60$ ps is 2.4 dB, while if the BER-minimization algorithm is used, the penalty is 2.2 dB.

IV. DISCUSSION

We note that our results do not fully agree with those in [5]. For example, [5] found a 1-dB additional penalty for FFE equalizers, which we do not observe. The discrepancies between our results and those of [5] may arise from differences in the system design. In particular, we have observed that the performance of electrical equalizers in optically amplified systems is sensitive to the types and bandwidths of optical and electrical filters. Generally, equalizers are more effective in systems using strong optical and/or electrical filtering.

At the high bit rates of optical communication systems, generating analog or multibit digital representations of samples (for example, ε_k in Fig. 2) is costly and may be impractical. Thus, the sgn–sgn LMS algorithm [1] which uses only single-bit digital samples, has been proposed as an alternative to the standard LMS algorithm. The adaptation formula for the sgn–sgn LMS algorithm is written as

$$\vec{C}^{(k+1)} = \vec{C}^{(k)} + \Delta \text{sgn}(\varepsilon_k) \text{sgn}(\vec{V}^{(k)}). \tag{4}$$

When we employ sgn–sgn LMS instead of standard LMS and scale Δ to realize the same adaptation speed, the CD and PMD compensation performance of the equalizer is virtually unchanged.

V. SUMMARY

We studied the performance of electrical equalizers in compensating CD and PMD in optically amplified NRZ-OOK and NRZ-DPSK systems. We found that FFE and DFE based on the LMS adaptation algorithm offer the greatest improvement when combating first-order PMD with NRZ-OOK. With NRZ-DPSK, the equalizers offer quite limited improvement, because of noise enhancement. We found that the performance of the LMS algorithm is only slightly inferior to that of a BER-minimization algorithm.

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